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The influence of shelf life on the integrated production scheduling and vehicle routing optimization for perishable products

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Abstract

Perishability is a characteristic that affects several production systems. The loss of the value of a product over time creates challenges and opportunities to integrate the production and distribution planning, increases the complexity of inventory management, and also influences the pricing practices. Such importance leads researchers to propose quantitative models to solve problems that consider perishability. Although there is evidence that the shorter the shelf life, the greater the benefits to solve the production and distribution planning in an integrated way, there are still few studies investigating how the shelf life influences the problem's solution. Therefore, this study aims to analyze how the shelf life influences the Integrated Production and Distribution Scheduling Problem For Perishable Products (IPDSP-P). To make it possible, we also proposed a metric named Normalized Shelf Life, which allowed us to compare several studies and propose a guideline to classify the shelf life as "long" and "short". Another contribution of this work is the proof of a theorem that allowed us to decompose the problem and create a model using the Logic-based Benders Decomposition approach. This theorem is also the basis of a genetic algorithm and an alternative form of the Mixed Integer Linear Programming (MILP) model, named in this study as MILP-Distribution. Besides these two models, we also developed a MILP model containing all production and distribution constraints (named MILP-Full), and the performance of the proposed models was compared. The findings suggest that shorter shelf lives make it more difficult for exact models to find a solution and prove its optimality. For the genetic algorithm, although there was a fast convergence to a single solution for the short shelf life instances, we observed a higher gap between the solution and the lower bound obtained from a commercial solver. Finally, the genetic algorithm could find the best solution for more instances when compared to the other solution approaches. Thus, this study contributes to understanding how shelf life impacts the solutions of IPDSP-P and the understanding of the performance of different approaches to solve the problem.

Keywords: Shelf life, Perishability, Scheduling, Distribution, Integrated Problems

Resumo

A perecibilidade é uma característica que afeta muitos sistemas de produção. A perda de valor ao longo do tempo cria desafios e oportunidades para integrar o planejamento da produção com a distribuição, aumenta a complexidade em gerenciar os estoques, e também influencia as práticas de precificação. Tamanha relevância levam pesquisadores a propor modelos quantitativos para resolver problemas que lidam com produtos perecíveis. Embora haja evidências de que quanto menor o *shelf life*, maiores são os benefícios de um modelo que resolva o planejamento de produção e distribuição de forma integrada, ainda há poucos estudos de como o *shelf life* influencia a resolução desse problema. Portanto, essa pesquisa tem como objetivo estudar como o *shelf life* influencia o Problema Integrado de Sequenciamento de Produção e Distribuição para Produtos Perecíveis (IPDSP-P). Para tornar isto possível, nós também propusemos uma métrica chama *Normalized Shelf Life*, que nos permitiu comparar diversos estudos a propor uma referência para diferenciar o *shelf life* longo do curto. Outra contribuição desse trabalho foi provar um teorema que nos permitiu decompor o problema e criar um modelo usando a técnica *Logic-based Benders Decomposition*. Esse teorema também foi a base para o desenvolvimento de um algoritmo genético e um modelo alternativo de Programação Linear Inteira Mista (MILP), identificado como *MILP-Distribution*. Além desses dois modelos, também desenvolvemos um modelo MILP contendo todas as restrições de produção e distribuição, que identificamos nesse estudo como *MILP-Full*. Por fim, o desempenho de todos esses modelos foram comparados. Os achados sugerem the quanto menor o *shelf life*, mais difícil fica para os modelos exatos a encontrarem uma solução e provar a optimalidade. Para o algoritmo genético, embora haja uma rápida convergência para as instâncias com *shelf life* curto, também foi observado um gap maior entre a solução do algoritmo e o limite inferior obtido pelo *solver* comercial. Por fim, quando comparado com os outros métodos, o algoritmo genético encontrou as melhores soluções para um número maior de instâncias. Portanto, esse estudo contribui para o entendimento de como o *shelf life* impacta as solução do IPDSP-P e para o entendimento do desempenho de diferentes métodos para resolver o problema.

Palavras-chave: *Shelf Life*, Perecibilidade, *Scheduling*, Distribuição, Problemas Integrados

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1 Introduction

This chapter contextualizes the subject studied; specifying the goals and the importance of the study. It also briefly describes the research method used.

1.1 Contextualization and Motivation

The idea of Enterprise-wide optimization (EWO) first appeared in Grossmann (2005a). This paper defines the EWO as a research area that aims to optimize the procurement, manufacturing, and distribution operations in an integrated way. Despite the computational difficulty in solving these integrated optimization problems, there are several reasons for the growth of this research area.

Computational studies show that integrated production and distribution planning can lead to cost reductions ranging from 3% to 20% when compared with sequential planning (Chandra; Fisher, 1994). Besides the benefits in costs, situations where the inventory is constrained also benefit from adopting an integrated approach. Examples of this are found in companies that adopt the make-to-order production system (Chen, 2010) or companies that handle perishable products (Marandi; Zegordi, 2017), which are goods that have physical deterioration or a decrease in the customer's value perception over time (Amorim et al., 2013). In this case, inefficiencies in the production planning will affect the distribution system. Therefore, several authors often highlight the attractiveness of the integrated planning (Amorim; Gunther; Almada-Lobo, 2012; Chen, 2010).

One characteristic that significantly influences operational planning for a perishable product is its shelf life. When analyzing the integration of production scheduling and vehicle routing problem for a product with a fixed shelf life, Amorim, Gunther and Almada-Lobo (2012) observed that the integrated approach benefit is leveraged for products with a higher perishability degree. Farahani, Grunow and Gunther (2012) reached the same conclusion on a similar integrated problem. Despite these encouraging results, there is little knowledge regarding how the problem's characteristics impact the integration of these two subproblems (Moons et al., 2017). This lead us to the following research question:

Does shelf life influence the expected performance of solution approaches that aim to solve the integrated production and distribution scheduling problem for perishable products (IPDSP-P)?

This research question is very broad as there are many ways to model the production and distribution. For example, Moons et al. (2017) present a literature review about the integrated production scheduling and distribution routing problem and classify the production environment by 10 characteristics and 15 for the distribution environment. However, the literature review conducted by this study showed that only a small number of articles were published about the IPDSP-P even for the simplest production environment, namely the single machine environment, which suggests a gap of theoretical knowledge. Therefore, this work focuses on analyzing the influence of shelf life for the IPDSP-P modeled in the most simple way found in the literature, i.e., considering several orders of a single perishable product that must be scheduled into a single machine and distributed to customers before the product expires using an unlimited fleet of homogenous vehicles. It is expected that the results from this specific problem can be evaluated for more complex cases in future studies.

1.2 Objectives

The main objective of this study is to evaluate the influence of the shelf life on the computational performance of several methods to solve the integrated production and distribution scheduling problem for perishable products (IPDSP-P). However, to meet this objective some specific objectives should be achieved:

- Provide a discussion of what an integrated problem is and an overview of how research addresses the integrated production-distribution problem, i.e., present what the common forms are to model and solve those problems.
- Provide an overview from the literature of how perishable products are modeled.
- Provide a simplification for the IPDSP-P when the problem consists of a single machine production environment without setup times or costs and the distribution must occur using a fleet of unlimited homogeneous vehicles that must deliver a single perishable product with a fixed shelf life before it expires.
- Propose a standard metric for shelf life to enable researchers and practitioners to distinguish the short from the long shelf life.
- Provide a comparison between different approaches to solve the IPDSP-P and analyze if the shelf life equally influences the problem for the several solution approaches.

1.3 Justification

About one-third (approximately 1.3 billion tons per year) of all food produced worldwide is lost during all the stages in the food supply chain (Gustavsson et al., 2011). Despite the fact that one of the leading sources of waste relies on the final consumer, a considerable share of that loss occurs from production to retailing stages. For example, in

Latin America, almost 200kg per capita per year of food is lost from production to retailing (Gustavsson et al., 2011). Futhermore, 492 million tons of fruits and vegetables were lost worldwide due to inefficient and ineffective food supply chain management (Zhong; Xu; Wang, 2017).

This loss can be reduced by using quantitative models. For example, optimizing production, distribution, or even both in an integrated way. However, a preliminary literature review shows that there are just a few studies that proposes quantitative approaches to address this category of problem. As will be presented in Chapter 2, between 2011 and 2022, 23 articles were published regarding quantitative models to optimize the production or distribution of perishable products. When we consider the IPDSP-P, the number of articles falls to 5, which indicates a need for theoretical investigation even for simple situations. As previously discussed, Farahani, Grunow and Gunther (2012) and Amorim, Gunther and Almada-Lobo (2012) observed that the integrated approach benefits over a decoupled approach are leveraged for products with a higher perishability degree, however, no study was found about how the shelf life influences the capacity of the implemented solution approach to reach a reasonable result.

To strengthen the findings of our computational experiments, two other solution approaches besides the MILP were tested: a Logic-Based Benders Decomposition (LBBD) and a Genetic Algorithm (GA). The Genetic Algorithm was chosen due to the ease of integrating additional techniques into the steps of the algorithm and the promising results found in different applications (e.g., see Shin, Lee and Lee (2022), Placido, Archetti and Cerrone (2022), Türkyılmaz et al. (2022)). The other solution approach analyzed, the Logic-Based Benders Decomposition, was selected because one major challenge of solving an integrated problem using an exact method is the size of the resulting model (Grossmann, 2005b; Papageorgiou, 2009; Garcia; You, 2015). Thus, decomposition methods are often applied to address the integrated problems (Grossmann, 2012). Moreover, the structural properties of the problem (presented in Section 3.2) suggested that the IPDSP-P could be decomposed into a master problem and subproblems, which supports the choice of the LBBD as a solution approach.

1.4 Research method

As described in Section 1.2, two research methods were adopted in the present study: literature review and quantitative modeling.

A literature review is a method that helps to organize and summarize the knowledge about a research topic. A review can support a researcher identifying research questions, develop contextualization for the research, and build understanding about concepts and terminology (Rowley; Slack, 2004). However, many studies conduct a review by simply collecting papers as a narrative element of the work, which may lead to inappropriate recommendations or conclusions.

Therefore, to minimize implicit biases of the researchers and provide a replicable and transparent process that can be audited by reviewers, some of the guidelines proposed by Tranfield, Denyer and Smart (2003) were followed. We used well-defined criteria to identify, appraise and synthesize the literature. This way of conducting a review provides high-quality evidence to answer research questions and may be the cornerstone for other research methods. For example, the results obtained through the literature review conducted for this study were used to delimit the problem that would be analyzed and the solution methods that would be tested to answer our research question.

Quantitative modeling is a method that consists of describing processes and systems in variables and mathematical relations. These mathematical representations are named models, and it is expected that solving the model can provide insights into the real problem. As the real world can be infinitely complex, the model must be sufficiently detailed to capture key elements that will influence the decision, but it must also be simple enough to be solved in an adequate amount of time (Arenales et al., 2015). The ability to solve a quantitative model is directly related to the computational processing performance, and the evolution of computing in recent years allows more difficult models to be solved by many options of solution approaches.

In this study, several approaches were tested: two different MILP formulations implemented in a commercial solver, a Logic-based Benders Decomposition and a genetic algorithm, whose formulations are detailed, respectively, in Sections 3.1, 3.3, 4.1, and 4.2.

As both the production scheduling and the vehicle routing problem are difficult problems to solve, it is expected that a commercial solver would hardly be able to find the optimal solution for IPDSP-P instances in a reasonable time. Nevertheless, this study evaluates the best solution a solver could provide for a limited amount of time, and compares that solution to other solution methods. Although this practice is not very common in the literature, the authors have already witnessed situations in companies in which the best solution of a commercial solver running for a amount of time was considered to support decision making.

After delimiting the problem and the solution methods, several computational experiments, detailed in Chapter 5, were conducted to answer the research question.

1.5 Structure of this document

The remainder of this study is organized as follows. Chapter 2 provides a literature review about perishable products and the integrated production-distribution scheduling problem for perishable products. Two different ways of modeling the problem as MILP and the alternative solution methods are presented, respectively, in Chapters 3 and 4. The experiments, results and discussions to answer the research question are presented in Chapter 5. Finally, conclusions and further research opportunities are given in Chapter 6.

2 Literature review

To determine how shelf life can influence quantitative modeling, we need to understand what a perishable product is and how perishability can be modeled. Moreover, since the issues described in this paper contain elements of integrated production and distribution problems, it is necessary to understand how these problems were handled previously. In this chapter, we review the literature to answer these questions and support our work.

This chapter is divided into four sections. Section 2.1 introduces what a perishable product is, presenting definitions and frameworks in the literature about how to model perishability. Section 2.2 provides the used method to conduct the review in a replicable form. The results, separated by how perishability is modeled, how problems are modeled and the solutions methods adopted are addressed, respectively, in Sections 2.3, 2.4 and 2.5. The findings from the literature review are discussed in Section 2.6.

2.1 Perishable products

It is simple to recognize a tomato or a strawberry or any other fruit or vegetable as perishable. However, there are items whose perishability factor is not so clear, such as newspaper or fashionable clothes. According to Wee (1993), one product is perishable when spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of a commodity results in a decreasing usefulness of that product. Although this definition helps us to understand what a perishable product is, a more detailed definition is needed, as it is still difficult to translate it into a mathematical formulation.

Nahmias (1982) proposes a classification that is more suitable for a mathematical formulation. The author classifies the perishable products in two categories: fixed lifetime and random lifetime. The former consists of products whose shelf life length is known and is a problem parameter. The latter category consists of products whose shelf life is stochastic and follows a probability distribution. This classification was extended by Goyal and Giri (2001), who also included one more category: the deteriorating product. That category consists of products whose quality decays proportionally to its age.

Another classification for perishable products was proposed by Amorim et al. (2013). They developed a framework that categorizes a perishable based on three dimensions: physical deterioration, descrease of customer value and limits imposed by authorities. A product that suffers from physical deterioration loses its physical status through time, for example, a medicine that loses its healing properties as it becomes older. The second dimension, the customer value, is another measure of perishability because there are products, such as a newspaper, that does not suffer from physical deterioration, but it still loses its value as there is little to none interest in old news. Finally, the authority limits reflect external regulations that limit the period a product can be sold or consumed. A product is considered perishable, if at least one of those dimensions applies to them. Figure 1 summarizes the framework.

