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Supplying Lean Assembly Lines: A Vehicle Routing Approach for the Milk Run System

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Master’s research document presented to the Industrial Engineering Postgraduation Program of the Universidade Federal de São Carlos, campus Sorocaba, for the Industrial Engineering master’s degree.

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For my son João Eduardo.

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ABSTRACT

The milk run system is inserted in the context of Lean Logistics, the logistics dimension of the Lean Production System. The combined pick-up between different suppliers characterizes it, optimizing the material's transportation and storage costs. Milk-run can be treated as a vehicle routing problem (VRP), one of the most explored problems of Operations Research in supply chain management. In this study, we address the Milk Run Routing Problem with Progress Lane (MRPPL), which consists of a VRP variant that models a milk-run system in the context of Lean Logistics. This problem encompasses the trade-offs between the lean performance criteria, such as pick-up frequency, vehicle efficiency, and inventory level at the manufacturer's plant. We present a literature review on the basic concepts of Lean Manufacturing applied to the logistics of the milk run pickup systems and the state-of-the-art VRP and its application to the automobile industry. Additionally, we develop a Systematic Literature Review (SLR) to identify the characteristics and peculiarities of the VRP application to lean logistics. To model and solve the addressed problem, we propose two mixed integer linear programming formulations, one based on a previous study from the literature, and the other originally proposed in this study, considering different types of decision variables and constraints. The results of computational experiments show that applying the second formulation considerably reduces the computation time to solve realistic instances. On average, the total computation time to solve an instance using the second formulation is 8% of the total time for the first formulation. We provide managerial insights considering several scenarios and different variations of parameters in the real problem presented in the case study. An optimal solution could be found for the real problem, indicating a low number of vehicles required and an increase in their efficiency.

Keywords: Lean Logistics, milk run, vehicle routing problem, automotive manufacturer.

FIGURES

TABLES

SUMMARY

1. INTRODUCTION

Lean Logistics is the logistics dimension of Lean Manufacturing, a production methodology developed in the Japanese automotive industry based on the ideology of maximizing productivity while minimizing waste within a manufacturing operation. The success of its application in the Japanese market after World War II spread it around the world, not only in the automobile environment but also in other productive sectors.

The basic principle of Lean Logistics is the “just in time” supply, assuring the correct inputs at the right time they are needed, without generating unnecessary inventories, and time and cost wastes. The “just in time” supply systems applied in the automotive industry consist of frequent deliveries to feed the manufacturers’ assembly line, considering the trade-offs between the number of routes their pickup frequency, and the stock levels at consumption points.

Based on the distribution of milk bottles in the USA, the “milk-run” concept was applied to logistics for achieving Lean goals. The “milk-run” logistics system consists of frequent pickups or deliveries at more than one point for goods supply and delivery. It can be used both for inbound and outbound logistics, despite the former being more common in the automotive industry, where the routes deliver empty boxes to suppliers and return them with parts for the manufacturing plant.

According to Brar and Saini (2011), the “milk-run” benefits, among others, consist of: (i) the transport network optimization by the pickup of goods in small batches and high frequency, (ii) reduction of the auto parts storage in suppliers and automakers, improving the operation efficiency of supply chain and reducing its incoming costs, and (iii) standardizing the parts supply with constant deliveries, scheduled routes, and standard vehicles and packages.

Because of the high volume of automobile components, the number of suppliers and their distance from the manufacturers, and the transaction fees, the logistics costs represent a considerable cost component for the automotive sector according to ANFAVEA (2023). In this sense, competitors have made many efforts to increase efficiency and reduce waste.

According to this background, optimization methods have been developed to achieve Lean Logistics goals. The “milk-run” system can be modeled as a Vehicle Routing Problem (VRP), a combinatorial optimization problem initially proposed in the 60’ by Dantzig and Ramser (1959) for a fuel distribution problem. Since its first application, many variations and solution methods have been presented, with a huge field of application in many areas of transportation and logistics. Some authors as Yingling (2005), Ohlmann et al. (2008), Sadjadi et al. (2009), and Mao et al. (2020) studied the application of VRP in the Lean Logistics environment, mainly considering cost reduction, vehicles' efficiency, and the pickup frequency optimization.

Despite researchers studying the application of the VRP to the "milk-run" systems in Lean Logistics environments, few studies considered an optimization problem analyzing, at the same optimization stage, the trade-offs between the lean manufacturing performance criteria. For example, many of them (OHLMANN et al., 2008, MAO et al., 2020) consider the route definition and scheduling, without analyzing the inventory levels at the manufacturer plant. Considering the stock level and the penalty for arrival delays, implying pars shortage risk, directly impacts the route schedule definition and its pickup frequency.

This literature gap has motivated the development of this work, which considers a case study in an automotive industry that uses the milk-run pickup system in the context of lean manufacturing. We propose optimization models that consider the lean performance criteria to define the milk run plan to support decision-making and reduce time spent on route planning.

