

# A Critical Review of MIP Models for Lot-Sizing and Scheduling in the Beverage Industry

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## 1. Introduction

The nonalcoholic beverage industry, typically involved with the production of soft drinks, fruit-based beverages, flavored waters, teas, and yogurts, has become a major global industry over the last century. By way of illustration, the global market for ‘Global Soft Drink & Bottled Water’ is expected to reach an estimated \$211.7 billion in 2024 (IBIS World, 2024). The competitiveness of this industry depends to a certain extent on the intertwined relationship between an effective response to market trends to grasp consumers’ overall expectations (Gupta et al., 2023), and an efficient production planning and control that helps optimize the usage of production resources and execute production plans and schedules that meet demands and deadlines (Toscano et al., 2019; Guimarães et al., 2012).

The challenges that may hinder efficient production planning and control activities are mostly related to the inherent characteristics of the beverage sector, including the wide variety of products, seasonality, and high demand variability (Jamalnia et al., 2017). These challenges also include – to a certain extent – typical barriers associated with production planning per se, such as limited resources (machinery, available time, and inputs), and specifics of the production processes, such as time-consuming/costly setups and changeovers, synchronization between production stages, raw material perishability, maintenance schedules, and others (Stefansdottir et al., 2017; Toscano et al., 2020a). Due to these complex factors, decision-making support tools that standardize, automate, and optimize production plans are exceptionally useful in the food and beverage industry (Bo et al., 2021).

In beverage production planning and scheduling (BPPS) and other similar production processes, decision-making primarily involves beverage production lot-sizing and scheduling. The former refers to determining which beverages to produce, including the quantities and time periods within a finite, multi-period planning horizon, whereas the latter concerns the sequencing and scheduling of those beverage production lots during each period of the planning horizon – specifically, when these lots will be produced and in which order. There is a well-recognized trade-off between lot-sizing and scheduling decisions. For example, minimizing scheduling-related costs may lead to the production of larger lots of a smaller variety of items per period, aimed at reducing the number of setups,

which is likely to generate high opportunity costs (inventory) for perishable products (Lee and Lee, 2023; Ferreira et al., 2012). Conversely, producing smaller lots of a greater variety of items can help reduce inventory and backlogging levels, but this strategy increases the number of product setups. This tradeoff can be adequately evaluated when lot-sizing and scheduling decisions are made simultaneously (Rohaninejad et al., 2023; Ferreira et al., 2010, 2009). Indeed, integrating these decisions may help to obtain better production plans which, in turn, optimize the usage of resources and balance decisions at different levels to achieve the best result at the lowest possible cost. Unsurprisingly, this approach therefore boosts the overall competitiveness of the organization.

For this reason, the scientific community has conducted an impressive number of studies over the past 20 years proposing mathematical models for integrated decisions on production lot-sizing and scheduling (L&S). These models have been applied to numerous types of production processes, including beverage production. Some review studies of L&S problems have been reported in the literature, including Copil et al. (2017); Díaz-Madroñero et al. (2014); Buschkühl et al. (2010); Jans and Degraeve (2008); Zhu and Wilhelm (2006); Karimi et al. (2003); Drexl and Kimms (1997); Potts and Van Wassenhove (1992). In particular, the related scientific literature on beverage production processes (for example, Ferreira et al. (2012); Kopanos et al. (2012); Baldo et al. (2014); Toscano et al. (2020a) and some references therein) has shown that their corresponding production planning and scheduling problems have specific characteristics that distinguish them from more general production processes. Therefore, addressing them requires the development of tailored mathematical models that cannot be straightforwardly adapted or extended from existing mathematical L&S models

Although the importance of these models has been acknowledged by scholars over the past decades, to the best of our knowledge, existing literature has neither focused on the intricacies of lot-sizing and scheduling within typical beverage manufacturing processes, nor has it discussed and analyzed the specificities of different beverage production processes from the perspective of their applicability to real-life situations. This makes each problem and its corresponding mathematical model unique, despite sharing some similarities with problems from other sectors. Thus, the main goal of this literature review is threefold

- To provide a comprehensive overview of mathematical models developed for L&S problems in the beverage industry. For this purpose, we identify the distinguishing characteristics of beverage production and scheduling problems, such as the existence of two main production stages, synchronization aspects between these stages, and perishability issues primarily related to the liquids prepared in the tanks. These characteristics have not been explored in any existing literature review of L&S models thus far.
- To propose a novel and extended classification of such models that accommodates the specific and important characteristics of beverage production processes, as aforementioned. This classification can help decision-makers choose the most appropriate approach for their problems.
- To contribute to the scientific literature in the broader field of L&S, not only with a comprehensive and up-to-date review of 48 mathematical models and their applicability to real-world problems faced by beverage businesses but also by pointing out opportunities for new avenues of research and practical problems that have been little explored so far.

This literature review is organized as follows: Section 2 explains the main characteristics of beverage production processes, focusing on the production of soft drinks, fruit-based beverages, beer, and yogurt. Section 3 then presents a classification of each process, focusing on setups and cleaning activities. Section 4 provides an overview of the origins and lineage of BPPS models within a timeline that spans the last three decades. Section 5 analyzes the corresponding mathematical models applied to beverage production processes in terms of objective functions, decisions and constraints. The main solution methods and data used to address these problems are also presented. Section 6 identifies some literature gaps, recent trends, and research opportunities in BPPS problems. Finally, Section 7 offers some concluding remarks.

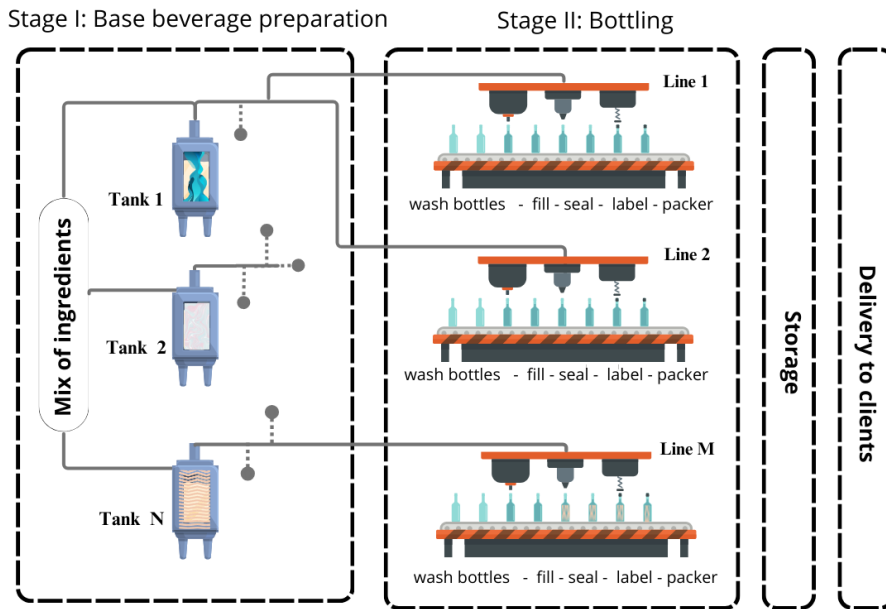
## **2. Production planning & process in the beverage industry**

Production planning in the beverage industry involves complex and intertwined decisions due to its convoluted manufacturing processes. This section describes the production planning and processes for soft drinks, fruit-based beverages, beers, and yogurts, based on the seminal papers of

Toledo et al. (2009); Ferreira et al. (2009) (soft drinks), Pagliarussi (2013); Toscano et al. (2019) (juices), Baldo et al. (2014) (beers), and Lütke et al. (2005); Doganis and Sarimveis (2008a) (yogurt). The full list of papers used throughout the text for understanding and analyzing each type of beverage production is given in Table A.5 of Appendix A.

The production of each beverage consists of a set of similar steps and may involve certain particularities, such as the need for maturation or periodic setups, which make production planning challenging. The general beverage production process flow is outlined in Figure 1, where we can see that beverage preparation and filling are the main and distinct production stages.

Fig. 1: General beverage production process.



In the stage I (base beverage preparation), a set of ingredients is mixed in tanks to produce a base beverage. Each tank has a minimum and a maximum capacity given in liters, which defines the size of each base beverage lot. The minimum capacity is related to the minimum quantity of liquid that should be prepared in tanks to assure liquid homogeneity (Ferreira et al., 2009). For each type of beverage, there are specific processes that transform the base beverage into the final product, which is then directed to filling machines. In some cases, the final beverage is obtained at

the end of the stage I (i.e., the base beverage is already the final product), e.g., in the production of some fruit-based juices. In the stage II (bottling), the bottles are cleaned, filled with the final beverage, packaged, and then stored before being distributed to clients. A final item is defined by the beverage flavor and bottle size. Beverages with the same flavor but bottled in different-sized bottles are considered different items. For example, Ale beer bottled in a can and in a glass bottle is considered to be two different items.

As shown in Figure 1, a tank can usually supply several lines simultaneously; in this case, the final item produced on each line will have the same flavor as the base beverage prepared in the tank. Figure 1 shows tank 1 connected to lines 1 and 2, with each line connected to only one tank at a time. It is worth noting that, as beverage products are perishable, it is necessary to perform a Clean-In-Place (CIP) procedure on all production process equipment before starting the production of a new flavor item to avoid cross-contamination. In several processes, CIP must be performed in the tanks after each base beverage preparation, even if the subsequent lot has the same flavor as the previous one. This implies that with each base beverage preparation, the tank must be empty and clean. Therefore, changeover times in tanks are composed of CIP and the preparation times of the beverage base. The changeover time in the stage II occurs between the production of different items and comprises CIP and mechanical machinery adjustment times.