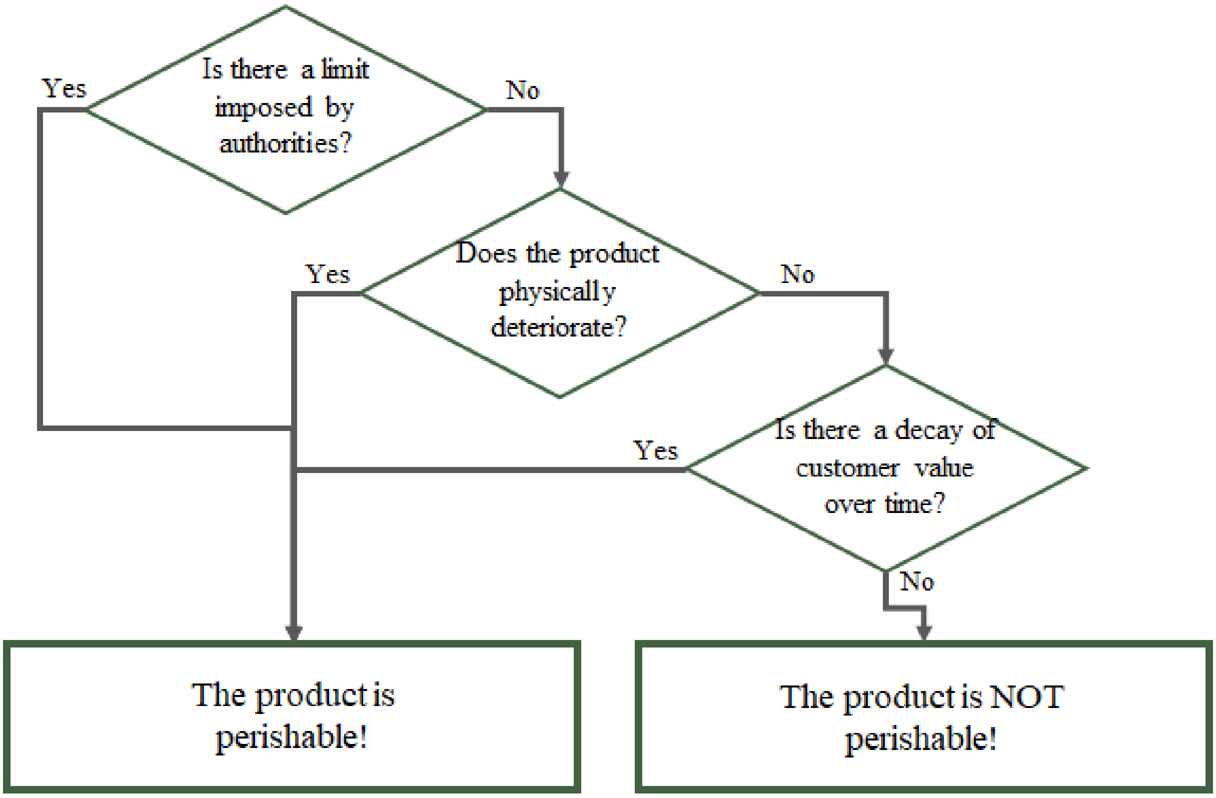


Figure 1 – Definition of perishability according to the framework proposed by Amorim et al. (2013)

This framework brings more clarity on how to represent perisable issues into mathematical formulations. For example, a product that is perishable by authority limits will probably have a fixed lifetime (Amorim et al., 2013). On the other hand, items such as fruits and vegetables that have physical deterioration but do not have authority limits, will have its perishability modeled according to their physical deterioration. Pahl and Voß (2014) summarize how the perishability can be modeled: fixed lifetime, discrete deterioration, continuous deterioration. Figure 2 presents how in time.

This study will model the perishable product with a single fixed lifetime, i.e., products that maintain their total value while there is still some shelf life, as in the first graph in Figure 2. In terms of mathematical formulations, the shelf life will be modeled as a parameter of our models.

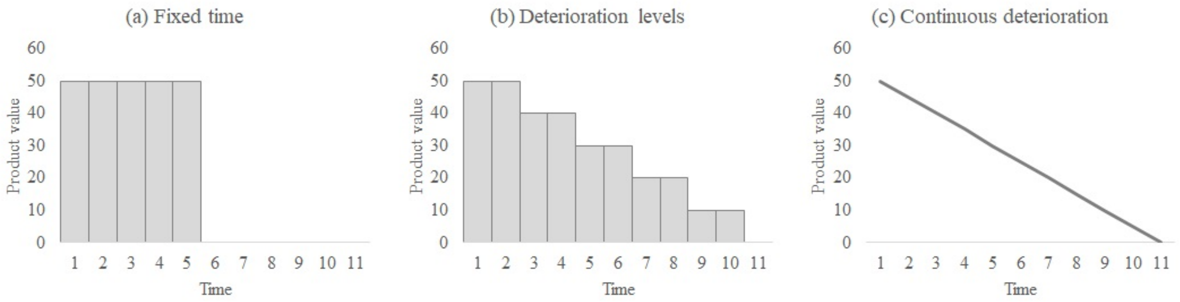


Figure 2 – Perishability models proposed by Pahl and Voß (2014)

2.2 Review method

In order to ensure a transparent and replicable review, some of the guidelines proposed by Tranfield, Denyer and Smart (2003) were followed. In the planning stage, we defined the research questions, the search strategy and the criteria to include or exclude studies. In this investigation, four questions were defined:

- How much activity has there been since 2011?
- How is shelf life modeled in studies about the IPDSP-P? How can one distinguish a long from a short shelf life?
- What are the most common characteristics from production and distribution that are modeled in studies about the IPDSP-P?
- What metrics do researchers aim to optimize when studying the IPDSP-P?

The search strategy consisted of a search using keywords in the bibliographic database Compendex (Engineering Village) of articles written in English and published in journals from 2011 to 2022. An article would only be located if its title explicitly indicated that the paper was about optimization of the production and/or distribution problem for perishable products. Therefore, to be a candidate for this review one of the following search terms for each subject had to be found in its title:

- perishable products: "perishable", "perishability" , "fresh", "freshness", "deteriorating", "decay", "short shelf life", "expiration date";
- optimization problems: "optimization", "planning", "optimisation";
- problem: "production", "production-distribution", "scheduling", "routing";

Through the database search, a total of 97 papers was obtained, and a content analysis was conducted. This analysis consisted of the first evalution of title and abstract, and if no exclusion criteria were met, the full content was reviewed. Some examples of papers that did not meet the inclusion criteria are studies that evaluated an integrated optimization of inventory-distribution problems, studies about strategic decisions, such as the facility location, and studies that the perishable products had no risk of expiration or decrease of value over time. Table 1 summarizes the inclusion/exclusion criteria:

Inclusion/Exclusion criteria	**Example of what will be disconsidered**
Only studies that present the quantitative optimization model for the production, distribution or integrated production-distribution problem will be considered	Replenishment; integrated replenishment-distribution problems; literature reviews
Only studies considering lot sizing, production scheduling, direct shipment or the vehicle routing problem.	Facility location problem; Choice of distribution channels; Implementation of new technologies
The perishabilty must represent a risk of expiration or decrease in the product value	Problems in which perishable products are considered, but the products do not run the risk of losing their value or expiring, e.g., cases where perishability only restricts the temperature and humidity levels of the operation
Only discrete production will be considered	Problems from process manufacturing

Table 1 – Inclusion/exclusion criteria adopted for the literature review

From the 23 found, whose distribution over time is presented in Figure 3, 6 evaluated problems related to the production of perishables, 6 considered the distribution of perishables and 11 considered the integrated problem, that is, considering production and distribution in an integrated way. Only two papers, which were published in 2012, compare the integrated approach with the sequential approach, i.e., the production-distribution being solved as a single problem versus the production problem solution being used as input for the distribution problem.

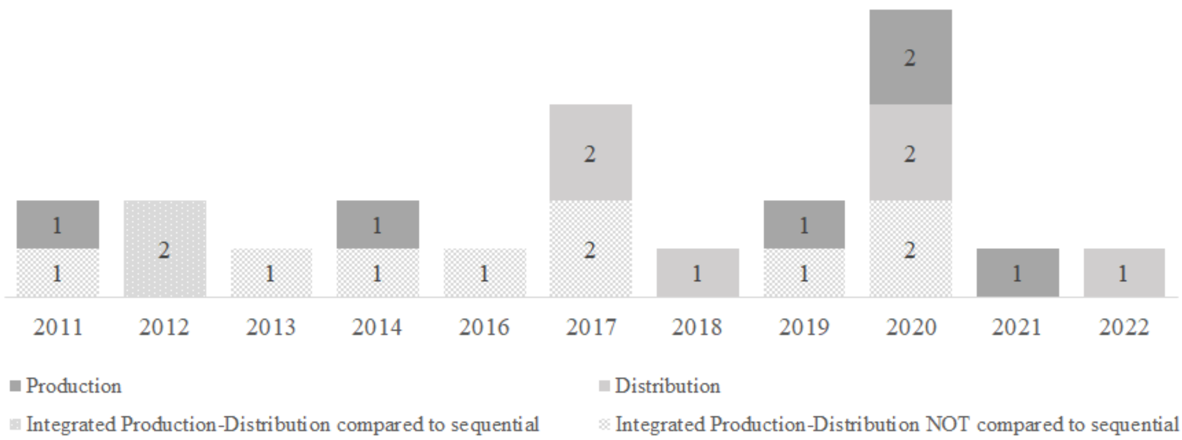


Figure 3 – Evolution of publications about production, distribution and integrated production-distribution planning between 2011 and 2022

A database was created to classify the papers based on the following dimensions:

- How is perishability modeled?
- Which production-related characteristics were included in the model?
- Which distribution-related characteristics were included in the model?
- What solution method was used to solve these problems? What metric does the study aim to optimize?

Regarding perishability dimension, the criteria used was based on the works from Amorim et al. (2013) and Pahl and Voß (2014). The former reference was used to classify the perishability according to the characteristics of the product and the latter to classify how the perishability is included in the model. As most authors did not classify the product according to the framework proposed by Amorim et al. (2013), on many occasions we had to assume a classification according to the product studied or according to how the perishability was modeled. This study also proposes a classification modeling perishability based on the articles reviewed. Table 2 presents the criteria considered for the perishability dimension.

Criterion	**Answer options**	**Observation**
Perishable product	-	When the product was not named, it was classified as "generic"
Perishability by authority limits	Fixed or Loose	Criterion based on Amorim et al. (2013)
Physical product deterioration	Yes or No	Criterion based on Amorim et al. (2013)
Customer value	Constant or Decreasing	Criterion based on Amorim et al. (2013)
How perishability is modeled	**$** - additional cost or revenue in objective function; **velocity** - the objective function aims to maximize the freshness of the delivered products; **dem** - demand variation according to the product quality; **tw** - time window constraint; **pt** - perishability impacts the processing time of the material; **lsl** - the shelf life length depends on a variable in the model	Classification proposed by authors based on the articles reviewed
What's the course of quality decay?	Fixed time or Levels of deterioration or Continuous deterioration	Criterion based on Pahl and Voß (2014)
Product lifetime	Known or Random or Undefined	Criterion based on Pahl and Voß (2014)

Table 2 – Criteria to classify how the perishability aspect was considered in the paper

Regarding the production and distribution dimensions, both were adapted from the classification proposed by Moons et al. (2017). However, instead of evaluating articles that handle only scheduling and vehicle routing, we also analyzed studies that consider the lot sizing and the direct shipment problems due to the close relation between those problems. In the literature, we found studies that integrate the production scheduling + direct shipment and lot sizing + vehicle routing problem. As the studies of lot sizing problems were also included, the "batch processing" nor the "Production release date" were considered because those characteristics are more related to the scheduling problem.

Tables 3 and 4 present the criteria used to classify the production and the distribution characteristics, respectively. Besides analyzing the problem, the solution method and the objective function were also registered and presented in Table 5.

Criterion	**Answer options**	**Observation**
Production problem	Lot sizing or Production Scheduling	-
Number of plants	Single plant or Multiple plants	criterion based on Moons et al. (2017) article
Machine environment	Single machine or Parallel Machines or Flowshop or Jobshop or Others	criterion based on Moons et al. (2017) article
Does it considers production cost?	Yes or No	criterion based on Moons et al. (2017) article
Setup times	Yes or No	criterion based on Moons et al. (2017) article
Setup cost	Yes or No	criterion based on Moons et al. (2017) article
Backorder	Yes or No	criterion based on Moons et al. (2017) article
Precedence setup	Yes or No	criterion based on Moons et al. (2017) article
Does it considers inventory?	Yes or No	criterion based on Moons et al. (2017) article
Limited inventory capacity	Yes or No	criterion based on Moons et al. (2017) article
Inventory holding cost	Yes or No	criterion based on Moons et al. (2017) article

Table 3 – Criteria to classify the aspects related to the production

2.3 How perishability is modeled

This section describes, it is described how perishability is modeled in the reviewed literature. For this analysis, we considered the 23 articles that met the inclusion and exclusion criteria, i.e., studies that model the production problem, the distribution problem, and articles that model the integrated production-distribution problem. This decision was due to the perishability concept that was not related to the production or distribution environment.

Besides the description, further analysis is conducted by linking the models' characteristics to the framework proposed by Amorim et al. (2013). This analysis was motivated by questions of how certain aspects of the framework should be included in optimization

Criterion	Answer options	Observation
Distribution problem	Routing or Direct shipment	-
Fleet type	Homogeneous or Heterogeneous	criterion based on Moons et al. (2017) article
Multiple trips	Yes or No	criterion based on Moons et al. (2017) article
Split delivery	Yes or No	-
Vehicle variable cost	Yes or No	criterion based on Moons et al. (2017) article
Vehicle fixed cost	Yes or No	criterion based on Moons et al. (2017) article
Loading times	Yes or No	criterion based on Moons et al. (2017) article
Unloading times	Yes or No	criterion based on Moons et al. (2017) article
Delivery constraint	Due dates or Time window or No	criterion based on Moons et al. (2017) article

Table 4 – Criteria to classify the aspects related to the distribution

Criterion	Answer options	Observation
What was the objective function?	-	Classified according to the article, but of studies considered cost or profit as the objective function
What was the solution method used?	**E** - Exact method; **D** - Decomposition method, i.e., any form to split the problem in two or more parts; **DP** - Dynamic programming; **H** - Heuristic; **MH** - Metaheuristic; **R** - Robust optimization; **SP** - Stochastic programming; **Sim** - Simulation technique employed to support an optimization method	-

Table 5 – Criteria to classify what the model's objective function was and the solution method

problems. For example, how a product that physically deteriorates should be modeled? Thus, a classification matrix is proposed and presented in Figure 4 at the end of the section.

In the reviewed literature, a common way to model shelf life is as a single known fixed parameter used as a deadline for the product to be delivered. All studies that modeled the shelf life this way conducted the computational experiments using random instances or instance generators from the literature (Amorim et al., 2013; Devapriya; Ferrell; Geismar, 2017; Seyedhosseini; Ghoreyshi, 2014; Marandi; Zegordi, 2017; Shirvani; Ruiz; Shadrokh, 2014).

Besides modeling a known and fixed shelf life as a time window, according to the literature, other ways to model this situation are revenue/cost in the objective function or the velocity that products are delivered (or production orders that are completed). Chan et al. (2020) modeled a meat product supply chain as a component in the multi-objective function by maximizing the food quality level provided to consumers. The authors also considered a time limit for perishablesl products because the food quality should not exceed a threshold value. LI et al. (2016) modeled a generic perishable food using a food quality index $q \in Q = \{0, 1, 2, Nbq\}$, where 0 is the initial quality of the newly produced food. For each quality level, there is a different selling price. As there is a limited number of levels, the product cannot fall below a minimum quality level (Nbq).

As presented in the previous examples, the product lifetime can also be a decreasing value over time, which can be modeled as continuous deterioration or levels of deterioration. According to the reviewed papers, the main difference between these two quality decays is that the former is usually modeled as a variable in the objective function, while the latter is usually modeled as one of the model's indexes. Albrecht and Steinrucke (2018) were motivated by a real-life German fresh produce distributor specializing in fruits. The product has several quality grades that are modeled as an index in the model. Each quality grade has a different sales price, and the product expires when its sales price reaches zero. Piewthongngam, Chatavithee and Apichottanakul (2019) studied the disassembly process for the meat processing industry. The disassembly process transforms an item into several subproducts, which can also be disassembled into other subproducts. The authors modeled these subproducts as different indexes, and specific holding costs and shelf lives are considered for each. The model handles perishability by minimizing the cost of expired items. Drenovac, Vidović and Bjelić (2020) studied the distribution of sugar beets, and the perishability was included as a decay function that must be maximized.

When the lifetime is not a known value or function, five articles did not model the product lifetime, and one article considered a random lifetime. In those cases, all articles considered continuous deterioration or levels of deterioration. When the lifetime is undefined, the perishability is included as a cost or quality decay in the objective function. For example, Zhao, Li and Zhou (2020) transformed the loss of quality into a cost that is included in the objective function. Matsumoto, Kashima and Ishii (2011) aimed to minimize both orders finished before the due date (earliness), and orders finished after the due date (tardiness). Farahani, Grunow and Gunther (2012) aimed to minimize food decay, which is measured by the difference between the delivery time of an order and its production finishing time. All these papers were classified as undefined lifetime because it was impossible to infer the lifetime.

Regarding the study that considers perishability as random lifetime, Rahbari et al. (2019) modeled the shelf life of the fresh products from their work as a decreasing function. This decreasing function is modeled both with and without uncertainties. When the uncertainties are present in the model, the authors solve them by robust optimization.