1.1. Objectives

The main objective of this work is to study the application of the VRP in the context of lean logistics applied to milk-run pickup systems and propose new formulations to support supply planning for an automobile manufacturer. We aim to evaluate the trade-offs between the lean performance criteria, considering the pickup frequency and inventory levels at the manufacturer's plant.

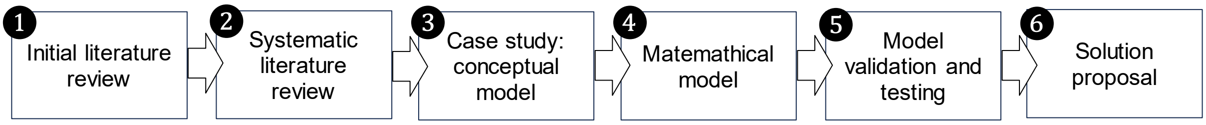
As a secondary objective, we intend:

a) to perform a Systematic Literature Review to find out what authors have studied about the VRP application in the Lean Logistics environment and the characteristics of the VRP variant, problems, and model applications.
b) to describe the detailed application of the VRP in a milk run system from an Original Equipment Manufacturer (OEM) under the perspective of Lean Logistics.

1.2. Methodology

According to Bertrand and Fransoo (2002), Operations Management (OM) research using quantitative modeling can be approached either axiomatically or empirically. The axiomatic approach starts with a concise problem description based on accepted concepts, focusing on either studying a new variant of a known problem or offering new solutions using established techniques. Its contribution lies in extending existing literature by analyzing variable behavior. The empirical approach, on the other hand, tests the validity of theoretical models and the performance of their solutions in real-life operational processes, aiming to improve operations by aligning models with real-world conditions and comparing results to actual performance. Therefore, this research can be classified as empirical quantitative, since it is motivated by a case study of an automobile manufacturer. To achieve the defined objectives of this study, the methodological structure is organized as shown in Figure 1 and explained in the following paragraphs.

Figure 1 – Methodology steps of this work.



Source: elaborated by the author.

The first step consists of an initial exploratory literature review aiming to identify, besides the Lean Logistics and VRP theoretical basement, the state-of-the-art application of VRP models in the Lean Logistics environment and milk-run systems.

In the second step, a Systematic Literature Review (SLR) is developed for a detailed investigation of the mentioned application. The SLR is conducted based on the methodology proposed by Tranfield et al. (2003) and Page et al., (2021) and divided into three phases: (i) planning, (ii) conducting and (iii) reporting.

The third step consists of a case study in a lean automobile manufacturer, focusing on the inbound logistics coordinated by a milk run system. As stated by Voss et al. (2002), case study research has been one of the most important methods in operations management, particularly in the development or testing of new theories. Its key purpose is theory building, where the researcher needs to identify and describe the key variables of the problem, the linkage between them, and why these relationships exist.

The fourth step considers the adaptation of a literature optimization model named Milk-run Routing Problem with Progress Lane (MRPPL) for real problem resolution. Besides the MRPPL, a new model was proposed, considering a simplified number of indexes and binary variables. To differentiate the models, we will name the literature model as named Milk-run

Routing Problem with Progress Lane – route-based (MRPPL-RB) and the proposed model as Milk-run Routing Problem with Progress Lane – arc-based (MRPPL-AB)

In the fifth step of our methodology structure, both models were validated with fictitious data for comparison. Results demonstrated the superior computational performance of the proposed model. The new model was then used for solving many instances of a representative scenario from the real problem analyzed in the case study. The main idea of this step is to vary the model's parameters to evaluate its impacts on the result from the lean logistics perspective. The last step consists of the resolution of the real problem. The MRPPL-AB was also used for the resolution of the real problem using the commercial solver IBM CPLEX Optimization studio within the GAMS software.

This work is organized as follows. In Chapter 2, the literature review is divided into three topics. The first presents an overview of the lean logistics and milk-run pick-up systems. The second is the state of the art of VRP problems and their applications to lean logistics. The last one consists of a Systematic Literature Review (SLR) considering the combination of the VRP and the lean logistics. Chapter 3 provides the problem statement by considering first, a general description of lean milk-run systems and then the detailed explanation of the real problem case study. In Chapter 4, we present the two proposed mathematical models, one adapted from literature and the other originally proposed in this study. Chapter 5 presents the tests and results, divided into three topics. The first aims to compare the efficiency of the two models considering the same scenario. The second intends to test the best-performing model through different instances for the same representative scenario to evaluate the impact of the model's parameters variation under a lean logistics perspective. The last one presents the application of MRPPL-AB to the real problem. Finally, Chapter 6 provides the research conclusions and perspectives for future works.

2. LITERATURE REVIEW

This chapter presents a literature review of the topics addressed in this work. The literature review is divided into three main topics. Firstly, in 2.1, we point out the Lean Logistics background and its most important concepts for the milk run pickup and delivery system explanation. Secondly, in Section 2.2, The VRP is defined, and its most important variants are mentioned. Then, in section 2.3, an SLR is conducted aiming at the relevant findings on the relationship between Lean Logistics and VRP.