In beverage production processes, synchronization between production stages is essential to avoid waiting times. Notice that because changeovers are usually costly and time-consuming, synchronization between stages becomes even more important. Consequently, production plans that do not consider this synchronization may require adjustments, such as the inclusion of changeover times, that impact the capacity of time periods and may even render the plan infeasible in practice.

In most processes, while a filling line is bottling the beverage, a tank containing the base beverage must be connected to this filling line; during this operation, the tank is unavailable for preparing the next beverage. Waiting times occur when the beverage from stage I is ready to be bottled, but there is no available filling line (due to a CIP, for example), or if the stage II line is ready to start bottling but there is no base beverage prepared for bottling.

The general production planning process can be described as follows: We must determine the

lot sizes of the base beverage and the final product to be produced in each tank and on each bottling line for each time period over a finite planning horizon. This ensures that the quantity of the base beverage is sufficient to meet the demand for the final product. In most processes, due to perishability, there is typically no inventory held for the base beverage. However, maintaining inventory and allowing backlogging for the final product is permitted. It is also necessary to assign specific tanks to specific lines, i.e., determining from which tank a line receives the base beverage.

To ensure a feasible production plan, it is necessary to consider that both tanks and bottling lines are capacitated, and that the planning horizon is finite, divided into periods with limited available work time. Additionally, mandatory changeovers at each stage, which are typically time-consuming operations, affect the per-period capacity. These changeovers may be sequence-dependent, based on the type of beverage and the stage, necessitating a defined sequencing of items. As beverages are a type of food, there is a requirement to clean the machinery at the beginning of each production period. Furthermore, both production stages must be synchronized to prevent waiting times on the filling lines.

Depending on the type of beverage to be produced, certain particularities of the production process may affect the feasibility of production planning. For example, such particularities could include buffer tanks between stages, maturation times, temporal cleanings, among others. For this reason, the subsequent sections detail the production processes of soft drinks, fruit-based juices, beer, and yogurt, respectively.

### *2.1. Soft drinks production*

The main difference between the soft drinks process and the general process is related to how the ingredients are mixed to obtain the final beverage. The base beverage, a syrup, is converted into the final liquid (soft drink) in the stage II (filling lines) by adding water and carbon dioxide. In soft drink production processes, both the syrup preparation and bottling steps may involve CIP procedures, which can range from a few minutes to several hours depending on the beverage and/or the process. The changeover times and costs are sequence-dependent at both stages. While the filling lines are bottling, the tanks must be connected to the line, rendering them unavailable

for starting the preparation of the next syrup lot. Moreover, the tanks should be completely empty and clean before any syrup preparation begins. This changeover time is necessary for each lot preparation, even if the next lot is the same type as a previous syrup lot. Therefore, while the maximum lot size of the stage II is restricted by the per-period capacity of the stage I, the maximum lot size is tank capacity, given in liters. Additionally, for technical reasons, there is a minimum lot size (in liters) required to produce syrup in tanks; see Toledo et al. (2009) for further details.

A changeover in the filling lines occurs only when a new item must be produced, which involves either introducing a new flavor that necessitates cleaning the line, or changing to a new package size that requires technical adjustments to the equipment. In stage I, changeover times consist of cleaning and any syrup preparation, whereas in stage II, changeover times are the sum of cleaning and technical adjustments to the equipment

Waiting times may occur when the syrup is ready in the tank, but there is no available line to bottle it immediately. Conversely, if the bottling line is available, i.e., cleaned and adjusted, but the syrup is still being prepared in the tank, then the bottling line must wait for the tank. Once the syrup is ready, it must be bottled without delay. Thus, waiting times occur in tanks while the tank is empty, before syrup preparation, not after the syrup is ready in the tank (Ferreira et al., 2009).

## *2.2. Fruit-based juice production*

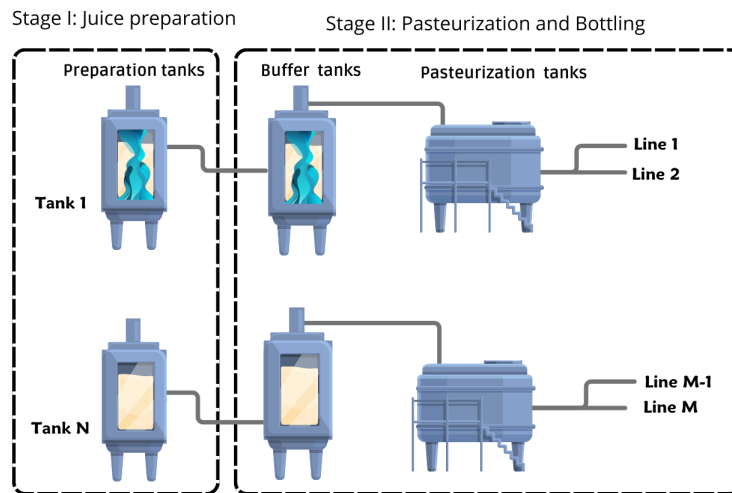
In the fruit-based juice production process, ingredients – consisting of powders and concentrated juices that arrive at the factory ready to be mixed – are combined in the preparation tanks during the stage I, resulting in the final juice (Pagliarussi et al., 2017). This process contrasts with that of fresh-fruit juices, which require extensive preliminary fruit treatment. In the fresh-fruit juice process, stages include fruit reception, washing, milling, pulp-pomace separation, pulp maceration, extraction, evaporation, filtration, packing, and cold storage (Blanco et al., 2020).

Before bottling, fruit-based juices are quickly transferred from a preparation tank to a buffer tank, allowing the preparation tank to become empty so that a new lot of syrup can be started.

Figure 2 illustrates the buffer tanks positioned between the preparation tank and the filling lines.

The buffer tanks are connected to pasteurizers, which extend the shelf life of the beverage. After pasteurization, the liquid is then bottled, sealed, packaged, and stored. It is important to note that the final beverage is ready after the stage I. Therefore, the stage II in the fruit-based production process involves buffer tanks, pasteurizers, and filling machines working together as a single unit to bottle the final beverage. These machines are even cleaned and set up together (Toscano et al., 2020a). Figure 2 illustrates the connections among these machines.

Fig. 2: Fruit-based beverage production process. Based on (Toscano et al., 2020a).



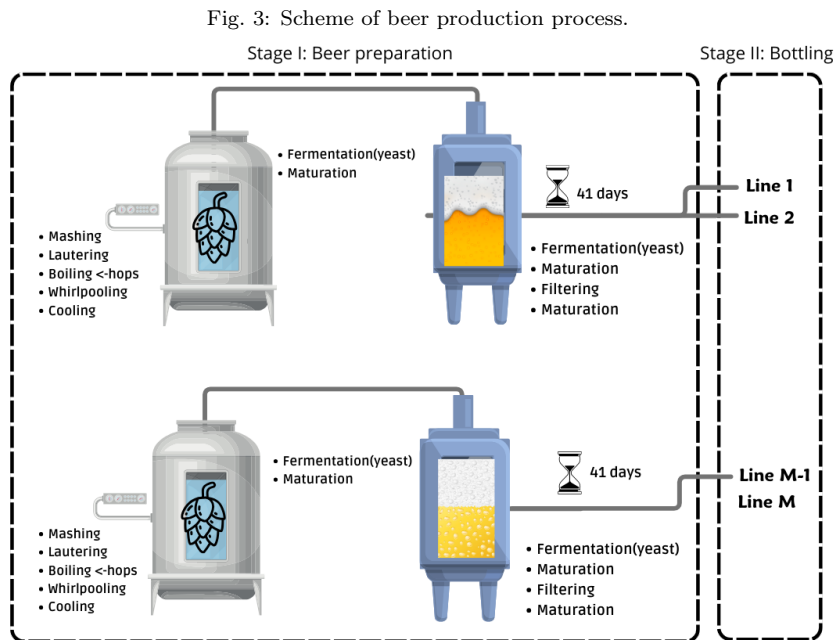
In fruit-based juice production processes, two types of CIP must be considered in the production planning. These often include changeover time and temporal cleaning time. Changeover is similar to that in other beverage production processes and is generally sequence-dependent in both stages. However, in some factories, the changeover in stage II can be considered not sequence-dependent (Toscano et al., 2020a)

A unique aspect of fruit-based juice production is the requirement for mandatory temporal cleaning. This cleaning occurs because a CIP in the production processes involving pasteurization has an expiration time. Specifically, it means that 24 hours after the last CIP in a tank, production must be halted for cleaning. In stage II, the requirement for temporal cleaning extends to approximately 48 hours. Cleaning of the tank or line is mandatory, regardless of production times,

machine stoppages, waiting periods, etc. Temporal cleaning is necessary only between lots of the same item, as cleaning routinely occurs at every changeover Toscano et al. (2020a).

### 2.3. Beer production

Beer is made from four main ingredients: water, malt, hops, and yeast. Different beer categories are obtained depending on the raw materials used and the required processing time at each production step. Despite these variations, all categories of beer essentially undergo the same processing steps (Baldo et al., 2017). As previously mentioned, beer manufacturing involves two production stages: base beverage preparation and bottling. It is worth noting that even the same type of beer can yield different final products depending on the bottling container, such as bottles, cans, or barrels. Figure 3 illustrates the sequence of steps in each stage.