Two studies modeled the perishability impacting the demand. Ahumada and Villalobos (2011) developed a model for agricultural products. In this study, the perishability impacts the demand because the customer has a preferred quality level, and if the delivered products pass a determined threshold, the lot can be rejected, or the supplier must offer a discount. Hsiao, Chen and Chin (2017) also considered the perishability impacting the demand because, in their study of planning for cold chain distribution, the customers demand the product at a specific quality level. The proposed model satifies the customers' demand by delivering a higher quality product, incurring a substitution cost that is calculated based on the price gap between the higher quality level delivered to customers and the lower quality level initially required.

Besides modeling perishability such as cost, freshness, variation in demand, or time window, Acevedo-Ojeda and Chen (2020) studied a situation where the feedstock has a limited shelf life and its processing time (and cost) increases as time passes. Amorim, Gunther and Almada-Lobo (2012) considered a scenario in which a model variable could vary the shelf life length: the temperature at which products are stored at the distribution centers. Furthermore, perishability was included in the objective function by maximizing the product freshness. Besides the described scenario, named by the authors as "loose shelf life", they also proposed and evaluated a model where the product has a fixed shelf life.

In the reviewed literature, we did not find the perishable products classified according to (Amorim et al., 2013) proposed framework. Therefore, the articles were categorized using the following steps: 1 - When the article refers to fresh produce or food, we indicated physical deterioration. 2 - For generic products, physical deterioration was considered when the content indicated it, for example, when the model aims to optimize freshness or when the need for temperature control is indicated; 3 - Fruits and vegetables were classified as loose authority limits; 4 - Most of the problems that limit the time to deliver the product were classified as having a limit imposed by an authority. This authority can be a governmental entity or a company policy restricting the minimum shelf life for the products to be delivered to consumers. 5 - When the optimization objective consists of maximizing the product freshness or different sales prices according to the quality grade, the problem was classified as decreasing value for customers. Figure 4 presents the classification matrix.

2.4 Integrated Production-Distribution Problems

To analyze the integrated production and distribution problems in the literature, the papers were divided according to the production environment, similarly to Moons et al. (2017). The justification for that division is that authors can decide to include elements in the model according to the production environment. The first category considered

			Autorithy Perishability / Deterioration Perishability / Value Perishability				
			Fixed			Loose	
			No	Yes		Yes	
Lifetime Classfication	**Quality Decay**	**Model**	Constant	Constant	Decreasing	Constant	Decreasing
Known	Fixed time	$ + TW			1		
		TW	5				
		Velocity + TW			1		
	Continuous deterioration	$					1
		$ + TW			1		
		$ + TW + PT				1	
		$ + Velocity + TW					1
		Velocity					1
		Velocity + TW			1		1
		Velocity + TW + LSL					1
	Levels of deterioration	$ + Dem + TW			1		1
		$ + TW		1	1		
Random	Continuous deterioration	Velocity + TW					1
Undefined	Continuous deterioration	$					3
		Velocity					2

$ - additional cost or revenue in objective function; **Velocity** - the objective function aims to maximize the freshness of the delivered products; **Dem** - demand variation according the product quality; **TW** - time window constraint; **LSL** - Loose Shelf Life; **PT** - Processing Time

Although 23 articles met the inclusion criteria, two articles considered two different forms to model perishability, resulting in 25 occurrences.

Figure 4 – Matrix linking the framework proposed by (Amorim et al., 2013) and the modeling of perishability

the simplest production environments: single machine, single production level, and cases where the supply connects directly to the demand, without an intermediary stage. The articles from this category are presented in Subsection 2.4.1. The second category, whose studies are presented in Subsection 2.4.2, consists of models considering a parallel machine production environment and also simple supply. The cases not included in those two groups are presented in Subsection 2.4.3. We present a classification matrix for each category that combines the production and distribution characteristics.

2.4.1 Single machine environment

Seyedhosseini and Ghoreyshi (2014) integrated the lot sizing and direct shipment problem considering a limited capacity inventory at the production site. The problem consists of a single plant producing a product with a fixed shelf life. The decision on production consists of defining the production level and how much to stock. When a batch is produced, a setup cost and a variable production cost should be considered. On the distribution aspect of the problem, a unlimited fleet of homogenous capacitated vehicles must transport the products from the plant to several distribution centers with an established demand at each period. The model must define the number of vehicles and how many products will be delivered to customers during each period. Each vehicle can supply one single distribution center at most, and each distribution center can receive at most a single vehicle. The objective consists of minimizing the supply chain costs, i.e., production costs, setup costs, holding costs, and the total trip costs.

LI et al. (2016) integrated the lot sizing and the vehicle routing problem, considering a limited capacity inventory. The production aspect of the problem is very similar to

Seyedhosseini and Ghoreyshi (2014), considering production, setup, and holding costs. On the distribution aspect, products must be delivered to several retailers by a limited number of homogeneous capacitated vehicles. The model objective is to maximize the total profit.

Devapriya, Ferrell and Geismar (2017) integrated the production scheduling and the vehicle routing problem. In this article, the objective is to minimize the total cost of distribution to satisfy the customers' demand considering a single plant that produces a single perishable product with a fixed shelf life. The decision variables are the production sequence and the routes for a limited number of vehicles. Multiple trips are allowed and there is no inventory to store the goods. The authors named this problem an integrated production and distribution scheduling problem.

Chan et al. (2020) integrated the lot sizing and the vehicle routing problem where the inventory has a limited capacity. Concerning the production aspect, the authors considered production costs, setup costs, and holding costs. For the distribution subproblem, the transport of orders is done by a limited number of vehicles that are allowed to make multiple trips. The authors also included the CO2 emission to be minimized in the objective function. Besides that, the model also aims to maximize the freshness of the delivered goods and minimize costs and delivery time.

Table 6 presents a classification matrix for the studies analyzed in this subsection.

2.4.2 Parallel machine environment

Amorim, Gunther and Almada-Lobo (2012) studied the benefits of solving the production-distribution problem in an integrated way over the sequential approach. The production part of the problem contains both characteristics of production scheduling (decide the production sequence) and lot sizing problem (how much to produce) on parallel production lines on multiple plants. Besides the production costs, the authors also considered both sequence-dependent and independent setup times and costs. Regarding the distribution aspect, the products are transported from the production site to distribution centers via direct shipment with unlimited capacity considering only the variable transport cost. During each period, there is a demand to be satisfied, and we considered that need to deliver a quantity of goods at each period as a "due date" constraint. Products at the distribution center can be used for customers' orders or be stocked for subsequent periods. The distribution center has unlimited inventory capacity, and there are no holding costs for storing items. The model addresses a multi-objective problem, aiming to minimize costs (production, transportation, and spoilage costs) and maximize the remaining shelf life of the delivered items.

Farahani, Grunow and Gunther (2012) studied the integrated production scheduling and vehicle routing problem for a catering company based in Copenhagen. The model

			Machine environment - Single machine Production characteristics											
			Lot sizing	Scheduling	Single plant	Multiple plants	Production cost	Setup times	Setup cost	Backorder	Precedence	Limited inventory capacity	Inventory holding costs	**Number of studies**
Distribution		Direct shipment	1	0	1	0	1	0	1	0	0	1	1	**1**
		Routing	2	1	3	0	2	0	2	0	0	2	2	**3**
	Vehicle fleet	Single vehicle	0	0	0	0	0	0	0	0	0	0	0	**0**
		Homogeneous fleet	3	1	4	0	3	0	3	0	0	3	3	**4**
		Heterogeneous fleet	0	0	0	0	0	0	0	0	0	0	0	**0**
		Unlimited number	0	1	1	0	0	0	0	0	0	0	0	**1**
		Limited number	3	0	3	0	3	0	3	0	0	3	3	**3**
	Other	Multiple trips	1	1	2	0	1	0	1	0	0	1	1	**2**
		Split delivery	0	0	0	0	0	0	0	0	0	0	0	**0**
		Variable transportation cost	2	1	3	0	2	0	2	0	0	2	2	**3**
		Fixed transportation cost	1	1	2	0	1	0	1	0	0	1	1	**2**
		Loading times	0	0	0	0	0	0	0	0	0	0	0	**0**
		Unloading times	0	0	0	0	0	0	0	0	0	0	0	**0**
		Delivery due date	1	0	1	0	1	0	1	0	0	1	1	**1**
		Time windows	0	0	0	0	0	0	0	0	0	0	0	**0**
		Number of studies	**3**	**1**	**4**	**0**	**3**	**0**	**3**	**0**	**0**	**3**	**3**	**4**

Table 6 – Matrix of distribution and production characteristics for single machine environment

aims to minimize the weighted sum of the total setup costs, food decay, and variable transportation costs. On the production side, the customers' orders are produced at a single production site that operates several identical ovens (parallel machines). Since each order requires a unique combination of temperature and processing time, the production decision consists of grouping customer orders with similar temperature requirements and processing times and then sequencing those grouped orders to be processed in one of the ovens. The temperature setting of the ovens create a condition of sequence-dependent setup times. When the production finishes, the distribution consists of a vehicle routing problem with time windows and a limited number of homogeneous vehicles that will deliver the orders to customers.

Amorim et al. (2013) compared two problems: the integrated batching scheduling vehicle routing problem with time windows (I-BS-VRPTW) and the integrated lot sizing vehicle routing problem with time windows (I-LS-VRPTW). The difference between the two models is that the first can determine only the production sequence, while the second also specifies how much to produce. Both models consider variable production cost and sequence-dependent setup time and costs. Concerning the distribution aspect, a set of limited homogeneous vehicles fleet delivers the customer orders. Each order must be delivered within strict time windows, and the delivery demands a certain service time when the vehicle arrives at the customer's destination. There is a fixed cost for each vehicle used and a variable cost proportional to the travel time.

Rahbari et al. (2019) studied the vehicle routing problem with cross-docking. We considered this an integrated problem because the schedule of vehicles arriving and departing from the cross-docking station is analogous to the production schedule problem. Moreover, in the cross-docking station, there is the option to stock goods incurring in holding costs. At the distribution part, it is considered a fixed cost for each vehicle and a variable cost to transport the customers' orders. This problem also considers the loading time at the cross-docking and the service time at the customer. The demand must be satisfied within a time window, and the objective function consists of minimizing costs and maximizing the freshness of the product delivered.

2.4.3 Other production environments

Two articles considered other production environments. Marandi and Zegordi (2017) integrated the flow-shop production scheduling to the vehicle routing problem. Their model aims to set production scheduling and the vehicles' routes to minimize the delivery and tardy cost when the production due date is violated. The model does not consider any setup time or cost. A heterogeneous fleet of vehicles is responsible for the delivery, and variable transportation costs proportional to travel time are incurred.

Ahumada and Villalobos (2011) presented an MIP model for optimally scheduling the harvesting and distribution operations for the fresh produce industry. Some of the decisions included in the model are the frequency of harvest, the number of products shipped to warehouses or customers, and the number of operator hours. The authors applied the model to a hypothetical fresh produce grower who grows tomatoes and bell peppers.

We did not find any article that proposes an integrated optimization of a job shop production environment and a distribution problem.

2.5 Solution approaches

This section describes the solution methods applied in the papers mentioned in the previous section. Those methods are summarized in Table 8.

			Machine environment - Parallel machine Production characteristics											
			Lot sizing	Scheduling	Single plant	Multiple plants	Production cost	Setup times	Setup cost	Backorder	Precedence	Limited inventory capacity	Inventory holding costs	**Number of studies**
Distribution	**Vehicle fleet**	Direct shipment	1	0	0	1	1	1	1	0	1	0	0	**1**
		Routing	1	3	4	0	2	3	3	0	3	0	1	**4**
		Single vehicle	0	0	0	0	0	0	0	0	0	0	0	**0**
		Homogeneous fleet	2	3	4	1	3	4	4	0	4	0	1	**5**
		Heterogeneous fleet	0	0	0	0	0	0	0	0	0	0	0	**0**
		Unlimited number	2	1	2	1	3	3	3	0	3	0	0	**3**
		Limited number	0	2	2	0	0	1	1	0	1	0	1	**2**
	Other	Multiple trips	0	0	0	0	0	0	0	0	0	0	0	**0**
		Split delivery	0	0	0	0	0	0	0	0	0	0	0	**0**
		Variable transportation cost	2	3	4	1	3	4	4	0	4	0	1	**5**
		Fixed transportation cost	1	2	3	0	2	2	2	0	2	0	1	**3**
		Loading times	0	1	1	0	0	0	0	0	0	0	1	**1**
		Unloading times	1	2	1	0	2	2	2	0	2	0	1	**3**
		Delivery due date	1	0	0	1	1	1	1	0	1	0	0	**1**
		Time windows	1	3	4	0	2	3	3	0	3	0	1	**4**
		Number of studies	**2**	**3**	**4**	**1**	**3**	**4**	**4**	**0**	**4**	**0**	**1**	**5**

Although 4 articles were classified as "Parallel Machine", Amorim et al. (2013) studied two problems in their paper: the I-BS-VRPTW and the I-LS-VRPTW. We counted each of them in this table, resulting in 5 studies

Table 7 – Matrix of distribution and production characteristics for parallel machine environment

Ahumada and Villalobos (2011), Amorim et al. (2013), and LI et al. (2016) conducted computational experiments using a commercial solver as a solution method. Ahumada and Villalobos (2011) applied the solver for a hypothetical fresh produce grower to test the validity of the proposed method. Amorim et al. (2013) compared two production situations, one that allows that model to set the production volume and the production sequence (lot sizing) and a second model that controls only the production sequence (production scheduling). The article concluded that in 5 out of 120 instances, the lot sizing model reached a better solution than just scheduling the production, caused mainly by the reduction in the setup times, which led to less product waste due to expiration. LI et al. (2016) evaluated the model structure by generating random instances varying several parameters, such as the minimum quality level, number of retailers, length of the planning horizon, and the number of available vehicles. Some of the conclusions are that the number of vehicles impacts the computational time, the profits increase when the shelf life is longer, and a limitation to optimally solve the problem using a commercial solver at around 15 customers.

Devapriya, Ferrell and Geismar (2017), Marandi and Zegordi (2017), and Chan et al. (2020) used metaheuristics as a solution approach for the proposed problems, in all studies, the authors validate the proposed metaheuristic comparing it to a MIP model implemented in a commercial solver. Devapriya, Ferrell and Geismar (2017) tested three variations of the genetic algorithm for randomly generated problems. The solution approaches were compared to CPLEX for small instances, while for large instances, they used a non-parametric statistical test to evaluate differences between the median of the percentage gap above a lower bound. Marandi and Zegordi (2017) proposed a metaheuristic derived from the Particle Swarm Optimization (PSO) that was called Improved Particle Swarm Optimization (IPSO). To evaluate the performance of the IPSO, the authors compared it to the MILP model implemented in a commercial solver for small and medium instances. For large instances, the algorithm was compared to the Genetic Algorithm, and statistical tests were conducted to validate the proposed approach. Chan et al. (2020) also proposed a metaheuristic derived from the PSO, which was named Multi-Objective Global Local Near-Neighborhood Particle Swarm Optimization (MO-GLNPSO). To evaluate the proposed solution method, the authors applied it in a case study of a meat product supply chain from the literature and for several random instances. In both cases, the proposed method is compared to other PSO variations. The authors also conducted a sensitivity test to evaluate the influence of parameters on the solution result.

Amorim, Gunther and Almada-Lobo (2012), Farahani, Grunow and Gunther (2012), and Devapriya, Ferrell and Geismar (2017) proposed hybrid solution methods that decompose the integrated problem into subproblems that are solved using different solution methods. The subproblems are connected using loop structures and applying a subproblem solution as an input for the other subproblems. Amorim, Gunther and Almada-Lobo (2012) evaluated the benefits of solving the production-distribution problem in an integrated way compared to a sequential approach. To conduct that analysis, the authors considered two cases of perishability: a case where perishable products have a fixed shelf life and a case with loose shelf life. Thus, four solution methods were proposed: An MILP model implemented in a commercial solver was proposed for the integrated approach considering a fixed shelf life. For the decoupled approach, the authors solved the production problem first and then used the solutions from the production subproblem as input for the distribution problem. For the integrated approach with loose shelf life, the authors combined the genetic heuristic that generated inputs for an MILP model implemented in a commercial solver. For the decoupled approach with loose shelf life, an MILP model generated solutions for the production subproblem that are used as input for the distribution problem that was solved using the genetic heuristic and commercial solver, similar to the integrated approach. As a result, the authors verified that more weight of the freshness reduces the differences between the integrated and the sequential approaches, while more perishability increases the benefits of the integration.

Farahani, Grunow and Gunther (2012) solved the proposed problem using a hierarchical modeling approach that divides the entire planning problem into sub-problems. The authors combined heuristic methods and the MILP model implemented in a commercial solver to solve the sub-problems and compare that integrated solution approach to a sequential one. Through computational experiments, the authors concluded that the bigger the instance, the more significant the benefits of the integrated approach (around 9.5 to 28.2% of improvement in the objective function). In addition, the authors verified that the integrated approach provided better results for more perishable products.

Seyedhosseini and Ghoreyshi (2014) also solved the integrated problem by decomposing the integrated model into two dependent submodels, the production submodel, and the distribution submodel. The production submodels are solved using a commercial solver, and the distribution submodel is solved using a metaheuristic: the Particle Swarm Optimization (PSO). Both subproblems are connected by a loop that feeds the result of one subproblem into the other until a stop criterion is met.

Lastly, Rahbari et al. (2019) studied the vehicle routing and scheduling problem with cross-docking (VRPCD) for perishable products. The authors modeled the problem in two ways: a deterministic model in a commercial solver to evaluate the effects of perishability and two robust models to consider the uncertainties of shelf life and travel time. The results of the robust models are compared to the "Soyster's approach" and "Ideal case." One conclusion is that the model increased the freshness of the delivered products by 74.14% on average without increasing the distribution costs.

2.6 Discussion

The use of optimization models in production and distribution problems, when considering perishability, is not very common. Compared to Moons et al. (2017) who found 20 studies integrating production scheduling and vehicle routing problems between 2011 and 2017, in our review, only five papers solved the same integrated problem for perishable products between 2011 and 2022.

Concerning how perishability is modeled, we verified many articles modeling perishability as a time limit to deliver the orders to consumers or to finish production orders. Another common way is to model it in monetary terms included in the objective function. This way to model the objective function differs from the standard metrics optimized in the scheduling literature, usually related to weighted completion time, makespan, earliness, or

Authors	E	D	DP	H	MH	R	SP	Sim	Objective function
Single machine									
LI et al. (2016)	x								Profit
Devapriya, Ferrell and Geismar (2017)					x				Cost
Seyedhosseini and Ghoreyshi (2014)	x	x			x				Cost
Chan et al. (2020)					x				Cost + Freshness + CO2 emission + Delivery time
Parallel machine									
Amorim et al. (2013)	x								Cost
Amorim, Gunther and Almada-Lobo (2012)	x	x			x				Cost + Freshness
Farahani, Grunow and Gunther (2012)	x	x		x					Cost + Freshness
Rahbari et al. (2019)						x			Cost + Freshness
Other									
Ahumada and Villalobos (2011)	x								Profit
Marandi and Zegordi (2017)					x				Cost + Tardiness
Sinha and Anand (2020)					x				Cost

E = Exact method; D = Decomposition method; DP = Dynamic Programming; H = Heuristic; MH = Metaheuristic; R = Robust Optimization; SP = Stochastic Programming; Sim = Simulation

Table 8 – Solution methods

tardiness (Pinedo, 2008; Fernandes; Filho, 2010). One possible explanation for optimizing profit or costs in the objective function is that most of the studies consider fresh produce, and for those items, consumers can distinguish the different quality grades by visual inspection. Another possible reason may be the integration to the distribution problem since Moons et al. (2017) also pointed out several articles that aim to optimize profit or costs.

We also noticed that a standardized form to describe perishable products could benefit future studies, similar to production scheduling studies where it is common to follow the classification proposed by Pinedo (2008). This proposal is motivated by the existing relation between the characteristics of perishable products and how they can be modeled. Therefore, using a standard classification, such as the framework proposed by Amorim et al. (2013), can make future works more transparent and help authors find and propose forms to model perishability.

Most of the problems that considered physical deterioration also modeled it as a decreasing perceived value by customers. Only two studies did not consider the relationship between physical deterioration and perceived customer value. According to Acevedo-Ojeda and Chen (2020), the physical deterioration impacts processing the perishable feedstock, increasing the processing time and cost as time passes. For Piewthongngam, Chatavithee and Apichottanakul (2019), the physical deterioration incurred in discarding costs for expired goods. Moreover, this review observed a prevalence of research on fruits, vegetables, meat, and other fresh produce. In a world of fast fashion and planned obsolescence, there is an opportunity to model perishability for other items such as clothes and electronics.

There is a key difference between how the authors face the challenge of integration: a first group of studies considers a set of performance measures on both sub-systems (e.g., Chan et al. (2020) adopt production and distribution costs). Another approach considers only the performance measures from one subsystem, and the integration arises when finding viable solutions (e.g., Devapriya, Ferrell and Geismar (2017) adopts the distribution cost as the objective function, and the solution space is given by a combination of production and distribution constraints).

When comparing studies that analyzed the single machine production environment to studies that considered the parallel machine environment, we noticed that the latter included more characteristics from production and distribution in their proposed models. Only one study of the single machine environment considered the production scheduling problem, and none considered setup times between orders. On the other hand, most studies from the parallel machine environment included sequence-dependent setup times in their models. The same was noticed in the distribution aspect. The single machine environment studies modeled the distribution as a vehicle routing problem with a homogeneous fleet considering fixed and variable transportation costs. In contrast, many studies from parallel machine environments also included service times at customer and time windows to deliver orders.

Another aspect noticed was that only three articles evaluated different forms to model a problem. Amorim et al. (2013) compared lot sizing to the batch scheduling. Amorim, Gunther and Almada-Lobo (2012) compared four forms of modeling an integrated production scheduling routing problem: they tested combinations of the decoupled approach to the integrated approach and fixed shelf life and loose shelf life. Farahani, Grunow and Gunther (2012) also compared the sequential versus the integrated planning approach. Studies such as those support future works when authors must decide how to model and implement the problem.

Regarding solution methods, this review reinforces that solving integrated problems are benefited by alternative approaches such as decomposition methods and metaheuristics

(Grossmann, 2005b; Papageorgiou, 2009; Garcia; You, 2015). The lack of studies considering stochastic elements identified by Amorim et al. (2013) is also found in this review where only Rahbari et al. (2019) implemented stochastic elements in the model.

This overview demonstrates that the literature on integrated production-distribution optimization is still in the early step. The number of studies is small, and a fraction of those test different forms to model the problem or solve it. Therefore, there is a gap in theoretical studies that provides insights into modeling the integrated production-distribution problem for perishable products, whether testing different forms and different solution approaches or studying structural properties from the problem that can provide results to simplify it. These kinds of studies could be a way to support future works and reinforce this research topic in academia.

3 Problem Characterization

This chapter presents the mathematical formulation and structural properties for the integrated production and distribution scheduling problem for perishable products (IPDSP-P). The quantitative model, presented in Section 3.1, was based on the work of Amorim et al. (2013), but we removed all setup times and setup cost from the original model. This adaptation was done to simplify the model because the focus of this work lies on building theoretical knowledge about a simple form to model the IPDSP-P, and we wanted to avoid any other component besides the shelf life influences the expected performance in our experiments.

This simple model can be considered an integrated problem because when the available time between the order placement and the delivery is short, such as problems involving perishable products or a make-to-order system, integration is necessary. Below, an example is provided to demonstrate the necessity of integration for time-sensitive products.

Consider a company that produces a single perishable product and at the beginning of the planning period, there are nine orders to be sequenced in a single machine. After production, the order must be delivered to customers using identical vehicles that should be routed by the optimization model. For each vehicle used, the company incurs a fixed cost of $250. The variable cost for each combination of origin and destination is presented in Table 9. The travel time is the same as the variable cost. Finally, Table 10 presents the processing time for each order.

	P	**1**	**2**	**3**	**4**	**5**	**6**	**7**
P	-	15	16	40	50	50	45	27
1	15	-	21	52	63	46	32	34
2	16	21	-	53	43	65	54	44
3	40	52	53	-	61	50	70	21
4	50	63	43	61	-	97	96	66
5	50	46	65	50	97	-	34	32
6	45	32	54	70	96	34	-	49
7	27	34	44	21	66	32	49	-

P = Plant

Table 9 – Travel time and cost matrix for the plant and customers for an integration example

Consider four levels of perishability to exemplify the necessity of integrated planning:

- Scenario 1: Non-perishable product
- Scenario 2: Product with long shelf life - Shelf life = 747 units of time

Order	Process time	Demand
1	40	40
2	1	1
3	55	55
4	48	48
5	46	46
6	53	53
7	43	43

Table 10 – Process time and demand for each order of the integration example

- Scenario 3: Product with short shelf life - Shelf life = 340 units of time
- Scenario 4: Product with very short shelf life - Shelf life = 149 units of time
- Scenario 5: Minimum shelf life possible, i.e., the order must be delivered as soon as the production is finished

Then, consider two solution methods to the problem:

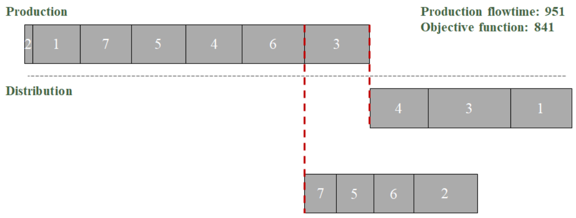
- Method A: Using the shortest processing time (SPT) rule to the production and optimization of the distribution
- Method B: Integrated optimization of the production and distribution

The results for scenarios are presented in Figures 5a, 5b, 5c, 5d. Figure 5a shows the result for the scenarios 1, 2 and 3 using solution method A, while Figure 5b is the result for the same scenarios using solution method B. For scenario 4, where the shelf life is very short, it is not possible to solve the production and distribution separately, because the optimal solution for the VRP, without the production problem, would be the same as Figures 5a and 5b, i.e., using two vehicles. However, when solving the problem in an integrated way, as presented in Figure 5c, the result shows that three vehicles are needed for a feasible solution. Finally, for scenario 5, when the shelf life is so short that the product must be delivered as soon as the production is finished, only the production problem must be solved, because necessarily each vehicle will necessarily deliver a single order.

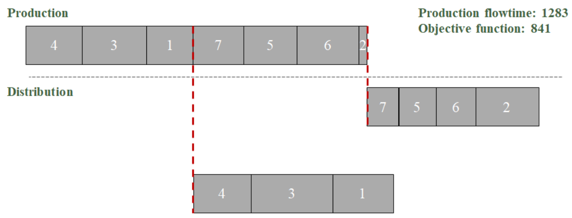
Section 3.2 presents a set of structural properties that allows one to, in Section 3.3, enhance the model described in Section 3.1.

3.1 Integrated production and distribution scheduling for perishable products

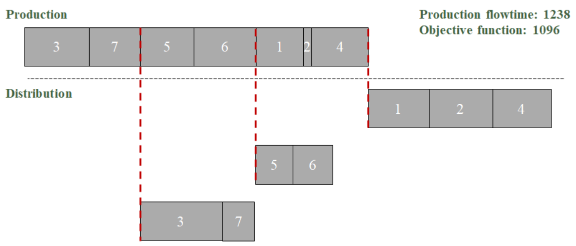
This problem consists of a single machine or plant, which produces a particular perishable product whose quality starts to decay right after the production starts. This



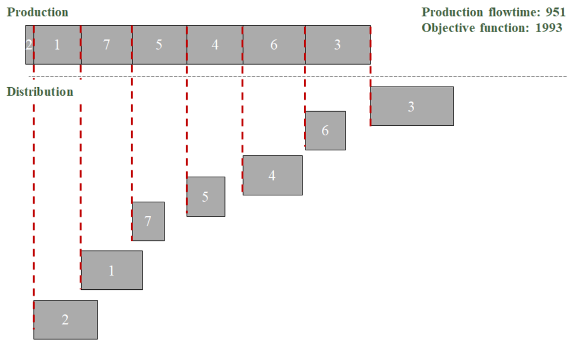
(a) Results for scenario 1 using solution method A (SPT rule for production and optimization of vehicle routing problem)



(b) Results for scenario 2 using solution method B (Integrated production-distribution scheduling optimization)



(c) Results for scenario 3 using solution method B (Integrated production-distribution scheduling optimization)



(d) Results for scenario 4 using the SPT rule for production

product must be delivered to a set of customers $N = \{1, \ldots, n\}$. The delivery can occur only while the product still has some shelf life and there is available capacity in one of the $|K|$ identical fixed capacity vehicles. These vehicles must be routed through a set A of arcs contained in a directed graph $G = (V, A)$. The customers and the plant represents the vertices of this graph (V), $V = N \cup \{0, n+1\}$, where vertices 0 and n+1 represents the production plant (start and ending of the route, respectivelly).

The notation adopted for this IPDSP-P model is:

Indices and sets

$(c, d) \in N$	Customers
$k \in K$	Vehicles

Parameters

dem_c	Demand at customer c (units)
pt	Processing time for one unit of product
$CapV$	Vehicle capacity
ct_{cd}	Cost to transport products from customer c to d
tt_{cd}	Travel time to transport products from customer c to d
$\overline{ft}$	Fixed cost associated to each vehicle k
sl	Product's shelf life

Variables

x^k_{cd}	assume the value of 1, if arc (c,d) is used by vehicle k (0 otherwise)
w^k_c	arrival time of vehicle k at customer c
V_{cd}	assume the value of 1, if customer d order is produced right after customer c order (0 otherwise)
Ct_c	completion time for customer c order

Based on these elements, the IPDSP-P may be formulated as follows:

$$Min \quad \overline{ft} \sum_{k \in K} (1 - x^k_{0,n+1}) + \sum_{k \in K} \sum_{c,d \in N} ct_{cd} x^k_{cd} \tag{3.1}$$

$$Subject\ to \quad Ct_0 = 0 \tag{3.2}$$

$$\sum_{c \in N} V_{cd} = 1 \qquad \forall d \in N \tag{3.3}$$

$$\sum_{d \in N} V_{cd} = 1 \qquad \forall c \in N \tag{3.4}$$

$$Ct_d \geq Ct_c + (pt_d \cdot dem_d) - M(1 - V_{cd}) \qquad \forall \begin{cases} d \in N \\ c \in N/\{0, d\} \end{cases} \tag{3.5}$$

$$\sum_{k \in K} \sum_{d \in N} x^k_{cd} = 1 \qquad \forall c \in N \tag{3.6}$$

$$\sum_{d \in N} x^k_{0d} = 1 \qquad \forall k \in K \tag{3.7}$$

$$\sum_{c \in N} x^k_{cd} - \sum_{c \in N} x^k_{dc} = 0 \qquad \forall k \in K; d \in N \tag{3.8}$$

$$\sum_{c \in N} x^k_{c,n+1} = 1 \qquad \forall k \in K \tag{3.9}$$

$$w^k_d \geq w^k_c + tt_{cd} - M(1 - x^k_{cd}) \qquad \forall k \in K; c, d \in N \tag{3.10}$$

$$\sum_{c \in N} dem_c \sum_{d \in N} x^k_{cd} \leq CapV \qquad \forall k \in K; c \in N \tag{3.11}$$

$$Ct_c - (pt \cdot dem_d) + sl - \sum_{k \in K} w^k_c \geq 0 \qquad \forall c \in N \tag{3.12}$$

$$w^k_0 \geq Ct_c - M(1 - \sum_{d \in N} x^k_{cd}) \qquad \forall k \in K; c \in N \tag{3.13}$$

$$w^k_d, Ct_c \geq 0 \qquad \forall k \in K; c, d \in N \tag{3.14}$$

$$x^k_{cd}, V_{cd} \in \{0, 1\} \qquad \forall c, d \in N \tag{3.15}$$

The objective function (3.1) minimizes the distribution costs, which are composed of variable and fixed costs, based on the total travel time and the number of vehicles used, respectively. The objective function does not consider production costs.

Constraints (3.2) to (3.4) are used to set the production sequence of customers' orders and constraints (3.5) establish when each order is completed.