Yeast converts sugar and other substances into alcohol during the fermentation/maturation step. The sugar that feeds the yeast is derived from wort, which is added along with hops. Wort is primarily produced from barley that has been transformed into malt. This transformation occurs in the stage I, where the liquid beer, as a beverage, is produced. In various processes, this stage lasts approximately 41 days, the majority of which is dedicated to fermentation and maturation.

The process begins with milled grain being mixed with water and subjected to a heating process (known as the mashing step). Afterwards, in the lautering step, the filtered mixture is boiled and mixed with hops to become wort (Georgiadis et al., 2021).

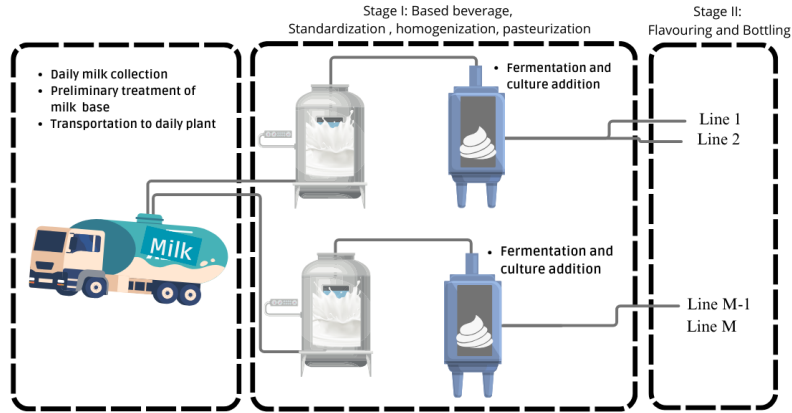
When proceeding to the fermentation step, the wort must be cooled before adding the yeast. This sequence of steps takes a few hours and yields a 'green beer.' This green beer is then transferred into fermentation tanks to begin the fermentation phase. Through the action of the yeast, fermentation occurs naturally in the tank and may take several days to complete. At the end of this phase, the beverage is decanted in the same tank; the yeast is separated, and the maturation phase starts. After maturation, the beverage must be filtered and carbonated before bottling. If the filling line is not available for bottling, there are two options: either the matured beverage can remain in the fermentation and maturation tanks, or it can be filtered and stored in buffer tanks while awaiting bottling. However, once filtered, the beer must be bottled within a few days.

In the stage II, the packaging is cleaned to accommodate the beverage destined for bottling, specifically glass bottles, aluminum cans, and barrels. The beverage is then sealed, pasteurized, labeled, and packed. Unlike in fruit-based juice production, where only the liquid is pasteurized, pasteurization here occurs in sealed containers

#### *2.4. Yogurt production*

Essentially, yogurt is made when microorganisms (bacteria) break down milk. The lactose sugars in the milk are converted into lactic acid by these bacteria during the fermentation process. After the milk becomes sufficiently acidic, the caseins (proteins found in milk) begin to clump together, changing the consistency of the milk and forming yogurt. The yogurt production process is presented in Figure 4.

Fig. 4: Scheme of yogurt production process.



In the automated process, raw milk is received in a silo and subjected to homogenization before being sent for fermentation. It is quite common to mix a milk-based ingredient, such as milk powder or skimmed milk, with the raw milk to aid in the homogenization process. The raw milk is then pre-heated in a plate heat exchanger and concentrated in an evaporator. Afterwards, the milk is pressed through a homogenizer, heated again, cooled, and then transferred to the fermentation tanks (Kopanov et al., 2010).

Fermentation is a natural process. Fermented foods are those in which microorganisms have transformed relatively complex substances into simpler ones. In stirred yogurt, starter cultures are added to milk in order to incubate it. The coagulum is then cooled in a plate cooler and stored in intermediate tanks. Although it is during the fermentation step that milk actually becomes yogurt, it is only in the flavoring and packaging step that it receives its flavor and becomes a ready-to-drink beverage. To flavor the yogurt, fruit and other ingredients are added. Before the cups are filled, they must be sterilized and cleaned

The changeover in this process is sequence-dependent. As in all food production processes, the machines that come into contact with the yogurt must be sterilized and cleaned. However, when the next flavor of yogurt is very different from the current filling flavor, the remaining yogurt in the machinery is lost, which makes the changeover more expensive. It can then be seen that the changeover costs are not proportional to the changeover times because, in some sequences, there is a loss of product that generates a cost greater than simply the downtime for cleaning.

In many factories, after being sealed with the lid, the yogurt undergoes a quarantine period where it is cooled to a temperature below 10°C for 48 hours until it is released as a final product ready for consumption. In some processes, the beverage often undergoes another stage of fermentation (temperature homogenization) after packaging that aims to increase the shelf life of the final product. For more details, please refer to Lütke et al. (2005), Doganis and Sarimveis (2008a).

Unlike the other mentioned drinks (fruit-based juices and beers), yogurt has a very short shelf life in the supermarket. Therefore, the production planning horizon considers a day as a period of time, whereas for other drinks it is typically a week. We also note that inventory is kept for only short periods of time.

### 2.5. Comparison between beverage production processes

The distinguishing characteristics of the production processes of the previous beverages are summarized in Table 1 and discussed as follows.

Table 1: Distinguishing characteristics of soft drinks, fruit-based juice, beer, and yogurt production process.

	Soft drink	Fruit-based juice	Beer	Yogurt
1 Sequence-dependent setup times/costs in stage I	✓	✓		✓
2 Proportional changeover times and costs	✓	✓	✓	
3 Final beverage obtained in stage I		✓	✓	
4 Final beverage is prepared and bottled within the same period	✓	✓		✓
5 Storage final beverage in process			✓	
6 Time periods longer than one day (e.g., weeks)	✓	✓	✓	
7 Loss of beverage in CIP				✓
8 Temporal cleanings		✓		
9 Connection between tanks and lines while bottling	✓		✓	✓

- 1. Sequence-dependent setup times/costs in stage I.** Although stage I of the beer production takes more than one month, the setup times are not sequence-dependent in stage I. On the other hand, the setup times/costs are sequence-dependent in stage II for all beverage production processes, including beer.
- 2. Proportional changeover times and costs.** Changeover costs are usually calculated based on the elapsed time to clean machines. They are evaluated as a potential loss or profit in that time interval. In the yogurt production process, beverage losses may also occur at the time of cleaning, and therefore, the changeover costs are higher than changeover times.

3. **The final beverage is obtained in stage I.** While in the juice and beer production processes the final beverage is obtained in stage I, in the soft drink and yogurt process they are not. In the case of soft drinks, carbon dioxide is added to the drink during bottling and in the yogurt case the flavoring is in the stage II.
4. **The final beverage is prepared and bottled within the same period.** All beverage, except beer, are produced to match the demand of that period. In the case of beer, the final beverage can take up to 41 days to be ready but the demands are usually given per week. This characteristic suggests that, in order to define production planning, it is necessary to consider the “future” availability of the bottling lines. Moreover, the production process is more fragile because if there is a loss of beverage in the tank for any reason, it is not possible to replace it within the same period, which can cause production backlogs.
5. **Storage final beverage in process.** In general beverages are perishable and cannot be stored for long periods while waiting for the bottling lines, but in the specific case of juice and beer, it is possible to storage the final beverage for while. In the preparation of beer before it goes through the filtering and carbonation process, it can be kept for a few days in the tanks before bottling. And in the case of juice, due the pasteurization step allows the drink to be kept for a longer period in the buffer tanks. This can make production scheduling more flexible, often avoiding waiting times.
6. **Time periods longer than one day (e.g., weeks).** In the specific case of yogurt, due to its perishability, the time period is usually given in days, differently from other drinks that are generally given in weeks.
7. **Loss of beverage in CIP.** As beverage loss can occur in yogurt production, this should be taken into account when planning the production to adequately meet the demand and adjust the changeover costs, as aforementioned.
8. **Temporal cleanings.** Only the fruit-based beverage production process requires stopping production to carry out temporal cleanings. Furthermore, also due to delays that can occur during temporal cleaning, the exact amount of time of temporal cleaning cannot be predicted or estimated by simply subtracting these values from the line/tank capacity in each period of

the planning horizon. That may affect synchronization between the production stages.

9. **Connection between tanks and lines while bottling.** Differently from the other beverages production process, in the case of juices, there is a buffer tank that allows tanks to be released to prepare new batches while the line fills the previous beverage batch. This affects the way in which waiting times are considered, as it is possible to advance to the next lot, which ends up affecting synchronization between stages.

Table 1 shows that an important aspect of beverage production is the need of performing setup and/or changeover. And because the type of setup/changeover may be unique, it is often regarded as a distinguishing characteristic of BPPS problems. Due to its importance, the next section suggests a classification framework for the different types of setups and cleaning operations based on Stefansdottir et al. (2017).