Constraints (3.6) to (3.11) refer to the distribution process. Constraints (3.6) ensure that each origin has only one destination and is visited only by one vehicle. Constraints (3.7) and (3.9) establish that each vehicle departing from the plant has only one destination and each vehicle returning to the plant has only one origin, respectively. Constraints (3.8) ensure that if a vehicle visits a client c, this node will be the next origin. (3.10) establishes the time when a vehicle that departs from c to d will arrive at node d. Constraints (3.11) enforces that the vehicle capacity is respected. As stated in (Amorim et al., 2013), $x^k_{0,n+1} = 1$ means that the vehicle was not used.

Finally, constraints (3.12) enforce that the product is delivered while it still has some shelf life, constraints (3.13) link the production to the distribution problem. The domain of variables are stated in equations (3.14) and (3.15).

3.2 Structural properties

In this section, we analyze structural properties of the IPDSP-P. We found that if any production sequence is feasible for a given distribution route, then the production sequence that has the same order as the distribution route is also feasible. As the production schedule does not impact the objective function, our goal is solely to find a production schedule that satisfies the shelf life constraints for the optimal distribution route. This simplification is possible if the following assumptions are true:

- Single machine environment;
- The size of each job is given
- Single product with a single shelf life length;
- No sequence dependent setup time or costs;
- The product's quality starts to decay at the starting time of the production.

Lemma 1. *Let $\sigma_k \subset A$ be a feasible route of vehicle k for the distribution part of IPDSP-P. E_i is the time between the start of the production of order i, $i \in \sigma_k$ and its departure from the plant, TSL is the total shelf life of the perishable product, and $Travel_i$ is the time elapsed between the departure from the plant and the delivery to the customer i for the given route. Considering this notation, the whole problem will be feasible only if $E_i \leq SL_i, \forall i \in \sigma_k$, where SL_i is given by equation 3.16*

$$SL_i = TSL - Travel_i \tag{3.16}$$

Proof. The route σ_k of the IPDSP-P will be feasible only if the total elapsed time between production and delivery times for each customer order is lower than the total shelf life, i.e.,

$$E_i + Travel_i \leq TSL \quad \forall i \in \sigma_k \tag{3.17}$$

Combining equations 3.16 and 3.17, it can be stated that IPDSP-P will be feasible only if $E_i \leq SL_i$.

Figure 6 presents an example of a scheduling with the variables $Travel_i$, E_i, SL_i and TSL.

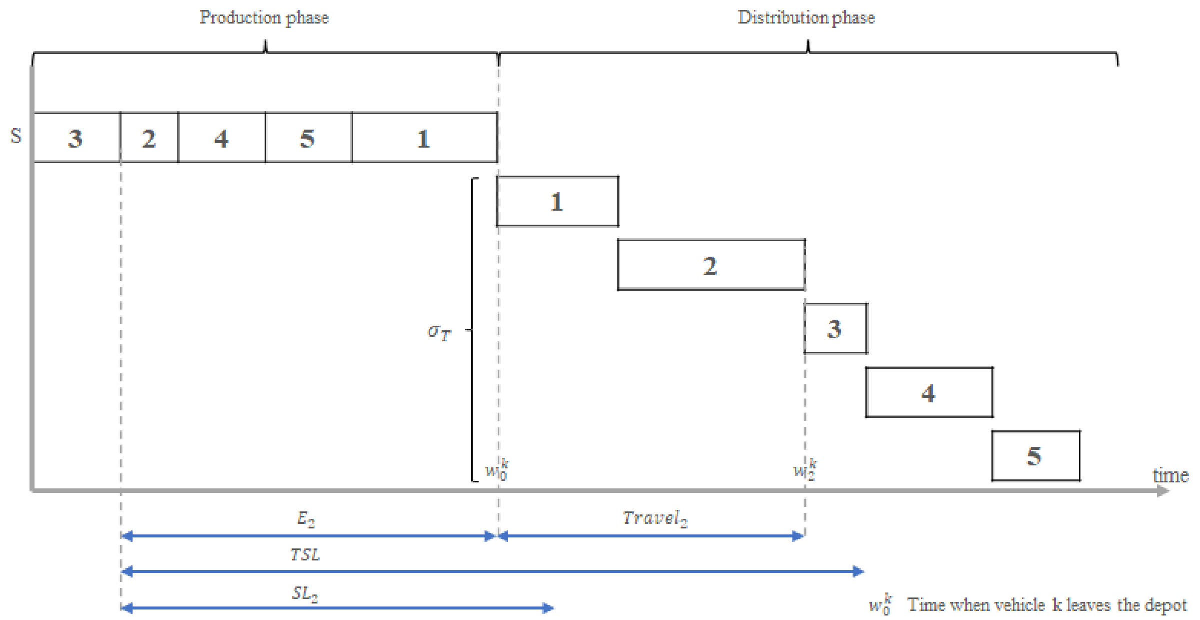


Figure 6 – A graphic representation of lemma 1

□

Lemma 2. *If the route $\sigma_k \subset A$ and the production sequence $S = \{x_1, x_2, ..., x_{|\sigma_k|}\}$ provide a feasible solution for IPDSP-P, then $E_i \leq SL_i$ for the orders $i = 1, 2, ..., x_1$ in production sequence $S' = \{1, ..., x_1, ..., |\sigma_k|\}$*

Proof. As the route is given and follows the order 1, 2, ..., $|\sigma_k|$, we may conclude that:

$$Travel_1 \leq Travel_2 \leq ... \leq Travel_{|\sigma_k|} \tag{3.18}$$

Since TSL is constant, the following relationship can be stated:

$$SL_{|\sigma_k|} \leq SL_{|\sigma_k|-1} \leq ... \leq SL_1 \tag{3.19}$$

Denoting Eo_i and E'_i as the E_i variable for S and S', respectively, and as x_1 is the first job of the sequence S', and S is feasible, we may assure that:

$$Eo_{x_1} = \sum_{i=1}^{|\sigma_k|}(pt * dem_i) \leq SL_{x_1} \leq SL_{x_1-1} \leq SL_{x_1-2} \leq ... \leq SL_1 \tag{3.20}$$

Where $(pt * dem_i)$ is the processing time of any job i.

As S and S' have the same number of jobs, and job 1 is the first job of sequence S', we may affirm that $E'_1 = Eo_{x_1}$, and for any other job in S', the following is true:

$$E'_j = E'_{j-1} - p_i \quad \forall j > 1 \tag{3.21}$$

As $p_i \geq 0$ then:

$$E'_{|\sigma_k|} \leq E'_{x_1} \leq E'_{x_1-1} \leq ... \leq E'_1 = Eo_{x_1} = \sum_{i=1}^{|\sigma_k|} p_i \tag{3.22}$$

According to (3.20) and (3.22):

$$E'_{|\sigma_k|} \leq E'_{x_1} \leq E'_{x_1-1} \leq ... \leq E'_1 = Eo_{x_1} \leq SL_{x_1} \leq SL_{x_1-1} \leq ... \leq SL_1 \tag{3.23}$$

Therefore,

$$E'_i \leq SL_i \quad \forall i \leq x_1 \tag{3.24}$$

Figure 7 presents a graphical illustration of the lemma.

□

Lemma 3. *If the route* $\sigma_k = \{1, 2, ..., |\sigma_k|\}$ *and the production sequence* $S = \{x_1, x_2, ..., x_N\}$ *provide a feasible solution for IPDSP-P. Then,* $E_i \leq SL_i$ *for any job i that* $x_1 + 1 \leq i \leq y$ *in sequence* $S' = \{1, ..., x_1, ..., y, ..., |\sigma_k|\}$*, where y is the next job in S that* $y > x_1$*.*

Proof. Since $y > x_1$, there will be no job y' scheduled after x_1 and before y that $y' > x_1$. Thus:

$$Eo_y = \gamma + \sum_{i=x_1+1}^{|\sigma_k|} p_i \leq SL_y \tag{3.25}$$

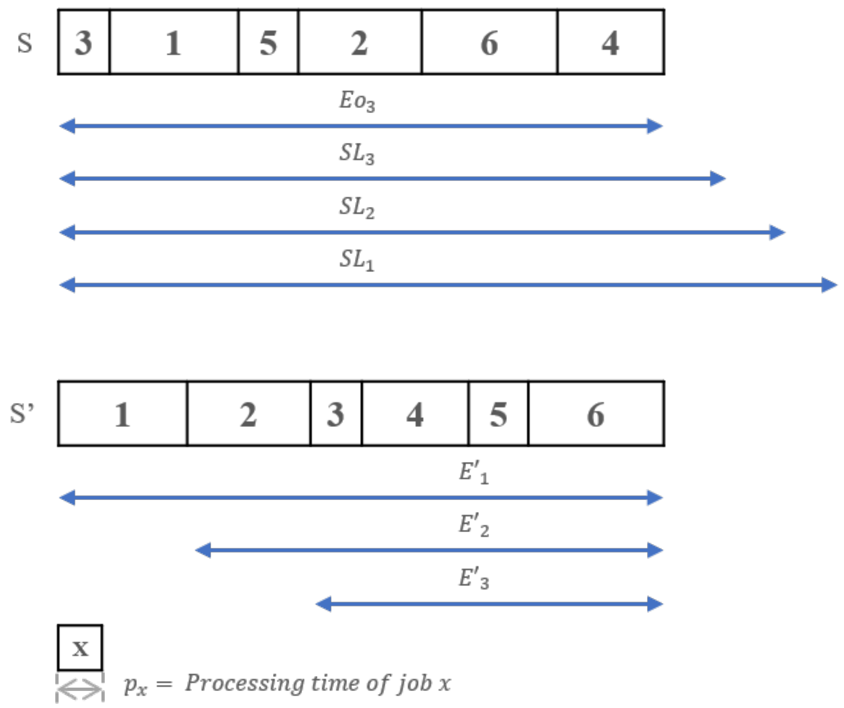


Figure 7 – A graphic representation of lemma 2

Furthermore,

$$E'_{x_1+1} = \sum_{i=x_1+1}^{|\sigma_k|} p_i \leq SL_{x_1+1} \tag{3.26}$$

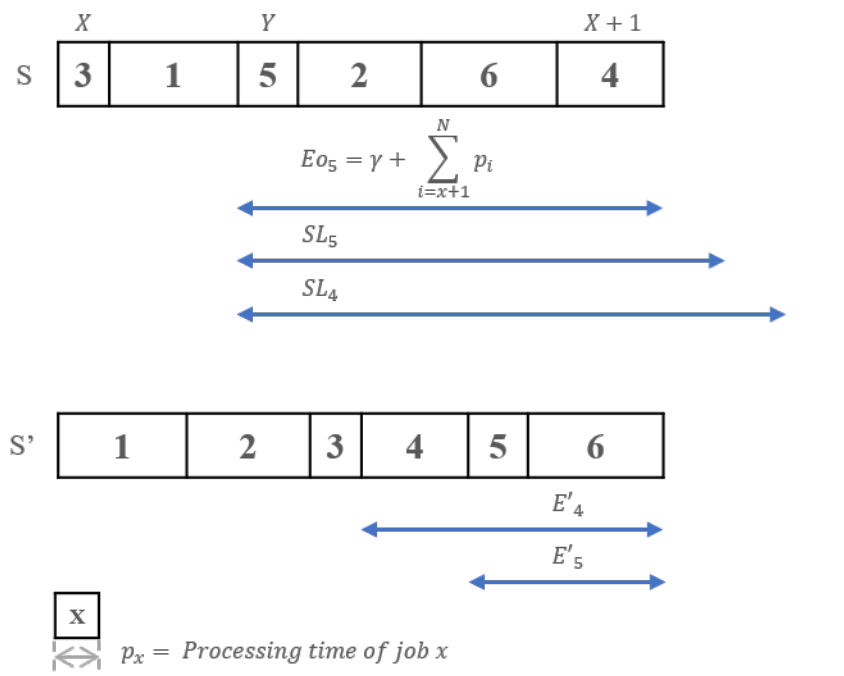


Figure 8 – A graphic representation of lemma 3

γ is a real number ≥ 0 and, from equations (3.25) and (3.26), $E'_{x_1+1} \leq Eo_y$. Figure 8 provides a graphical support to prove that (3.26) is true. Finally, by the same logic presented in 3.23:

$$E'_{|\sigma_k|} \leq E'_y \leq E'_{y-1} \leq E'_{x_1+1} \leq Eo_y \leq SL_y \leq SL_{y-1} \leq ... \leq SL_{x_1+1} \tag{3.27}$$

Therefore,

$$E'_i \leq SL_i \quad \forall \quad x_1 + 1 \leq i \leq y \tag{3.28}$$

□

Theorem 1. *If the route $\sigma_k = \{1, 2, ..., |\sigma_k|\}$ and the scheduling sequence $S = \{x_1, x_2, ..., x_{|\sigma_k|}\}$ provide a feasible solution for IPDSP-P, then the route σ_t and the production sequence S' = {1,2, ..., $|\sigma_k|$} will also be feasible.*

Proof. Lemma 2 proves that the production sequence S' is feasible for all jobs $i < x_1$. Then, by repeating the lemma 3 until y = $|\sigma_k|$, the theorem is proved. □

The following corollary then follows immediately.

Corollary 1. *If the route $\sigma_k = \{1, 2, ..., |\sigma_k|\}$ and the scheduling sequence $S = \{x_1, x_2, ..., x_{|\sigma_k|}\}$ provide the optimal solution for IPDSP-P, then the route σ_k and the production sequence $S' = \{1, 2, ..., |\sigma_k|\}$ will also be optimal.*

Proof. As the production sequence does not directly influence the objective function of IPDSP-P, the objective function will be the same for S and S'. Thus, S' is optimal. □

3.3 Simplifying the model formulation

Given the corollary presented in the previous section, the model from Section 3.1 can be simplified by removing all constraints related to production, i.e, constraints (3.2) to (3.4), and add the following constraints:

$$Ct_d \geq \sum_{k \in K} (pt * dem_d * x^k_{0d}) \qquad \forall d \in N \quad (3.29)$$

$$Ct_d \geq Ct_c + (pt * dem_d) - M(1 - \sum_{k \in K} x^k_{cd}) \qquad \forall \begin{cases} d \in N \\ c \in N/\{0, d\} \end{cases} \quad (3.30)$$

$$\sum_{k \in K} w^k_c - Ct_c + (pt_c * dem_c) \leq sl \qquad \forall c \in N \quad (3.31)$$

$$w^k_0 + M(1 - \sum_{d \in N} x^k_{cd}) \geq Ct_c \qquad \forall k \in K; c \in N \quad (3.32)$$

The main goal of those constraints from the new formulation is to assure the same sequence for production and distribution and that orders will be delivered before the product expires. More specifically, constraints (3.29) and (3.30) are responsible to keep the production sequence the same as the distribution sequence. Constraint (3.31) assures that the orders will be delivered while products still have some shelf life. Constraint (3.32) links the production and distribution problems.

The reason to expect that the reformulated model outperforms the original model formulation is related to the removal of the variables responsible for the production sequence (V_{cd}). Even though, in Chapter 5, we compare the performance of this reformulated model against the complete model from Section 3.1 and evaluate if there were any improvements.

Besides this new formulation that can be implemented in any commercial solver, the corollary from the previous section also favors alternative solution approaches, such as the genetic algorithm and Logic-Based Benders Decomposition, which will be detailed in the next chapter.

4 Solution approaches for IPDSP-P

As presented in the literature review, metaheuristics and decomposition methods are popular approaches to solve the integrated production-distribution problem. Moreover, as pointed out by several studies, one major challenge in integrating the production problem into the distribution problem is the difficulty of solving the resulting model (Grossmann, 2005b; Papageorgiou, 2009; Garcia; You, 2015). This challenge was also observed during the computational experiments when the IPDSP-P was implemented in a commercial solver. Therefore, in addition to the MILP model, we also considered alternative approaches: the Logic-Based Benders Decomposition (LBBD) and the Genetic Algorithm (GA). LBBD is an exact approach proposed by Hooker and Ottosson (2003) that partitions the problem into a master and subproblems. The method and the implementation of that approach for IPDSP-P are explained in Section 4.1. The other alternative approach, the Genetic Algorithm, is a metaheuristic procedure proposed by Holland (1975) that emulates an evolutionary process to search for the best solution to an optimization problem. The method and the implementation of GA for IPDSP-P are explained in Section 4.2. By partitioning the problem or emulating an evolutionary process, it is expected that LBBD and GA can find solutions for problems that a MILP model would not solve in a feasible time.