### 3. Classification according to setups and cleanings

Stefansdottir et al. (2017) present a general classification scheme for setup and cleaning operations in the process industry, such as food and beverage. The first part of their classification relies on “Changeover Class”, which is divided into three different classes: batch-dependent, time-dependent, and volume-dependent. This class is further differentiated based on its “General Characteristics” (Separability, Substitutability, Reference Point, and Flexibility), and classified according to the “Changeover Matrix” (product dependency and batch-size dependency) for times and costs. Figure 5 shows this classification scheme and Table 2 proposes the classification for each type of beverage production we are dealing with. To the best of our knowledge, this is the first attempt to classify the setups associated with beverage production processes based on this categorization. Depending on the characteristics that are used to model the lot-sizing and scheduling problem of the same production process, the classification of Stefansdottir et al. (2017) can vary for the same type of beverage. This occurs with yogurt, for example. In the production process modeled by Doganis and Sarimveis (2007) and Doganis and Sarimveis (2008a), the changeover and cleaning are classified as changeover products – type 1 and time-dependent changeover – type 1, according to

Table 2. In Kopanos et al. (2010), the batch-dependent changeovers are classified as type 2 and the time-dependent changeover as type 1, according to Table 2. Moreover, in Kopanos et al. (2011) the batch-dependent changeovers considered are classified as type 3 and 4, and the time-dependent changeover is classified as type 2, according to Table 2.

Fig. 5: Changeovers classification criteria and categories. Adapted from Stefansdottir et al. (2017).

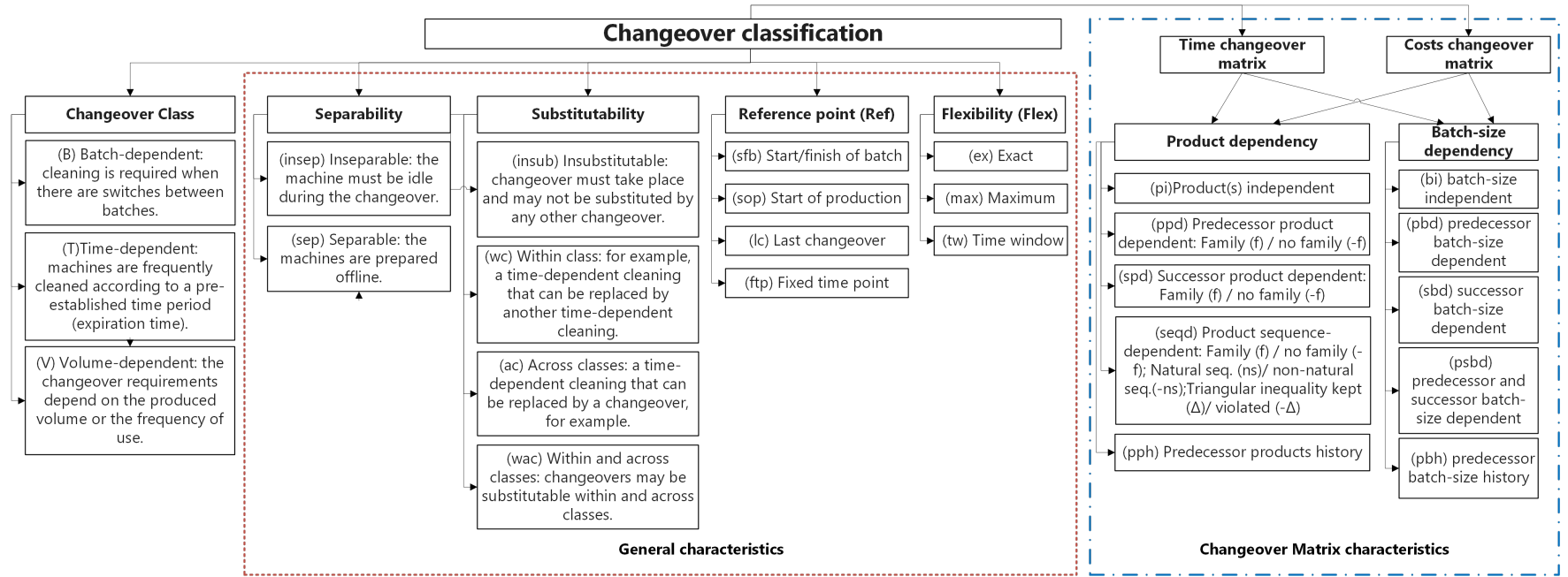


Table 2: Classification of changeovers in the literature of L&S in beverage industry.

Beverage	Process	Class	Changeover Classification		
			General Characteristic (Sep/Sub/Ref/Flex)	Changeover-matrix Time (Prod/Size)	Changeover-matrix Costs (Prod/Size)
Soft Drink	Changeover flavors (Stage I)	B	(insep/insub/sfb/ex)	$t(\text{seqd}(-f;ns;\Delta) / bi)$	$c(\text{seqd}(-f;ns;\Delta) / bi)$
	Changeover recipients (Stage II)	B	(sep/insub/sfb/ex)	$t(\text{seqd}(-f;ns;\Delta) / bi)$	$c(\text{seqd}(-f;ns;\Delta) / bi)$
Fruit-based	Changeover between items	B	(insep/insub/sfb/ex)	$t(\text{seqd}(-f;ns;\Delta) / bi)$	$c(\text{seqd}(-f;ns;d) \Delta bi)$
	Time changeover (Temporal Cleaning)	T	(insep/ac/lc/max)	$t(pi/bi)$	$c(pi/bi)$
Beer	Changeover liquid (Stage I)	B	(insep/insub/sfb/ex)	$t(pi / bi)$	$c(pi / bi)$
	Changeover recipients (Stage II)	B	(sep/insub/sfb/ex)	$t(\text{seqd}(-f;ns;\Delta) / bi)$	$c(\text{seqd}(-f;ns;\Delta) / bi)$
Yogurt	Changeover products – Type 1	B	(insep/ac/sfb/bmax)	$t(\text{seqd}(-f;ns;\Delta) / bi)$	$c(\text{seqd}(-f;ns;\Delta) / bi)$
	Time dependent changeover – Type 1	T	(insep/ac/ftp/ex)	$t(pi/bi)$	$c(-/-)$
	Changeover products – Type 2	B	(insep/ac/sfb/bmax)	$t(\text{seqd}(f;ns;\Delta) / bi)$	$c(\text{seqd}(f;ns;\Delta) / bi)$
	Changeover products – Type 3	B	(insep/insub/sfb/max)	$t(\text{seqd}(f;ns;\Delta) / bi)$	$c(\text{seqd}(f;ns;\Delta) / bi)$
	Changeover products – Type 4	B	(insep/insub/sfb/max)	$t(\text{spd}(-f)/bi)$	$c(-/-)$
	Time-dependent changeover – Type 2	T	(insep/insub/ftp/ex)	$t(pi/bi)$	$c(-/-)$

#### 4. Origins and lineage of BPPS models

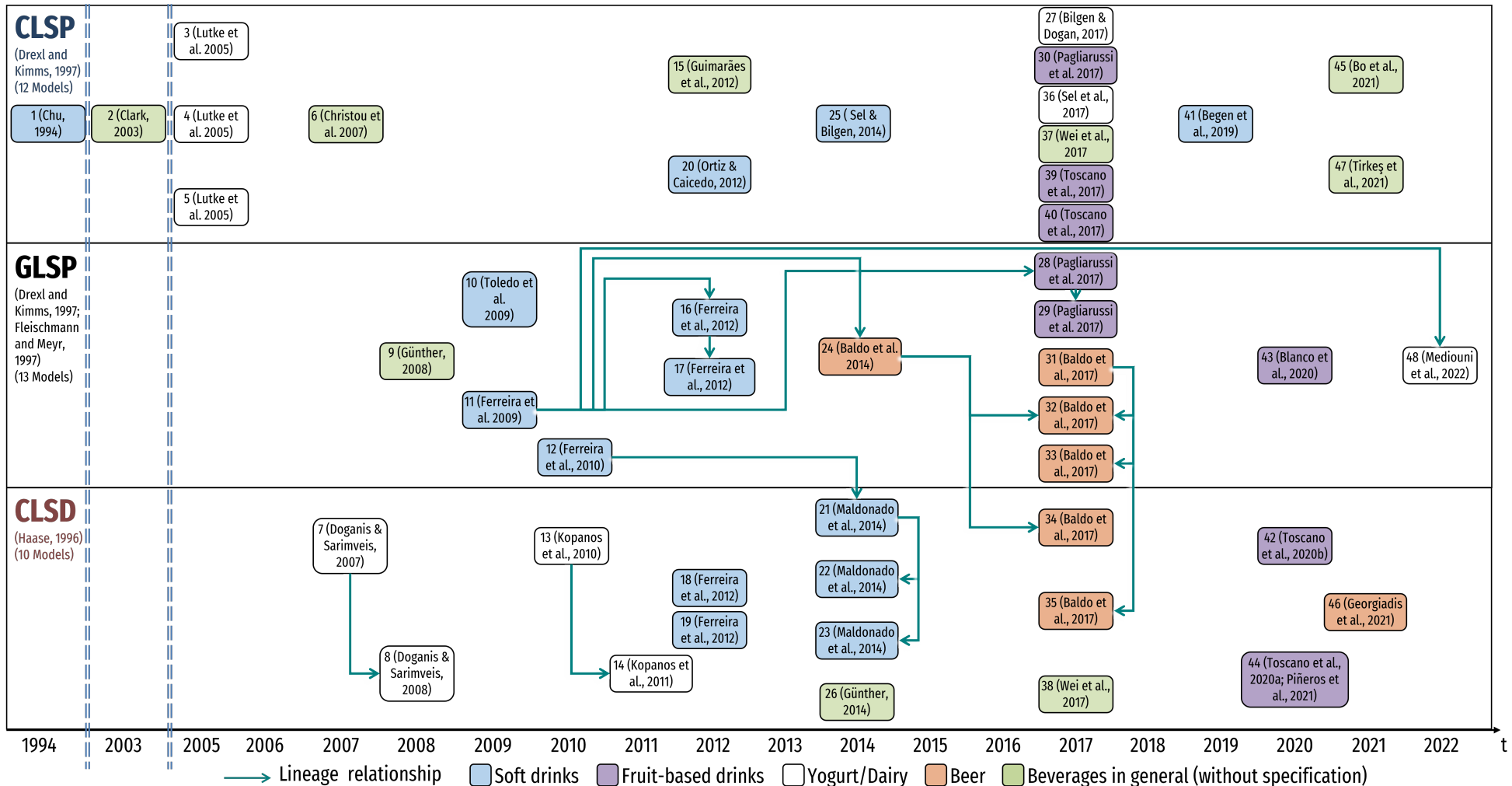
The integration of scheduling decisions into lot-sizing models can vary significantly, leading to three classical approaches: the Capacitated Lot-Sizing Problem (CLSP) (Drexl and Kimms, 1997), the General Lot-Sizing and Scheduling Problem (GLSP) (Drexl and Kimms, 1997; Fleischmann and Meyr, 1997), and the Capacitated Lot-Sizing Problem with Sequence-Dependent Setups (CLSD) (Haase, 1996). The CLSD model, in particular, modifies the CLSP to include both scheduling choices and sequence-dependent setup costs. Figure 6 illustrates the temporal evolution of these approaches and delineates the ‘lineage’ relationships between BPPS models using arrows. We have numbered the formulations from 1 to 48 and provided citations where each formulation was first developed or adopted. For instance, formulation 11 (Ferreira et al., 2009) is a GLSP-type and inspired formulations 16 (Ferreira et al., 2012), 24 (Baldo et al., 2014, 2017), 28 (Pagliarussi et al., 2017), and 48 (Mediouni et al., 2022). A summary of all 48 formulations is also provided in Table A.5 of Appendix Appendix A.