4.1 LBBD approach to solve the IPDSP-P

From the corollary presented in the previous section, we found an opportunity to create a model based on the decomposition of the integrated problem. For this reason, we came up with a model using the LBBD. Our expectations for this model are that it can solve larger instances of IPDSP-P compared to the MILP model.

As presented in Hooker and Ottosson (2003), the LBBD approach extends Benders Decomposition strategy of "learning from one's mistake" to a broader class of problems. This approach partitions the problem into a master problem and one or more subproblems. The subproblems are easier to solve, and their solution provides information to the master problem, making it easier to solve. This information, which is called Benders' cuts, can be some set of variables' values that makes the master problem infeasible (infeasibility cuts) or can be a function that provides new lower bounds on the objective value for the master problem (optimality cuts) (Kloimüllner; Raidl, 2017). Then, there is a new iteration to solve the master problem with those further cuts that create a set of new subproblems. Then, the whole process is repeated until it reaches the optimal solution.

To apply the LBBD to the IPDSP-P, we consider as the master problem a relaxed version of the model presented in Section 3.1, dropping all constraints related to the production scheduling, namely constraints (3.2)–(3.5), and solve it using a commercial MILP solver. When a solution for the distribution problem is found, we generate a candidate solution $S \subset A$, where A is the set of arcs, for the complete problem considering the same production schedule as the vehicles' routes, based on the corollary from Section 3.2, and evaluate if each tour σ_t from solution S satisfies the shelf life constraint. If the evaluation shows that the solution is infeasible, we add cuts presented in constraints (4.1), which forbid any infeasible route and the solution procedure of the master problem resumes. Algorithm 1 presents the described process.

$$\sum_{\{i,j\} \in S} x_{ijk} \leq |S| - 1 \quad \forall k \in K \tag{4.1}$$

Following the lemma presented by Chu and Xia (2004), the Benders Decomposition will finitely converge to the optimality of the original problem only if the cuts introduced by the subproblems satisfy two conditions:

1. If an infeasible solution is found in the master problem, the cut must exclude at least that solution.
2. The cut must not exclude any feasible solution for the IPDSP-P.

The cut from constraints (4.1) satisfies the conditions because it only excludes solutions that contain infeasible routes. This statement is true only because, based on the corollary from Section 3.2, it is impossible to exist any other production schedule that makes the IPDSP-P feasible when the schedule that uses the same sequence of the vehicles' routes makes the IPDSP-P infeasible.

4.2 Evolutionary approach to solve IPDSP-P

In this section, we describe the implementation of the genetic algorithm for the IPDSP-P, which consisted of applying the Split algorithm to produce feasible solutions for the problem.

4.2.1 Split algorithm for IPDSP-P

The Split algorithm was introduced by Prins (2004), and is based on the method proposed by Beasley (1983). The algorithm splits the complete route into sub-routes based on vehicles' capacity constraints from a complete tour containing all customers, which creates feasible solutions. Then, the algorithm selects the best one. Figure 9 illustrates

Algorithm 1 Logic-based Benders Decomposition for IPDSP-P

repeat
  Run a MILP model considering just the distribution part (VRP problem) of IPDSP-P until a solution is found
  if solution is found **then**
    for all subtour in solution **do**
      ProductionScheduling = route
      for all customer in route **do**
        TotalTime ← Time elapsed between the start of the production and the delivery of the customer order
        if TotalTime > Available shelf life **then**
          Create a cut to eliminate the route
        end if
      end for
    end for
  end if
until Solution - LowerBound ≤ Maximum GAP
  return OptimalSolution

this process and Algorithm 2 presents the adapted Split algorithm for IPDSP-P. The use of the Split algorithm in the genetic algorithm is presented in Algorithm 3.

Algorithm 2 Split algorithm for IPDSP-P

TSL ← Product's shelf life
$BestCost_0 = 0$
for all i in N **do** ▷ N is the set that contains all customers
  $BestCost_i = \infty$ ▷ Contains the cost of the best solution containing customers 1 to i
end for
for all i in N **do**
  load = 0; cost = 0; j = i
  repeat
    Add customer j to subtour (σ_{ji})
    Update the vehicle load
    Calculate σ_{ji} distribution cost ($Cost_{\sigma_{ji}}$)
    for all c in σ_{ji} **do**
      $Travel_c$ ← Time between the depart from depot to arrival on customer c
      E_c ← Time between the start of customer c order production and the depart from depot, considering the production schedule = σ_{ji}
      $RSL_c = TSL - E_c - Travel_c$ ▷ Remaining shelf life for customer c order
    end for
    if load < VehicleCapacity AND ($RSL_c > 0 \ \forall c \in \sigma_{ji}$) **then**
      if $Cost_{\sigma_{ji-1}} + Cost_{\sigma_{ji}} < BestCost_j$ **then**
        Sol_j ← Tour containing σ_{ji-1} and σ_{ji} ▷ best tour from 1 to j
        $BestCost_j = Cost_{\sigma_{ji-1}} + Cost_{\sigma_{ji}}$
      end if
      j = j+1
    end if
  until ($j > size(N)$) or (σ_{ji} is not feasible)
end for
ProductionScheduling ← The same order in Sol_n
  return Sol_n ▷ Selected tour, which contains all customers
  return ProductionScheduling

Prins (2004) focuses on variations of the Vehicle Routing Problem (VRP) such as distance-constrained VRP (DVRP) and Vehicle fleet mix problem (VFMP). To apply this

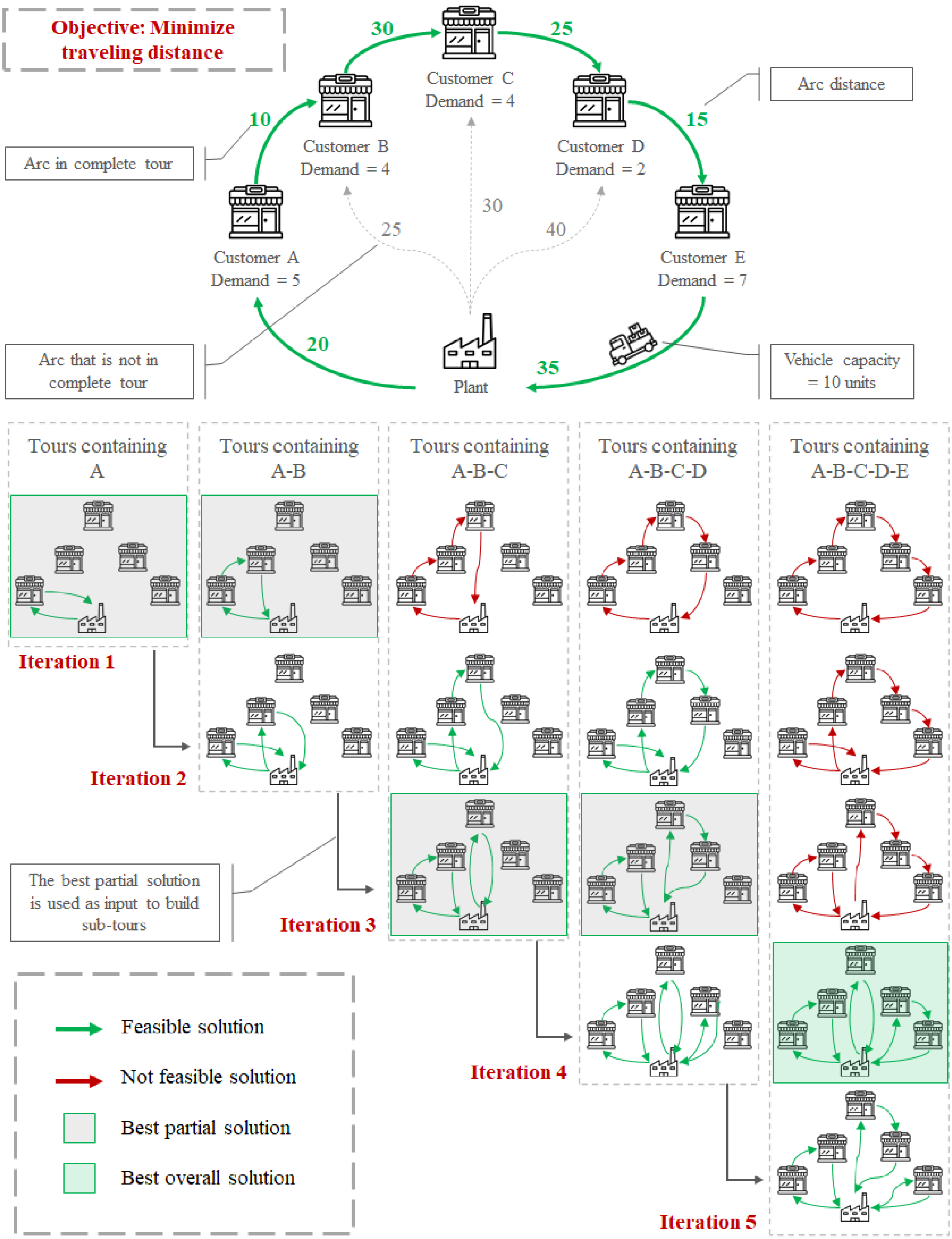


Figure 9 – A graphical example of Split algorithm [a]

[a] Icons made by Monkik and Surang from flaticon.com

method for perishable products, an additional procedure had to be included to ensure that products still have some shelf life remaining when delivered to apply this method for IPDSP-P (lines 12-17 in Algorithm 2).

Referring to the production component of the problem, we propose that customers' orders must be scheduled in the same order as sub-tours, based on the corollary presented in Section 3.2. For example, consider an IPDSP-P with three vehicles $K = (k_1, k_2, k_3)$, and each vehicle k departs from the depot at time W_0^k, where $W_0^{k_1} < W_0^{k_2} < W_0^{k_3}$. If vehicle k_1 serves customers a, b and c in that order, i.e., $\sigma_{k_1} = (a, b, c)$; and $\sigma_{k_2} = (d)$ and $\sigma_{k_3} = (e, f)$, then the production schedule sequence will be seq = (a,b,c,d,e,f).

4.2.2 Genetic algorithm for IPDSP-P

The genetic algorithm is a family of models based on the population evolution process. Its implementation starts with a population of random chromosomes. Those chromosomes are data structures that represent a solution to the target problem. The evolution process occurs by combining those chromosomes to produce new (and possibly better) solutions. This combination process is called crossover. Another form to obtain new solutions is through random changes in an existing chromosome called mutation. This process runs until a user-defined limit of generations is achieved.

Considering our particular problem, a chromosome (σ) is a complete tour containing all customers, and each customer is a gene. When a chromosome is generated, the Split algorithm, presented in Section 4.2.1, is employed to create feasible sub-tours (σ_j), and consequently, a feasible solution for IPDSP-P. The fitness value is given by the total distribution cost, as stated in Equation 3.1 in Section 3.1. Figure 10 provides an example of a chromosome and its solution obtained from the Split Algorithm

The first-generation chromosomes are randomly generated. For the following generations, the roulette wheel selection process based on relative fitness selects a set of parents. Each couple of chromosomes generates two children by a single-point crossover operator based on a crossover probability. Every child has a chance, given by mutation probability, to suffer mutation, performed by the SWAP algorithm. We used two stopping criteria: 1 - a fixed number of generations and; 2 - the number of iterations without improvement. Figure 11 ilustrates the crossover and mutation operators. Algorithm 3 provides the procedure, and Table 11 summarizes the operators used in this study.

Operator	**Value**
Parents selection operator	Weighted roulette wheel
Number of children per couple	2
Crossover operator	Single-point crossover
Mutation operator	SWAP Algorithm

Table 11 – Selected operators for the genetic algorithm

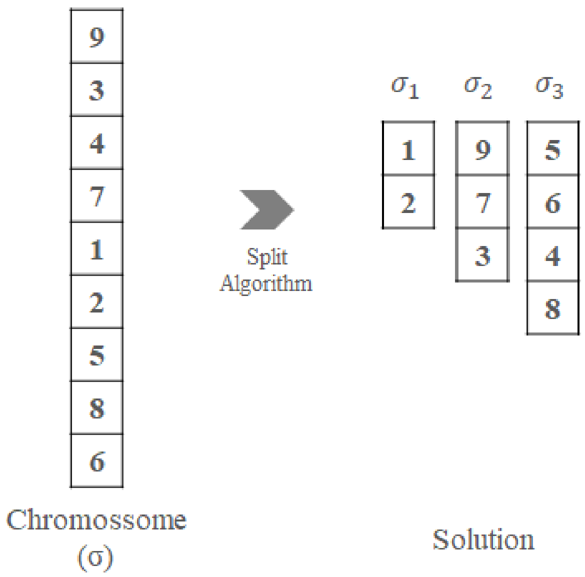


Figure 10 – A graphical example of one chromosome and its solution for a problem with nine customers/orders

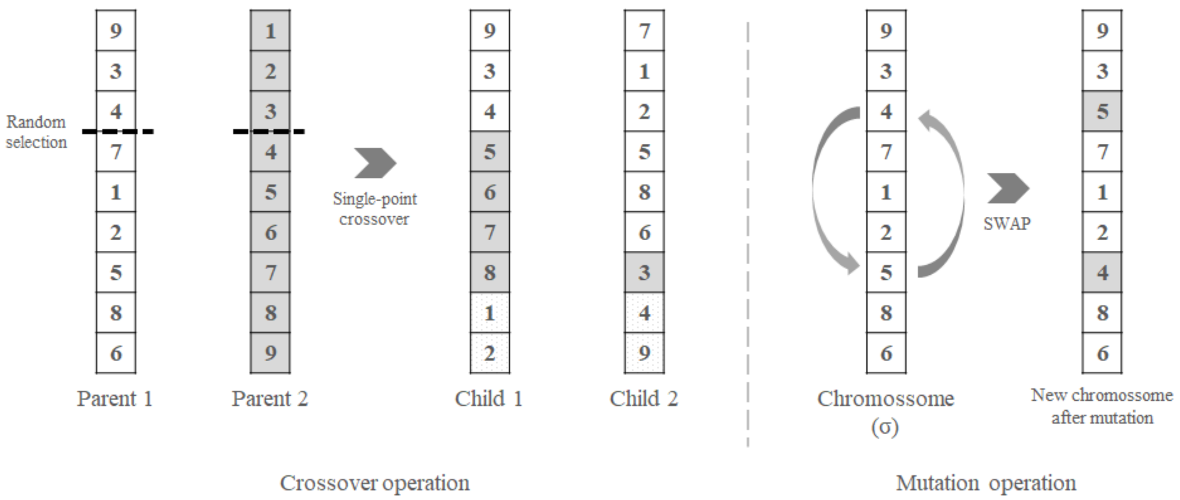


Figure 11 – Illustration of the crossover and the mutation operations used in this study

Algorithm 3 Pseudo-code for Genetic algorithm

```
PopulationSize = 0
BestSolution = ∞
while PopulationSize < MaxPopulation do                    ▷ Generate initial population
    σ ← random tour containing all customers
    PopulationSize = PopulationSize + 1
end while
repeat
    for all σ in Population do                                ▷ Find feasible solutions
        Find a feasible sub-tour using Split algorithm
        Calculate fitness value (Total distribution cost)
    end for

    BestGen ← Best fitness value of generation                    ▷ Evaluate solutions
    if BestGen is better than BestSolution then
        BestSolution ← BestGen
    end if

    PopulationSize = 0                                        ▷ Create next generation
    for i = 1 to MaxPopulation do
        Select parents through Roulette Wheel process
    end for
    for all Parent do
        Perform crossover considering the crossover probability
        if Crossover was performed then
            PopulationSize = PopulationSize + 1
            Perform mutation using SWAP algorithm considering the mutation probability
        end if
    end for
    while PopulationSize < MaxPopulation do
        Copy the best chromosomes from previous generation
        PopulationSize = PopulationSize + 1
    end while
until Limit for number of generations or iterations without improvement is reached
    return BestSolution
```

5 Computational Experiments

This chapter presents the design and results of the computational experiments designed to understand if the shelf life does actually influence the model's expected performance to solve the IPDSP-P. The experiments were conducted by implementing the four solution approaches presented previously: 1 - The MILP model from Section 3.1 that contains all constraints to solve IPDSP-P, which will be referred to in this chapter as "MILP-Full", 2 - The simplified MILP model from Section 3.3 that does not contain production sequence variables (MILP-Distribution); 3 - The Logic-Based Benders Decomposition model (LBBD) from Section 4.1; 4 - Genetic Algorithm (GA) from Section 4.2. Those readers who are interested can access data from the instances and the experiments results in the following GitHub repository: <https://github.com/HerculesDantas/dissertation_data>

The remainder of this chapter is organized as follows. In Section 5.1, the parameters for the computational experiments are presented. As we are analyzing the influence of shelf life, we need to be able to distinguish a short from long shelf life. Since we could not find any good suggestion in the literature, this study proposes a metric called Normalized Shelf Life (NSL). This metric is explained in Section 5.2. The parameters for the genetic algorithm, selected by the IRACE package (López-Ibáñez et al., 2016), are presented in Section 5.3. In Section 5.4, we describe how the solution approaches were implemented and the results obtained from the computational experiments. Finally, discussions on the results are presented in Section 5.5.