Figure 6 reveals that the three seminal models – CLSP, GLSP, and CLSD – have significantly inspired novel formulations in beverage production planning and scheduling. In total, 18 formulations are based on CLSP, 14 on GLSP, and 16 on CLSD. The adoption of the CLSD approach is relatively more recent. One reason for using CLSD-based models is the design of more ‘efficient’ formulations, which typically involve fewer variables and constraints than those based on the GLSP (Ferreira et al., 2012). However, a major drawback of CLSD-based models is that they require strategies to eliminate disconnected subtours. Among the strategies reviewed, the most common method for subtour elimination involves the use of Miller-Tucker-Zemlin (MTZ) constraints, originally proposed in Miller et al. (1960) for the Asymmetric Traveling Salesman Problem (ATSP), which have been shown to achieve shorter computational times than other ATSP-based constraints (Guimarães et al., 2014; Maldonado et al., 2014).

Although the classical models have inspired the inclusion of lot-sizing and scheduling decisions in BPPS problems, to accurately represent the specificities of beverage production, it is essential to define new decision variables and constraints that end up characterizing a new class of formulations specifically tailored for certain types of beverages. As an illustrative example, we present

a mathematical model for the production planning and scheduling of fruit-based juice (Toscano et al., 2020a) in Appendix B. Note that while constraints (B.2) to (B.11) are typically found in general lot-sizing and sequencing problems, several new constraints (B.12) to (B.37) are specifically designed to represent the fruit-based juice production process. Since the feasibility of the problem heavily depends on these constraints, we cannot regard the fruit-based juice mathematical formulation as merely an adaptation or extension of the general problem. Therefore, the next section analyzes the existing literature on BPPS mathematical models, focusing on their distinguishing characteristics (see Table 1) to further evidence the necessity of recognizing them as a distinct class of problems, separate from the classical lot-sizing and scheduling (L&S) ones.

Fig. 6: Lineage and evolution of BBPS models over time.



## 5. Overview of BPPS models and their characteristics

This section delves into the details of existing literature on BPPS problems. Unlike mainstream L&S literature, which primarily discusses the broad aspects of these formulations in general applications, this review aims to shed light on specific aspects of the beverage production process, such as scheduling decisions, setup and/or changeover times, synchronization between stages, perishability, optimization criteria, and solution methods. For this purpose, Table 3 summarizes the most specific characteristics of the 48 selected BPPS models and serves as a guideline to help decision-makers identify the most suitable approaches for their problems. Additional details on these models can be found in Table C.8 in Appendix Appendix C.

### 5.1. Scheduling decisions

As mentioned in Section 2, beverage production planning problems involve scheduling decisions in at least one of the production stages. For example, in the beer production process, these decisions occur only in stage II, while in the production processes of soft drinks, fruit-based juice, and yogurt, scheduling decisions are made in both stages. To accurately define the scheduling of production lots, it is essential to consider tank and line cleaning times, base beverage preparation times, and their associated costs. Including scheduling decisions implies defining specific variables and constraints to manage sequence-dependent times and costs, which generally leads to more complex mathematical models. In this regard, Table 3 indicates that models 10 (Ferreira et al., 2009; Toledo et al., 2014), 11 (Ferreira et al., 2010), 16-19 (Ferreira et al., 2012), 42 (Toscano et al., 2020b), 44 (Toscano et al., 2020a; Piñeros et al., 2021), and 48 (Mediouni et al., 2022) consider sequence-dependent setups in both stages. Conversely, models 37-38 (Wei et al., 2017) exclusively consider sequence-dependent setups in stage I, while models 7 (Doganis and Sarimveis, 2007), 8 (Doganis and Sarimveis, 2008b), 12 (Ferreira et al., 2010), 21-23 (Maldonado et al., 2014), 24 (Baldo et al., 2014, 2017), 31-35 (Baldo et al., 2017), 36 (Sel et al., 2017), and 46 (Georgiadis et al., 2021) include such setups only in stage II.

Table 3: Comparison between BPPS models based on specific characteristics of the beverage production process.

Model	Stp/chg - Tanks	Stp/chg - Lines	T. Mult-con.	Sync.	Perish.	Other objective function terms
1	No	No	No	No	No	Maximizes total demand satisfaction.
2	No	No	No	No	No	No
3	No	SI	No	No	Yes	Profit per sale, costs of: production, per resource use, material lost in each cleaning, overtime.
4	No	SI	No	No	Yes	Profit per sale, costs of: production, per resource use, material lost in each cleaning, overtime, non-maintenance of the setup.
5	No	SI	No	No	Yes	Profit per sale, costs of: production, per resource use, material lost in each cleaning, overtime, non-maintenance of the setup.
6	No	SI	No	-	Yes	Production costs.
7	No	SD	No	-	No	Cost of regular and overtime machine utilisation.
8	No	SD	No	No	No	Cost of regular and overtime machine utilisation.
9	SI	SI	No	-	No	Production costs, Major and minor clean-out setup costs.
10	SD	SD	Yes	Yes	No	Cost of production in the two stages, cost of syrup intermediate storage.
11	SD	SD	No	Yes	No	No
12	No	SD	No	No	No	No
13	SI	SD	No	Yes	No	Costs of operating units, unit utilization.
14	SI	SD	No	Yes	No	Fixed and variable costs of production, external production, use of units of production.
15	No	SI	No	-	No	Cost of product transfer, cost of production, cost of overtime.
16	SD	SD	No	Yes	No	No
17	SD	SD	No	Yes	No	No
18	SD	SD	No	Yes	No	No
19	SD	SD	No	Yes	No	No
20	No	No	No	-	No	Production costs.
21	No	SD	No	No	No	No
22	No	SD	No	No	No	No
23	No	SD	No	No	No	No
24	No	SD	Yes	Yes	No	No
25	SI	SI	No	-	No	Production, transportation costs.
26	SI	SI	No	-	No	Makespan minimization.
27	No	SI	No	No	Yes	Benefit per product sold, cost of resource use and cost of loss of sales.
28	No	SI	No	-	No	Benefit per product.
29	No	SI	No	-	No	No
30	No	SI	No	-	No	No
31	No	SD	Yes	No	No	No
32	No	SD	Yes	Yes	No	No
33	No	SD	Yes	Yes	No	No
34	No	SD	Yes	Yes	No	No
35	No	SD	Yes	Yes	No	No
36	No	SD	No	-	Yes	Makespan minimization
37	SD	No	No	Yes	No	Cost of transportation, loss of sale and exchange of line process type
38	SD	No	No	Yes	No	Cost of transportation, loss of sale and exchange of line process type
39	SI	No	No	No	No	Temporal cleanings costs
40	No	SI	No	No	No	Temporal cleanings costs
41	No	No	No	-	No	Costs of production and transportation.
42	SD	SD	No	No	No	Temporal cleanings costs
43	No	No	No	No	No	Costs of production, raw material, energy and natural gas, profit per flow rate of juice.
44	SD	SD	No	Yes	No	Temporal cleanings costs
45	No	SI	No	-	No	Costs of transportation to clients and external warehouses, external storage and unsatisfied demand.
46	No	SD	No	No	No	No
47	No	No	No	-	No	Benefit of product sold in companies X and Y minus costs of raw materials, labor, water and electricity.
48	SD	SD	No	Yes	Yes	No

“Stp/chg - Lines”: presence of setup/changeover in the stage I; “Stp/chg - Tanks”: presence of setup/changeover in the stage II; “SD/SI”: Sequence Dependent/Independent; “T. Mult-con.”: if there are tank multiple/simultaneous connections; “Sync.”: if there is synchronization between stages; “Perish.”: if there is perishability consideration; “Other terms in OF”: if there are other terms in the objective function rather than inventory/backlogging and setup/changeover. The symbol “-” corresponds to the models that address only one production stage.

Table 3 shows that 35 models (73%) include setup times, which reduce the available time capacity. Specifically, 16 models consider sequence-dependent setups, 14 models account for sequence-independent setups, and 5 models altogether disregard setup times. It is important to note that disregarding setup times, or assuming them to be sequence-independent, implies that the sequence of operations does not significantly affect capacity utilization. This assumption can lead to impractical production plans in real-world scenarios.

Another key observation is that, with the exception of models 37-38 (Wei et al., 2017), the models considering sequence-dependent setups in stage I also consider them in stage II. Note that scheduling decisions in stage I and stage II are interdependent; the preparation of the base beverage in stage I depends on the lot sizing and scheduling of the final product in stage II. However, if sequence dependency in stage I is not considered, changeovers between products with different flavors may result in excessive waiting times due to the necessary preparation time in the tanks. This phenomenon could also compromise the feasibility of the final production plan, rendering it impractical in real-world scenarios.

Other aspects related to capacity utilization also affect lot-scheduling decisions. As pointed out in Section 2.2, the fruit-based production process involves a special type of cleaning, often referred to in the literature as temporal cleaning. Temporal cleaning poses a particular challenge because it cannot easily be accounted for in modeling by simply reducing capacity or limiting batch size. This type of cleaning relates to the maximum time elapsed since the last cleaning and clearly impacts scheduling decisions. As evidenced in models 39-40 (Toscano et al., 2019), 42 (Toscano et al., 2020b), and 44 (Toscano et al., 2020a), incorporating temporal cleanings requires the introduction of new variables and constraints, making the models more intricate. Despite the importance of scheduling, some mathematical formulations simplify the problem to reduce computational difficulties by assuming that scheduling is performed a priori through pre-established sequences of items or between families of items. For example, a strategy seen in model 37 (Wei et al., 2017) relies on including decision variables to choose from predefined sequences of items. These predefined sequences integrate the consideration of sequence-dependent setups

In other models, the authors standardize setup times/costs and treat them as sequence-independent,

or they do not include them in the formulations at all. This is the case with models 1 (Chu, 1994), 20 (Ortiz and Caicedo, 2012), 27 (Bilgen and Dogan, 2015), 30 (Pagliarussi et al., 2017), 39-40 (Toscano et al., 2019), and 47 (Tirkeş et al., 2021). These models, which are based on the Capacitated Lot-Sizing Problem (CLSP) framework, do not take scheduling decisions into account. Therefore, since CLSP does not integrate scheduling, the efficiency of production plans obtained using this modeling paradigm depends heavily on the combination of pre-established sequences with lot-sizing. As expected, the models that disregard scheduling also overlook sequence-dependent changeover times/costs, or they require that the existence of sequence-dependent times and costs be evaluated a priori to construct the predefined sequences, if applicable.

### *5.2. Synchronization between stages*

In the beverage production process, lot-sizing decisions are closely tied to demand. Therefore, it is crucial to consider that the lot sizes of the initial stage (base beverage) depend on the lot sizes of the subsequent stage (final product). When addressing scheduling decisions, the production sequences in both stages are closely linked because the flavor of the base beverage in the tank must match the flavor of the required final product. This connection does not necessarily imply that setup and changeover times/costs are of the same type (sequence-dependent or sequence-independent); however, it emphasizes the need for synchronization between the stages. Failure to synchronize the stages can lead to issues such as the base beverage oxidizing in the tanks while waiting to be bottled, or the production line wasting capacity awaiting the readiness of the base beverage to be bottled. See Section 2 for additional details.

It is therefore necessary to define which tank supplies which line in each period. In addition, a tank can supply more than one line at the same time, a feature known as "tank multiple connections". Allowing tanks to supply multiple lines simultaneously can be beneficial for optimizing the available capacity as it helps to avoid double preparation of tanks to make separate lots in the stage II from the same flavor of the base beverage. In a general problem, the line can even switch to another tank within the same period if the lot size of the base beverage in the tank is smaller than the batch size of the line.

As mentioned above, in terms of mathematical models, this implies new sets of variables and constraints to control which tank supplies which lines for the production of which lots of final products. In addition to the large number of new variables and constraints, the variables are generally binary, which can make the model very complex. In fact, we can say that modeling synchronization between stages is the most challenging task in the mathematical modeling of beverage production processes.

To avoid dealing with synchronization, several authors adopt a simplification strategy by organizing the lines into sets and assigning each tank to a set of lines. However, this approach can result in less accurate calculations of changeover times and costs. This is the case for models 1 (Chu, 1994), 6 (Christou et al., 2007), 7 (Doganis and Sarimveis, 2007), 9 (Günther, 2008), 12 (Ferreira et al., 2010), 14 (Kopanos et al., 2011), 15 (Guimarães et al., 2012), 16-19 (Ferreira et al., 2012), 20 (Ortiz and Caicedo, 2012), 21-23 (Maldonado et al., 2014), 25 (Sel and Bilgen, 2014), 26 (Günther, 2014), 28-30 (Pagliarussi et al., 2017), 39-40 (Toscano et al., 2019), 41 (Begen et al., 2019), and 45 (Bo et al., 2021). Some of these models, such as 16-19 (Ferreira et al., 2012), require the longest preparation time between the two stages, ensuring that the changeover times and costs are not underestimated. Conversely, models 10 (Toledo et al., 2009, 2015), 11 (Ferreira et al., 2009; Toledo et al., 2014), and 48 (Mediouni et al., 2022) utilize a microperiod scale to achieve synchronization. This strategy allows for precise control of changeover times and waiting periods at each stage.

The models that explicitly allow tanks to be connected to multiple lines simultaneously are models 10 (Toledo et al., 2009, 2015) and models 24 and 31-35 (Baldo et al., 2017). Among these, the models that do not dedicate production lines to specific tanks include model 10 (Toledo et al., 2009, 2015), model 27 (Bilgen and Dogan, 2015), and models 24 and 31-35 (Baldo et al., 2017). These models represent the production processes of soft drinks and beers, respectively. Model 10 (Toledo et al., 2009, 2015) divides the planning horizon into three time scales: periods, slots, and microperiods. The microperiod scale is used to synchronize the two stages. It allows multiple connections of tanks to lines, and a line can switch to another tank at any instant within the planning horizon. Models 24 and 31-35 (Baldo et al., 2017) also allow multiple tank connections but with dedicated tank assignment. The synchronization between the stages is achieved through a

microperiod structure similar to that in model 10 (Toledo et al., 2009, 2015). During bottling, tanks are dedicated to specific lines, and it is not possible to switch a line to a different tank within the same microperiod. These models do not include considerations for perishability. This is justified because beer, unlike other beverages, has a longer shelf life.

### *5.3. Perishability and planning horizon*

Although the base beverage is usually perishable, only into 7 formulations of the studied BPPS problems the perishability are explicitly incorporated (models 3-5 (Lütke et al., 2005), 6 (Christou et al., 2007), 27 (Bilgen and Dogan, 2015), 36 (Sel et al., 2017), and 48 (Mediouni et al., 2022)) (see the sixth column of table 3). This is generally achieved by defining variables such as waiting times and storage durations, and limiting them according to the shelf life of the item.

Models 3, 4, and 5 presented in Lütke et al. (2005) integrate shelf-life issues into production planning and scheduling by including a freshness factor in the objective function and a group of constraints to specifically meet the demand, respecting the product shelf life. A similar approach presented in model 6 (Christou et al., 2007) but here the objective function penalty associated with the perishability increases exponentially as the meet of the demand gets further from the production date. Model 27 Bilgen and Çelebi (2013) incorporates the shelf life using the same strategy previously developed by Lütke et al. (2005) (models 3, 4, and 5). In model 36 (Bilgen and Dogan, 2015), to address perishability a 'random' shelf life time is determined using a probability distribution and it is used to limit the time elapsed between packaging and processing. Finally, the model 48 (Mediouni et al., 2022) differentiates between Stages I and II, and the perishability of the liquid is considered in the Stage I. There are constraints to guarantee the product's shelf life.

Other strategy to consider perishability is to define shorter planning horizons. They are defined in the scale of the final beverage shelf life. In the case of yogurt, for example, the periods are defined as one day, due to the product's shelf life being only few days. This is the case of 23 models (see Table 4 for more details).

In the rest of the models, fifteen (15) models either not consider or not even mention perishability as an issue. Most of them were designed for strategic-tactical decision and perishability is not a

critical issue, as in the case of beer production process.

A key observation is that, despite the importance of perishability, 85.41% of the models do not consider the base beverage or final product perishability explicitly in their mathematical formulations due to the complexity of the resulting model.