5.1 Data generation

To achieve the aim of this paper, random instances were generated by the following steps: Firstly, we have set the number of customers and the NSL, which is the metric we developed to describe perishability in an operational context (see Section 5.2). The other parameters were randomly generated by the same procedure used in (Amorim et al., 2013), details of which are presented in Table 12.

Since several studies consider 5, 10, and 15 customers to perform the computational experiments for the integrated production and distribution problem (LI et al., 2016; Noroozi et al., 2018; Belo-Filho; Amorim; Almada-Lobo, 2015), we also chose to use these customer numbers for our computational experiments. We also included instances with 7 customers to provide a better understanding of the problem. The NSL values were chosen based on the analysis presented in Section 5.2. Then, five random instances were generated for each combination of the number of customers and Normalized Shelf Life, resulting in a total of 60 different instances.

Symbol	Parameter	Generation method
K	Number of vehicles	Equal to N (number of customers)
dem_c	Customer's demand	75% of demand follows a uniform distribution in the interval U[40,60] and 25% is set to 0
pt_c	Production time (unit)	1
$CapV$	Vehicle capacity	$0.5 * \sum_c dem_c$
c,d	Node locations	Customers were positioned randomly in the x-y plane from (0,0) to (100,100) and the plant is located at position (50,50)
tt_{cd}	Travel times	Travel times were determined by the Euclidean distance between nodes
ct_{cd}	Variable costs	Variable costs are the same as the travel times
$\overline{ft}$	Fixed cost	250 for each vehicle used

Table 12 – Procedure to generate the instances

We also wanted to simulate a real planning environment where there is a time limitation to obtain a good operational plan. Therefore, for each instance we tested our models considering several limits that seem adequate to an operational planning process. Table 13 summarizes the chosen instances' parameters.

Number of customers	5, 7, 10, 15
Execution time limits	30, 60, 300 and 1800 seconds
Normalized shelf life	short = 1.55 long = 4.56 very long = 10

Table 13 – Instances' parameters

5.2 Normalized shelf life definition

The shelf life of a perishable product may vary from hours to weeks. In fact, the shelf life in operational planning is relative and it is possible that a product that expires in two days can be considered less perishable than a product that expires in a week. For example, considering that the former product is delivered across the neighborhood and the second is delivered all over the state, passing through several distribution centers, the product that expires in two days may be considered less perishable in an operational planning process. Therefore, time is not sufficient to characterize the shelf life length in operational planning.

To come up with a standard metric, we tested metrics such as the ratio between shelf life and the planning horizon, and the ratio between shelf life and the time of an average route, i.e., the average trip time, but we discarded both options because the variation of the planning horizon should not affect the perishability degree of the product and it should not be related to a decision variable of the problem.

Thus, we propose the following way to calculate the Normalized Shelf Life:

$$NSL = \frac{ShelfLife}{AvgTT} \tag{5.1}$$

The AvgTT is the average travel time from the depot (or plant) to customers. This is a good alternative to calculate the Normalized Shelf Life because the travel time is a model's parameter and it shows how long a product can be stored in the most favorable case, which is when the product goes directly from the depot to the customer.

The equation (5.1) fits situations where the shelf life starts to count after the production finishes, however there are cases when the shelf life starts to count right after the production starts (Amorim et al., 2013). In these cases, we must also consider the average processing time (AvgPT), as shown below:

$$NSL = \frac{ShelfLife}{AvgTT + AvgPT} \tag{5.2}$$

To validate these metrics and to understand what a short shelf life is and a long shelf life is, the equations (5.1) and (5.2) were tested on several works in the literature, considering models containing either the vehicle routing or the direct shipment for the distribution component of the problem. The procedure to convert the shelf life to the Normalized Shelf Life for each paper is described in the following paragraphs. Table 14 summarizes the obtained values after applying the equations. Except for (Wang et al., 2017), all analyzed papers considered more than one perishability degree. This helped to understand what literature considers a short or long shelf life.

The models proposed by Amorim, Gunther and Almada-Lobo (2012), LI et al. (2016), Li et al. (2020), Seyedhosseini and Ghoreyshi (2014), Coelho and Laporte (2014) considered a direct shipment delivery, and there was no delay between the decision and the delivery of the product. Thus, the AvgTT in those cases were set as 1.

To calculate the Normalized Shelf Lives for Amorim et al. (2013), Marandi and Zegordi (2017), we generated a random instance following the instructions in the paper and calculated the shelf life, the AvgTT, and AvgPT based on the generated instance.

In Albrecht and Steinrucke (2018), the authors analyzed the supply chain planning for a perishable product. In this problem, the product departs from the supplier stage and may pass through up to 3 distribution stages before arriving at the customer stage. The supplier stage comprises five supplier sites, and each distribution stage is composed of

Reference	Short shelf life	Long shelf life	AvgTT	AvgPT	Short Normalized Shelf Life	Long Normalized Shelf Life
(Amorim et al., 2013)	112.5	187.5	39	37.5	1.47	2.45
(Amorim; Gunther; Almada-Lobo, 2012)	2	8	1	-	2	8
(LI et al., 2016)	1	5	1	-	1	5
(Li et al., 2020)	1	5	1	-	1	5
(Albrecht; Steinrucke, 2018)	4	6	5.27	-	0.76*	1.13*
(Marandi; Zegordi, 2017)	96.92	241.53	69.80	-	1.39	3.46
(Seyedhosseini; Ghoreyshi, 2014)	2	3	1	-	2	3
(Wang et al., 2017)	28	28	0.75	-	37.48*	37.48*
(Coelho; Laporte, 2014)	2	5	1	-	2	5
		Average without outliers			**1.55**	**4.56**

* Outliers

Table 14 – Normalized shelf life for several models in literature

three warehouses and the customer stage comprises five customer sites. To calculate the AvgTT, the average travel time between each stage was calculated and every possible route was tested, for example, a route where the product is delivered directly from supplier to customer or routes passing across one, two or three distribution stages before arriving at the customer. The AvgTT was defined as the average travel time between all these routes.

In Wang et al. (2017), the authors provided the depot location and all customers' locations as well. Thus, the AvgTT was calculated by dividing the average Euclidean distance between the depot and customers by the vehicle's average velocity.

The short and long Normalized Shelf Lives used in computational experiments (1.55 and 4.56, respectively) were obtained from the average of Normalized Shelf Lives calculated from the literature, not considering the values from the references Albrecht and Steinrucke (2018) and Wang et al. (2017). Albrecht and Steinrucke (2018) was not considered because in their problem, the delivery options had to be analyzed route by route and not as an average, much so the short shelf life is lower than the AvgTT. The reason why Wang et al. (2017) was not considered because the Normalized Shelf Life of this work was an outlier compared to the others. We also added a very long shelf life (Normalized Shelf Life = 10) to support our analysis.

5.3 Parameter tuning for genetic algorithm

The selection of the genetic algorithm parameters may highly influence the performance of the genetic algorithm. Due to this, the parameters have to be tuned. In this study, we used the IRACE package (López-Ibáñez et al., 2016), which is an automatic procedure that selects the best set of parameters for a given experiment.

We generated a set of random instances of different sizes using the same procedure presented in Section 5.1, and the package selected the best parameters. These parameters are presented in Table 15.

Parameter	**Value**
MaxPopulation	790 chromosomes
Stopping criterion	87 generations
Crossover probability	73.86%
Mutation probability	01.27%

Table 15 – Parameters selected by IRACE package for Genetic algorithm

5.4 Computational Results

The LBBD model and the MILP models (MILP-Full and MILP-Distribution) were implemented with CPLEX 20.1 and were run on a PC with an Intel Core i5 8265U @ 1.6GHz 1.8GHz CPU and 8.0 GB of RAM. The genetic algorithm, implemented in Python, was run in the same PC, and each instance was run 30 times to assure statistical relevance of the results.

5.4.1 Comparison between solution approaches

Since there were several instances for which the exact models could not find a feasible solution and the genetic algorithm found feasible solutions for every instance, we decided to present the results of the exact models separated from the genetic algorithm results. This visualization provides a better understanding of how the shelf life impacts the expected performance of the exact solution methods.

For the exact approaches, i.e., the MILP-Full, MILP-Distribution and the LBBD model, we computed how many times they could find and prove the found solution was optimal, how many times they found a solution but without proving the optimality and how many times they found no solution. This result is presented in Table 16. As we are interested in the impact of the shelf life in those models, we grouped the data by the Normalized Shelf Life and time limit. Thus, for each Normalized Shelf Life and each time limit, 20 instances were solved (5 random instances * 4 different number of customers).

MILP-Full				
Normalized Shelf Life	Time Limit	No solution found	Solution found	Optimality proven solution
1.55	30	10	5	5
	60	10	5	5
	300	10	5	5
	1800	10	4	6
4.56	30	5	6	9
	60	3	7	10
	300	3	7	10
	1800	3	7	10
10	30		10	10
	60		10	10
	300		10	10
	1800		8	12
MILP-Destribution				
Normalized Shelf Life	Time Limit	No solution found	Solution found	Optimality proven solution
1.55	30	9	1	10
	60	9	1	10
	300	9	1	10
	1800	9	1	10
4.56	30		10	10
	60		10	10
	300		10	10
	1800		9	11
10	30		10	10
	60		10	10
	300		7	13
	1800		7	13
Logic-Based Benders Decomposition				
Normalized Shelf Life	Time Limit	No solution found	Solution found	Optimality proven solution
1.55	30	10	5	5
	60	10	2	8
	300	6	4	10
	1800	5	5	10
4.56	30		14	6
	60		13	7
	300		10	10
	1800		10	10
10	30		15	5
	60		13	7
	300		10	10
	1800		10	10

Table 16 – Exact models' results categorized by solution status grouped by Normalized Shelf Life and time limit

Number of customers	Normalized Shelf Life	GAP (%) 30s	60s	300s	1800s
5	1.55	0.00	0.00	0.00	0.00
	4.56	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00
7	1.55	29.01	29.01	29.01	29.01
	4.56	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00
10	1.55	87.51	87.51	87.51	87.51
	4.56	26.33	26.32	26.30	26.30
	10	21.62	21.62	21.61	21.61
15	1.55	78.50	78.50	78.50	78.50
	4.56	21.11	20.95	20.67	20.47
	10	20.65	20.46	20.26	20.06

Table 17 – Results for the genetic algorithm

Regarding the genetic algorithm, we computed the gap between the average solution from all runs and the instance lower bound provided by CPLEX after running the solver for 3600s. Equation 5.3 shows how the GAP was calculated, and Table 17 summarizes the results.

$$GAP = \left| \frac{Avg(GA\_runs)}{CPLEX\_Lower\_Bound} - 1 \right| \tag{5.3}$$

One impact of shelf life on the genetic algorithm is that it converged faster to a single solution when the shelf life was shorter. Figure 12 shows a boxplot for one of the instances with 15 customers that illustrate this finding. Other instances with the same number of customers had a similar behavior. This effect is not clear on instances with fewer customers because they had a fast convergence for the different shelf lives.

Finally, we provide a performance comparison among all models developed in this study. To perform this test, we evaluated all the previous tests to obtain the best result among all solution methods for each of the 60 instances. Then, we ran the models one more time and registered the runtime each model took to achieve the best result. As the genetic algorithm has a stochastic component, we ran it 30 times for each instance and computed the best and the worst runtime. Moreover, the runtime was limited to 3600s for all models. The results of the performance test are presented in Figure 13.

5.4.2 Effect of the shelf life on solving IPDSP-P

Another aspect that was analyzed in our study was the effect of the shelf life on the capacity to find a feasible or optimal solution when solving the IPDSP-P. This analysis was conducted by running 15 different random instances of IPDSP-P for each tested level of the Normalized Shelf Life. In those experiments, the focus was on the influence of the

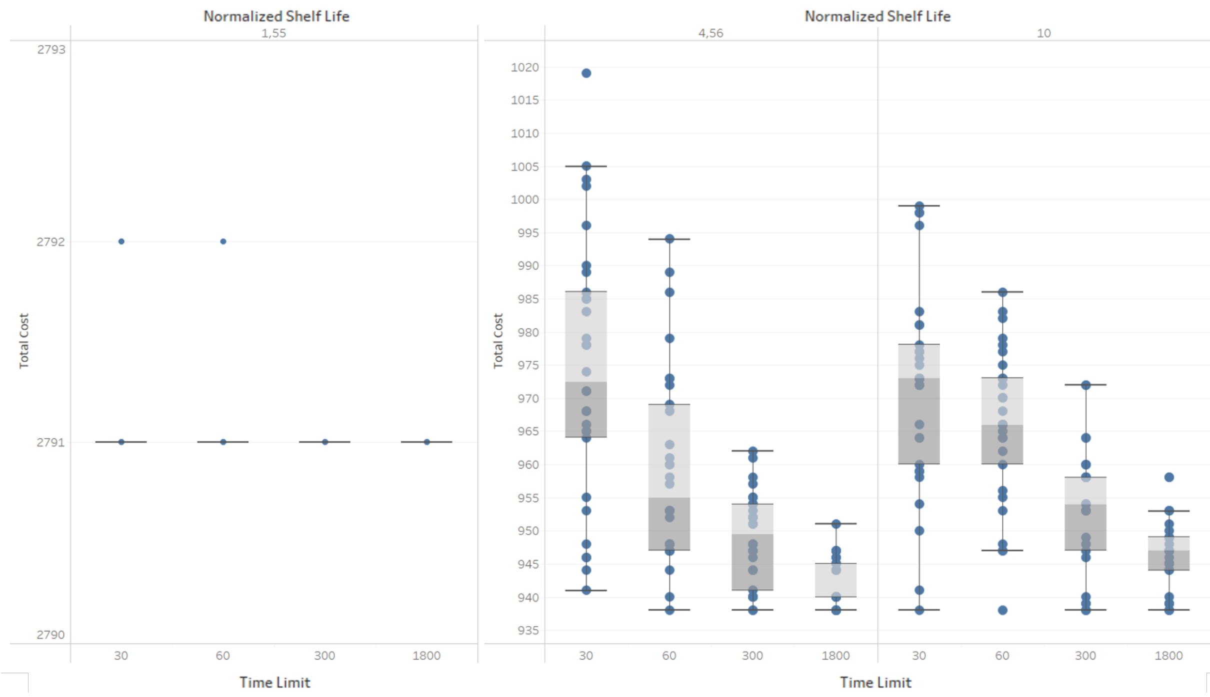


Figure 12 – Boxplot of random instance "1" with 15 customers for genetic algorithm computational experiment

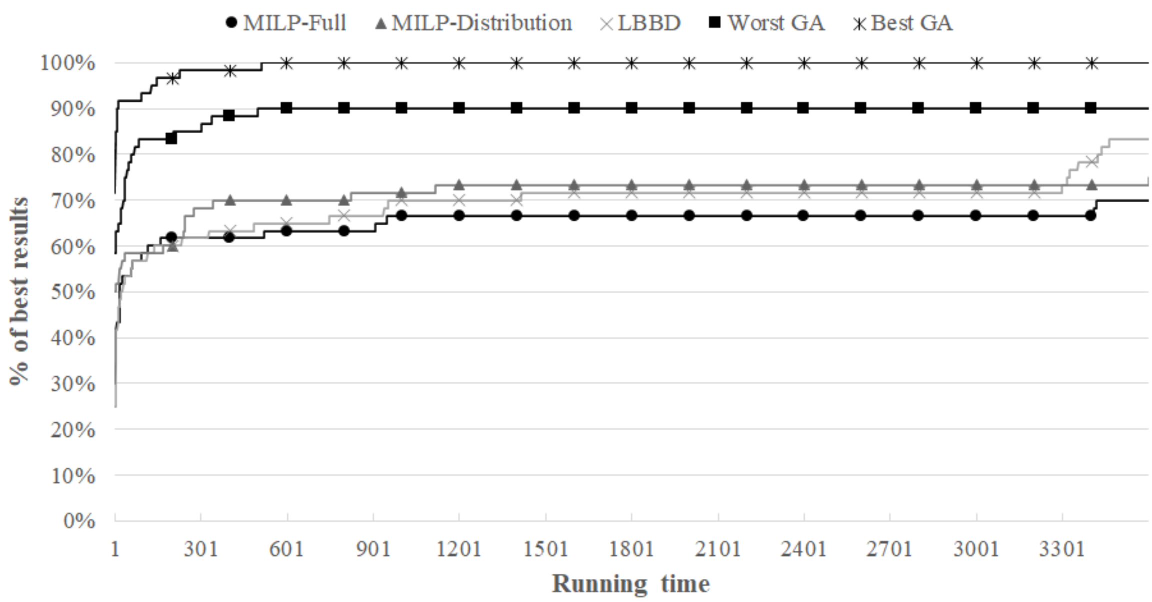


Figure 13 – % of instances that each solution method finds the best solution as runtime increases

shelf life and not how fast the several solution approaches could convert to the optimal solution. Thus, a fixed time limit of 1800 seconds was considered and only instances with 10 customers. That number of customers was chosen because, based on the previous experiments, this size was neither too small to make optimal solutions easy to be found, nor too big, so the solver could not find any feasible solution. Regarding the levels of Shelf Life, two scenarios were tested:

- Scenario 1 - This scenario consisted of varying the Normalized Shelf Life in a range between the short (1.55) and the very long shelf life (10) from experiments in the previous section;

- Scenario 2 - This scenario consisted of varying the Normalized Shelf Life in a broader range, i.e., between 4.56 and 65.55. This new scenario was motivated by the question if there is a level of Normalized Shelf Life that the implementation of MILP-Distribution reaches the same results as the implementation of a classic Capacitated Vehicle Routing Problem (CVRP).

All instances were solved using the MILP-Distribution implemented in CPLEX. This method was selected because the gap is a metric provided by the commercial solver, and the implementation of this model had a better performance when compared to the other two exact solution approaches. The details of this computational experiment are summarized in Table 18.

Solution method	MILP-Distribution in CPLEX
Number of customers	10
Number of random instances	15
Execution time limits	1800 seconds
Normalized shelf life	Scenario 1 = 1.55 to 10.1 Scenario 2 = 4.56 to 65.55

Table 18 – Details of experiments to evaluate the Shelf Life

Figure 14 shows the percentage of instances classified in three categories: proven optimal solution, feasible solution found, or no solution found for scenario 1. Another result from that scenario is presented in Figure 15. This figure shows the mean gap from the commercial solver for each Normalized Shelf Life level. A similar analysis for scenario 2 is presented in Figure 16. In this visualization, we also included two baseline levels: the GAP for the MILP-Distribution when the Normalized Shelf Life is large (9999) and the GAP for the "pure" CVRP problem. After all, without the shelf life constraint, the production sequence becomes irrelevant.

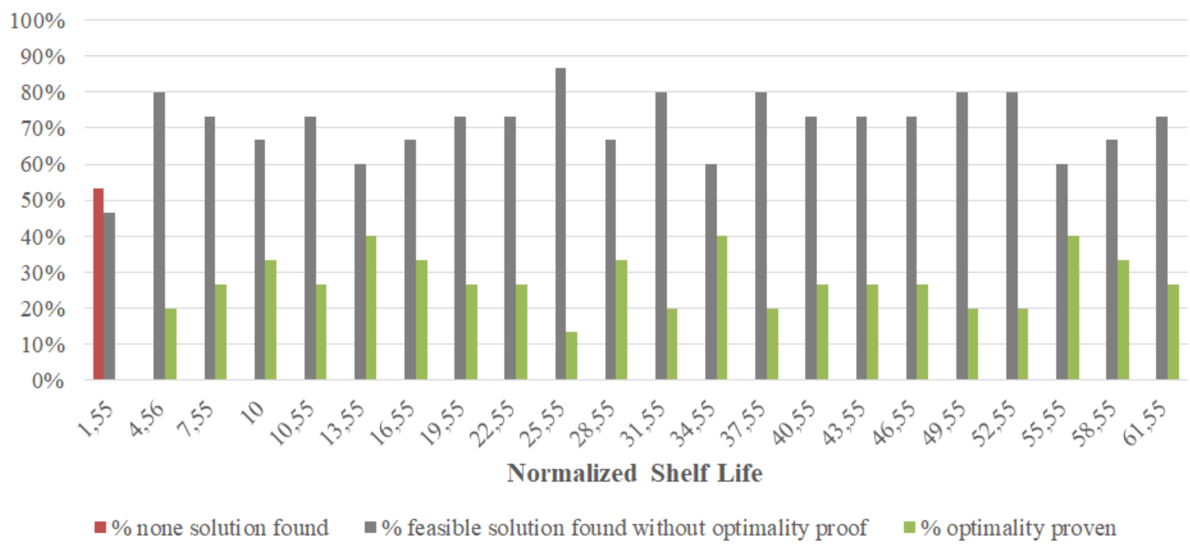


Figure 14 – The proportion of solution status for the 15 tested instances as the Normalized Shelf Life levels increase

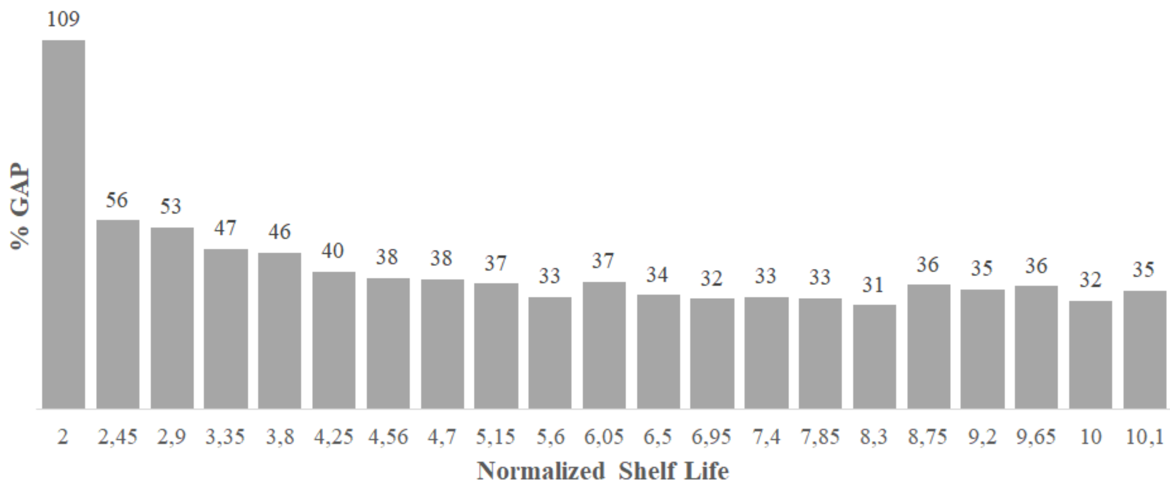


Figure 15 – Evolution of % gap, obtained from commercial solver, for Normalized Shelf Lives in a range between 2 and 10.1

5.5 Discussions

Although several studies have shown that the integrated approach leverages the results for products with shorter shelf lives (Amorim; Gunther; Almada-Lobo, 2012; Farahani; Grunow; Gunther, 2012), Table 16 also shows that a shorter shelf life makes the IPDSP-P harder to solve using exact approaches. This conclusion is based on the increase in the number of instances for which no solution was found as the shelf life becomes shorter, which happens for both the MILP and LBBD models.

The same conclusion cannot be drawn for the genetic algorithm. Although the GAPs become higher as the shelf life gets shorter (Table 17), we also noticed that the algorithm converged faster when the shelf life was shorter (Figure 12). This can indicate

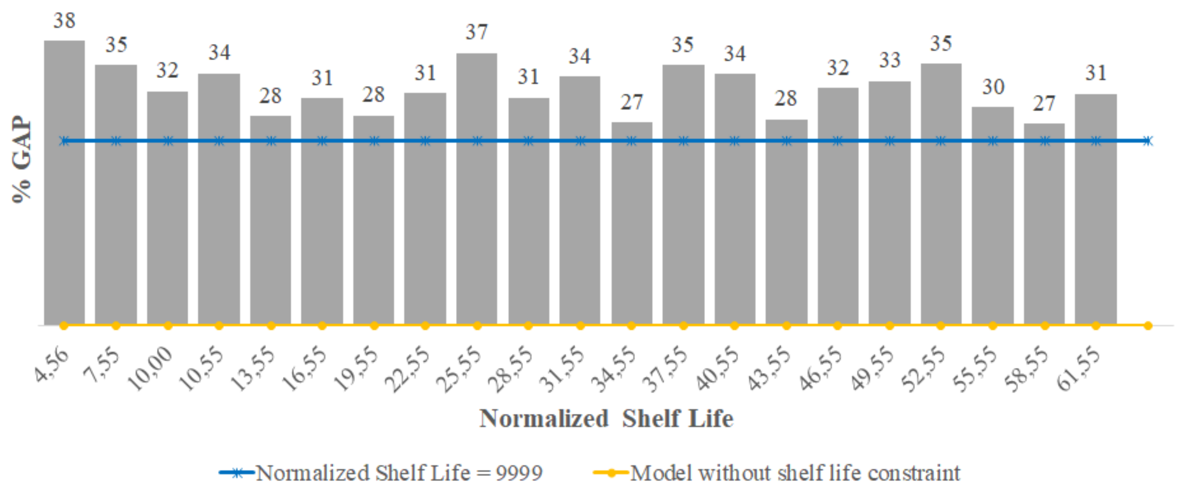


Figure 16 – Evolution of % gap, obtained from commercial solver, for Normalized Shelf Lives in a range between 4.56 and 65.55

one of two things:

1. As can be seen in Table 12, the number of vehicles is not a constraint of the problem. Therefore, the Split Algorithm will always find a feasible solution. However, the solutions may be converging to a local optimum, which explains why the GAP is higher for the shorter shelf lives.

2. Since the shorter shelf life makes it harder to find a feasible solution by means of an exact approach, the lower bound found in CPLEX may be very unrealistic for the short life situation.

The explanation for the difficulty to find a feasible solution using exact models when the shelf life is very short may be due to the lower number of feasible solutions, and any small change in the solution of a relaxed problem can violate the shelf life constraint, which makes the problem infeasible. This may be the same reason why the genetic converges so fast to a single solution. Therefore, solution methods that have good mechanisms to avoid the solutions to get stuck in local optima can be good options when solving the IPDSP-P for a product with a short shelf life.

Therefore, the shelf life of a product seems to be a relevant variable to be taken into account when deciding between an integrated approach, or a decoupled one, or even an iterative procedure.

When we compare the exact models and the genetic algorithm, considering both the short and long runtime, the genetic algorithm found more best results than the other models, as seen in Figure 13. This result may be explained by the difficulty for the exact models to find even a feasible solution for instances where the shelf life is short and for bigger instances. On the other hand, the Split Algorithm always provides a feasible solution for the problem studied in this paper, which benefits the genetic algorithm.

Comparing the exact methods: for most of the experiments, the MILP-Distribution model outperformed the other methods. However, the LBBD model had an improvement of performance when the solver was kept running for longer times, i.e., approximately 3300 seconds. This improvement was not seen in the other approaches, and the functioning of the LBBD approach may explain that improvement. The LBBD breaks the problem into a master problem and subproblems, which may cause many infeasible solutions to be considered at the beginning of the execution. However, as the model runs for more time, the subproblems eliminate many infeasible solutions, and the master problem's solution space resembles more the IPDSP-P 's solution space. However, the master problem is still a CVRP, which is already a difficult problem to solve. This may explain why the genetic algorithm outperforms the LBBD model even when we leave the models running for a long time.

Regarding the experiments that were designed to evaluate the effect of the shelf life on solving the IPDSP-P, we verified an improvement in the mean gap, and the number of optimal solutions as the Normalized Shelf Life increased from 1.55 to 4.56. From that level, the visual analysis could not show any substantial improvement in those metrics. Even when the shelf life was a large number, the solver could not find the same gap between the MILP-Distribution and the CVRP. We expected a similar result because when the shelf life is large, the shelf life constraint should become useless, and without that constraint, the production component of the problem should become irrelevant, which would transform the problem into the classic CVRP. However, this result tells us that the constraints that connect the production problem to the distribution problem continue to influence the search process for solutions even when the shelf life is large.

6 Conclusions

This research investigates how the shelf life length impacts the solving process of the integrated production-distribution scheduling problem (IPDSP-P). We considered a single machine environment that produces one perishable product that must be delivered by homogenous vehicles. There are two decisions regarding the routing strategy: the allocation of client orders to vehicles and the delivery sequence of each route. This paper proposes and implements a MILP model, a Logic-based benders decomposition model, and a genetic algorithm to solve this problem. Several computational experiments were conducted by varying the number of customers, the Normalized Shelf Life, and the time available to run the model.

The Normalized Shelf Life is a metric proposed to unify the various definitions of shelf life found in the literature into a single comparable measure. Therefore, this metric may be valuable in future studies about operational planning for perishable products.

Our experiments showed that when the Normalized Shelf Life is short, the number of instances for which both MILP and LBBD models could not find a feasible solution was higher when compared to the instances with a long or very long Normalized Shelf Life. This result contributes to the conclusion that the shelf life influences the difficulty of solving the IPDSP-P by exact models. Regarding the genetic algorithm, our experiments showed that the GAP between the lower bound obtained for the objective function and the genetic algorithm solution was higher for the short shelf life instances. However, we also found that the solutions converge to a single solution faster, in those cases. This result may indicate that the genetic algorithm got stuck in a local optimum.

This work contributed to the literature by discussing an important variable for the integrated production and distribution planning of perishable products: the shelf life. We provided a common metric that can be useful to distinguish a short from long shelf life. We evaluated the influence of shelf life on the expected performance for several solution methods to solve the integrated production and distribution scheduling problem. We also showed that is possible to simplify the problem by considering the same production scheduling as the vehicles' routing and developed an exact model and a genetic algorithm based on this result.

However, there are still several opportunities for future research. For example, one hypothesis for why the short shelf life makes the IPDSP-P harder to solve is that even minor adjustments in a solution may violate the shelf life constraint, which makes the problem infeasible. This may lead to the conclusion that the feasible solutions are more scattered over the solution space. Therefore, we believe that there is a research opportunity in combining linear programming to heuristics (or metaheuristics) to either obtain a favorable initial solution or assist the exploration of the search space. Moreover, we believe that an MIP-Heuristic could be derived by dynamically adjusting the normalized shelf life, and thus allowing the solver to enhance its response. Another research direction is to analyze the influence of shelf life on more complex scenarios (or even empirical situations).

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