#### 5.4. *Objective function*

As expected, the most common objective function criterion in the beverage production process problem is the minimization of total costs, which typically include inventory, setup, changeover, and backlogging costs. These costs are chosen due to the classical optimization trade-off between lot-sizing and scheduling, as explained in Section 1. Although the service level related to meeting demand is somewhat implicitly addressed through backlogging minimization, it appears that no paper explicitly optimizes different types of service level functions (Tomazella et al., 2023), which is a promising future study direction. Only eight models have a maximization objective function related, for instance, to demand satisfaction in model 1 (Chu, 1994); six of which also include the minimization of certain costs by subtracting them from profit or pursuing a similar objective. These cases include models 3-5 (Lütke et al., 2005), 27 (Bilgen and Dogan, 2015), and 47 (Tirkeş et al., 2021).

Other cost minimization models with non-typical L&S objective functions include production costs, overtime, and transportation costs. Meanwhile, other models incorporate penalties that are less common. For example, models 3-5 (Lütke et al., 2005), 39-40 (Toscano et al., 2019), and 42 (Toscano et al., 2020b), as well as model 44 (Toscano et al., 2020a; Piñeros et al., 2021), include penalties such as loss of material during each cleaning. Model 27 (Bilgen and Dogan, 2015) and models 37-38 (Wei et al., 2017) consider penalties for lost sales. Furthermore, model 10 (Toledo et al., 2009, 2015) addresses both production costs and intermediate syrup storage costs at stage I. Other studies focus on different objectives, such as minimizing production makespan, as seen in models 9 (Günther, 2008), 26 (Günther, 2014), and 36 (Sel et al., 2017).

### 5.5. Classification

Upon reviewing the bibliographic surveys (Brahimi et al., 2006; Copil et al., 2017) concerning lot-sizing and scheduling models, we observe that the commonly employed classification criteria, such as the number of machines, periods, and items, are very general. However, for the appropriate selection of a model to be applied in real-world beverage settings, it is crucial to examine and classify their distinguishing characteristics. Therefore, we propose a novel set of classification criteria that considers these characteristics, as depicted in Figure 7. Based on these criteria, we have classified the 48 models, which are summarized in Table 4.

Fig. 7: Model classification criteria.

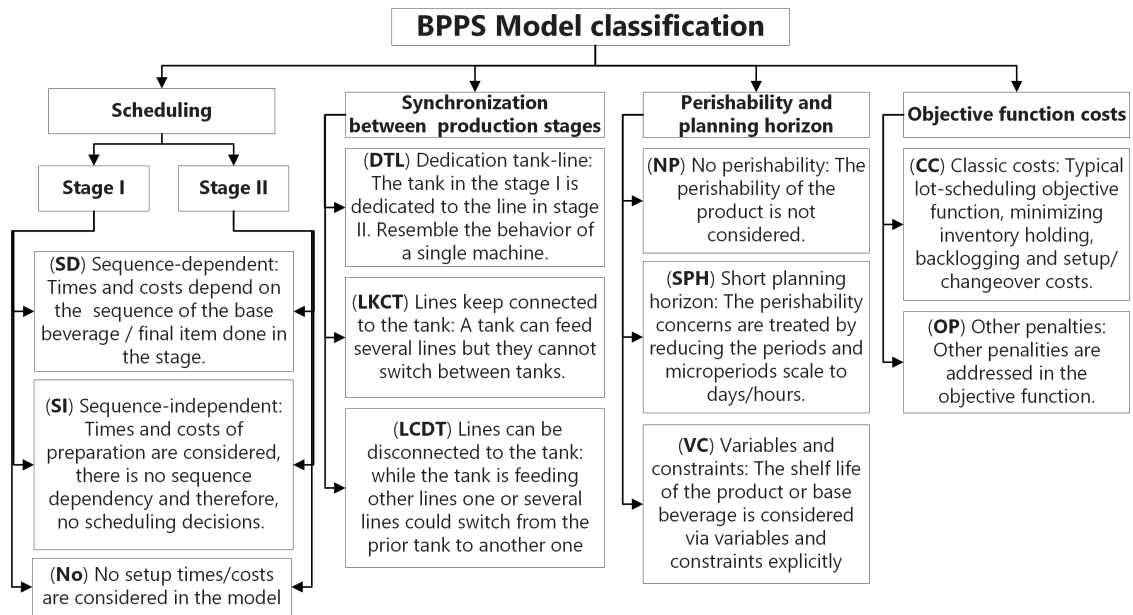


Table 4: Model classification.

Model	Classification	Model	Classification	Model	Classification
1	No/No/DTL/SPH/OP	17	SD/SD/DTL/SPH/CC	33	No/SD/LKCT/No/CC
2	No/No/DTL/SPH/CC	18	SD/SD/DTL/SPH/CC	34	No/SD/LKCT/No/CC
3	No/SI/DTL/VC/OP	19	SD/SD/DTL/SPH/CC	35	No/SD/LKCT/No/CC
4	No/SI/DTL/VC/OP	20	No/No/DTL/No/OP	36	No/SD/DTL/VC/OP
5	No/SI/DTL/VC/OP	21	No/SD/DTL/SPH/CC	37	SD/No/DTL/No/OP
6	No/SI/DTL/VC/OP	22	No/SD/DTL/SPH/CC	38	SD/No/DTL/No/OP
7	No/SD/DTL/SPH/OP	23	No/SD/DTL/SPH/CC	39	SI/No/DTL/SPH/CC
8	No/SD/DTL/SPH/OP	24	No/SD/LKCT/No/CC	40	No/SI/DTL/SPH/CC
9	No/SI/DTL/SPH/OP	25	No/SI/DTL/SPH/OP	41	No/No/DTL/No/OP
10	SD/SD/LCDT/SPH/CC	26	No/SI/DTL/No/OP	42	SD/SD/DTL/SPH/OP
11	SD/SD/DTL/SPH/CC	27	No/SI/DTL/VC/OP	43	No/No/DTL/SPH/OP
12	No/SD/DTL/SPH/CC	28	No/SI/DTL/SPH/OP	44	SD/SD/DTL/SPH/CC
13	SI/SD/DTL/SPH/OP	29	No/SI/DTL/SPH/CC	45	No/SI/DTL/No/OP
14	SI/SD/DTL/SPH/OP	30	No/SI/DTL/SPH/CC	46	No/SD/DTL/No/CC
15	No/SI/DTL/No/OP	31	No/SD/LKCT/No/CC	47	No/No/DTL/No/OP
16	SD/SD/DTL/SPH/CC	32	No/SD/LKCT/No/CC	48	SD/SD/DTL/VC/CC

### 5.6. Solution methods and data

The development of computerized systems with great processing capabilities has boosted the use of commercial optimization packages to solve lot-sizing and scheduling models. However, these models still present difficulties and challenges associated with the specificities of real-world problems, which have stimulated the development of tailored solution methods. Indeed, incorporating scheduling decisions can significantly increase the complexity of these models, thus directly affecting the performance of solution approaches, particularly exact ones. The increase in difficulty is primarily related to the computational complexity of the base models. For example, it is well-known that the Capacitated Lot Sizing Problem (CLSP) model is NP-hard (Bitran and Yanasse, 1992). Turing reduction theory (Cormen et al., 1990; Garey and Johnson, 1979) demonstrates that models based on foundational models – such as CLSD and GLSP, which in turn are based on CLSP – and featuring additional characteristics, are at least as complex as their underlying basic models. Therefore, the models analyzed in this review are at least NP-hard.

This has encouraged scholars to develop specific exact or heuristic methods for solving large-scale, real instances of L&S problems based on beverage companies (Piñeros et al., 2021). These methods typically involve heuristics, including MIP-based heuristics or hybrid approaches that combine exact and approximate methods, such as memetic algorithms or hybrid evolutionary ap-

proaches. Table D.9 presents the solution methods used in each study and highlights the type of dataset employed to construct the problem instances: real-world data, data based on real-world situations, or randomly generated synthetic data. Datasets marked as ‘based on real-world data’ contain both actual and synthetic elements. For example, such an instance may include all data derived from a real situation (e.g., capacity and demand parameters), except for cost parameters, which may have to be estimated due to confidentiality concerns. The ‘Solution approach’ column displays the techniques used to solve the models, such as MIP-based heuristics or commercial solvers. The specific commercial solver used to address subproblems or linear relaxations within a given method is also indicated in parentheses. Notably, two papers (referenced in lines 11 and 47 of Table D.9) do not specify how the mathematical models were solved.

Interestingly, most approaches fall within the first two categories of datasets – either real-world or based on real-world data – which renders the insights more practical and applicable. This also serves as strong evidence of the problem’s relevance in practical settings. As indicated in Table D.9, many authors begin by using optimization packages to solve the models. Some studies have explored fine-tuning strategies to identify better configurations for the solver (CPLEX), such as turning cutting planes on or off, and/or implementing heuristics (Ferreira et al., 2009; Pagliarussi et al., 2017). Not surprisingly, the solver alone often fails to deliver high-quality solutions within a reasonable amount of time (Baldo et al., 2014; Ferreira et al., 2009; Toledo et al., 2009). For this reason, approximate methods are commonly proposed, including two of the most popular approaches: relax-and-fix (R&F) and fix-and-optimize (F&O) heuristics, along with local search and evolutionary metaheuristics, aimed at reducing optimality gaps and/or computational times. Other studies (Clark, 2003; Toledo et al., 2014) present hybrid algorithms that combine mathematical programming with heuristics and metaheuristics for a more efficient search within the solution space. For instance, Clark (2003) employed heuristics that alter the exploration of the branch-and-bound tree, while Toledo et al. (2014) developed a multi-population genetic algorithm. Additionally, other techniques based on decomposition heuristics have also been proposed to solve the models, leveraging the mathematical structure of MIP formulations (Ferreira et al., 2012; Baldo et al., 2017; Toscano et al., 2020a,b).

## 6. Future research opportunities

This literature review also helped to identify some current gaps and recent trends in the study of BPPS problems, such as (i) maintenance, (ii) perishability, (iii) uncertainty, and (iv) the integration of production planning and scheduling with other relevant decisions. In what follows, we will discuss these four topics, as they also present opportunities for future research in the beverage sector.

### 6.1. Maintenance

A beverage filling line consists of various machines arranged in series, including bottle washers, conveyor belts, quality control sensors, beverage injectors, labelers, wrappers, and palletizers. Mixing tanks also incorporate numerous filters, sensors, and individual components that require calibration and preventive adjustments. All these machines require constant maintenance. Such components can be maintained individually by breaking this operation down into small, separate maintenance tasks for each machine. These maintenance tasks may involve calibrations, adjustments, and the replacement of small parts and sensors. As observed in real plants, depending on the type of component, such preventive maintenance can take from a few to many minutes. Although maintenance tasks can be performed outside of production periods in beverage plants that do not operate 24/7 (on weekends, for example), these tasks typically require overtime pay because they are usually carried out by the same employees who manage the machinery during production hours, a scenario common in small-scale fruit-based beverage plants. Additionally, in peak seasons like summer, when beverage production often extends into weekends, employees may work up to 14 hours a day, seven days a week. Consequently, machine maintenance becomes even more critical. Maintenance is particularly crucial when the production involves highly perishable syrups, such as in fruit-based beverages. A machine breakdown could result in the need to discard all prepared syrup and re-prepare the line and tank after repairs. Thus, correctly scheduling line and tank maintenance can further optimize the use of resources and consequently contribute to the competitiveness of the company. Various approaches have been used to address maintenance in production planning problems, including non-linear models (Dellagi et al., 2017; Liu et al., 2015, 2017), statistical control combined with constructive heuristic methods for maintenance plans (Ohunakin and Leramo, 2012),

iterative algorithms for production scheduling and preventive maintenance (Sortrakul et al., 2005; Zhao et al., 2014; Yildirim and Nezami, 2014; Glawar et al., 2018), and mixed-integer programming (Aghezzaf and Najid, 2008; Najid et al., 2011; Ramezani et al., 2013; Yalaoui et al., 2014). Several studies (Wang, 2001; Sortrakul et al., 2005; Wu, 2011; Ohunakin and Leramo, 2012) focus solely on maintenance programming, overlooking production lot-sizing and/or scheduling. Applying these approaches to BPPS problems represents a promising avenue of research.

### *6.2. Perishability*

Perishability in supply chains has gained focus in recent research. Amorim et al. (2011) explored integrated production and distribution planning for perishable products through a multi-objective framework. They compared the benefits of an integrated approach, in terms of economic gains and product freshness, against a decoupled one. Focusing on the highly perishable food processing industry, the study tackled challenges like integrating L&S, defining setup families based on setup times and costs, and managing multiple non-identical production lines. Although the study used illustrative examples, it suggested economic benefits from integration depend heavily on the delivered products' freshness level. Due to the intrinsic characteristics of different food production processes, it is crucial to consider perishability in some lot-sizing and scheduling settings to ensure optimized production scheduling (Alipour et al., 2020). Some studies address perishability in lot-sizing by imposing upper bounds on lot sizes, which necessitates more frequent scheduling of the same product and increases the complexity of sequencing. On a more tactical level, particularly in sectors like beverages where demand is affected by seasonality and production capacity is limited, it may be essential to produce seasonal stocks. A recent study by Alipour et al. (2020) proposed a general lot-sizing and scheduling problem for perishable food products, considering critical factors such as lifespan and shelf life. In the context of beverage production, perishability has been considered only in yogurt production, as previously shown.

Exploring the explicit consideration of perishability issues (e.g., base beverage perishability) in BPPS problems represents another promising avenue for future research.

### *6.3. Beverage production planning and scheduling under uncertainty*

Production planning and scheduling are complicated by two main types of uncertainty: environmental and system uncertainty (Ho, 1989; Mula et al., 2006; Alem et al., 2018). Environmental uncertainty includes external factors like fluctuating demand, supply variability, and changing costs, while system uncertainty involves internal variabilities such as production and changeover times, and operation yield. Scholars typically manage these uncertainties using stochastic programming (SP) or robust optimization (RO), with SP often preferred for its effectiveness in dealing with uncertainties in production planning and scheduling. Production planning applications were particularly influenced by Bertsimas and Thiele (2006), which introduced robust optimization to deal with a special class of inventory problem under demand uncertainty. To date, no studies specifically address beverage production planning and scheduling under uncertainty, although research on L&S problems using stochastic programming (SP) or robust optimization (RO) has increased. Pioneering works include Hu and Hu (2016), who developed a scenario-based two-stage SP model for a lot-sizing and scheduling problem with sequence-dependent setups and uncertain demands primarily in the automotive industry. Following this, Hu and Hu (2020) introduced a hybrid stochastic-robust lot-sizing model considering uncertainties in overtime costs and customer demand. Additionally, Alem et al. (2018) compared RO and SP models for lot-sizing and scheduling under demand uncertainty, providing a decision-making framework based on various performance indicators. Alem et al. (2020) explored risk aversion in lot-sizing within SP, specifically in the beverage sector but omitted scheduling issues. Mostafaei and Oliveira (2021) utilized RO to handle uncertainties in processing and changeover times for a paper machine’s production scheduling problem. Lastly, Chen and Su (2022) approached a lot-sizing and scheduling problem by multi-stage stochastic programming, addressing uncertainties in machine eligibility and sequence-dependent setups, demonstrating the model’s effectiveness with small test cases. While this paper does not cover all literature on L&S problems under uncertainty exhaustively, key observations emerge. A notable observation is the complete absence of studies on beverage production planning and scheduling under uncertainty. Similar to other manufacturing areas, the beverage industry faces environmental and system uncertainties, prominently influenced by consumer preferences and seasonal changes affecting demand

for products like soft drinks, juices, and beer. Government regulations, such as restrictions on soft drinks in schools (Hawkes, 2010), add further uncertainty in demand estimation. Thus, more research focusing on real-world cases of beverage L&S under uncertainty would be valuable, particularly to justify the use of stochastic programming (SP) and robust optimization (RO) approaches considering factors like data availability, computational tractability, and decision flexibility.

#### *6.4. L&S problem integrated with other stages*

Another recent trend in beverage L&S problems is the integration of production planning decisions with other stages of supply chain management. For example, model 45 (Bo et al., 2021) investigated a beverage problem involving production, distribution, and warehouse logistics, where due to the problem's complexity, a greedy heuristic and metaheuristic were developed to solve it. Similarly, model 41 (Begen et al., 2019) presented a tool to determine the production, distribution, and inventory plans of a Canadian beverage manufacturer and distributor using a MIP model, though the integrated problem lacks detailed consideration of the production process, such as setup or changeover times. Additionally, there has been increasing interest among practitioners and scholars in the field of sustainable or green supply chains (Seuring and Müller, 2008; Brandenburg et al., 2014), which encompasses sustainable production planning and scheduling. Despite long-standing discussions on sustainable supply chains, few academic studies address L&S issues from a sustainability perspective, with most focusing on manufacturing (Darom et al., 2018; Mishra et al., 2021) or pure scheduling (Piroozfard et al., 2018). Therefore, integrating lot-sizing and scheduling with other decisions represents a promising research area, particularly for addressing sustainability concerns in beverage production processes.

## **7. Final remarks**

This article reviews the scientific literature on integrated lot-sizing and scheduling (L&S) problems in the beverage industry, approached from two perspectives: integrated L&S problems in general, and those specific to the beverage industry. It analyzed dozens of studies on Beverage Production Planning and Scheduling (BPPS) applied to various production processes and companies,

identifying 48 Mixed Integer Programming (MIP) models for BPPS problems. The review indicates that most studies focus on specific types of beverage production because creating a decision support system that addresses all beverage types is not trivial. That is why most studies adopt simplifications to reduce the complexity of the actual problem, enabling its resolution through less complex mathematical formulations without losing generality. The most common simplifications include dedicating each tank to a specific production line, disregarding the perishability of the base beverage, aggregating lines, assuming preset production sequences, grouping products into families, and ignoring the availability for machinery maintenance. The specific cases of BPPS problems arise primarily from real-world settings in the production of soft drinks, fruit-based juices, beers, and yogurts. Because existing classifications of L&S models have overlooked the distinguishing characteristics of their production processes, we introduced a novel set of classification criteria. These criteria are based on four aspects: scheduling decisions, synchronization, perishability, and objective function. These elements enable us to further classify up to 48 MIP models developed for L&S models in beverage production. An important observation is the widespread use of approximate solution techniques due to the complexity of L&S models in beverage production. Although exact solution methods are in use, methods like decomposition heuristics and hybrid algorithms need further study due to their demonstrated improvements in problem-solving efficiency and effectiveness. Current research remains crucial, particularly as companies trend towards decentralizing production to multiple plants to cut costs and reduce distribution lead times, necessitating flexible decision-support tools. Recent studies also show an emerging trend of integrating distribution decisions into these models. We also discuss promising research directions based on maintenance, perishability, and beverage production planning and scheduling under uncertainty.

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