2.1. Lean Logistics and the Milk-run Systems

In this section, we present the fundamental principles of Lean Manufacturing and Lean Logistics. These concepts will be important for a better understanding of the case study presented in this work and its specific characteristics.

The main objectives of Lean Logistics, according to Baudin (2004) can be stated as: (i) delivering the materials needed, when needed, in the exact quantity, and conveniently presented, to production for inbound logistics and customers for outbound logistics, and (ii) without degrading delivery, pursuing the elimination of waste in the logistics process.

While keeping stocks is a way of preventing materials shortage, the lean approach is to hold the minimum needed to support production but monitor it closely. Planning the production to smooth the consumption ratio of each item over the planning period, organizing inbound logistics to make replenishment lead times predictable, and responding with countermeasures at the first sign of problems (WOMACK et al., 2007).

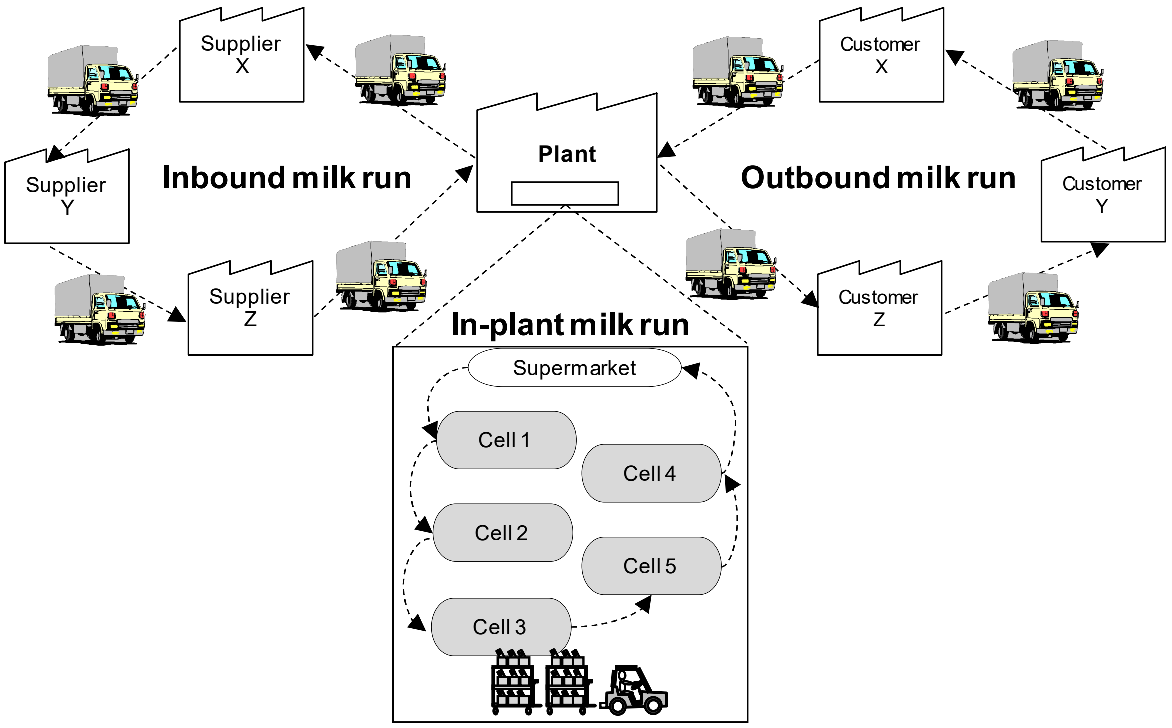
Moving small quantities of many items between and within plants with short, predictable lead times requires pickups, and deliveries at fixed times along fixed routes called “milk-runs”. According to Meyer (2015), the milk run can be defined as:

- A concept to serve supplier relations with regular volumes.
- A fixed tour with fixed sequences of stops serving at least one supplier and being executed cyclically or according to a fixed schedule.
- The volumes are determined daily by an order policy aiming for leveled load sizes or simply by aggregating the demand until the next shipment.
- The milk run is planned and communicated to the suppliers by the consignee. The execution is usually outsourced by a freight forwarder.

- The milk run plan is generally valid for several weeks or months to achieve positive savings.
- The milk run can be – but does not have to – a round tour starting at the receiving plant to allow the exchange of full and empty returnable containers.

As shown in Figure 2, the milk run system can be used for different logistics proposals such as inbound logistics, where vehicles collect components at suppliers and deliver them to the manufacturer, in-plant logistics, where components are delivered to production lines and outbound logistics, where vehicles are used for final products distribution to customers.

Figure 2 – Inbound, outbound, and in-plant milk runs.



Source: adapted from Baudin (2004).

Usually, more than one supplier is served in the same milk-run route to improve transportation efficiency and enable high pickup frequencies, reducing the necessary stock of components in the manufacturer plant. In general, as pointed out by Meyer (2015), the milk-run concept applied with a supplier kanban control is the result of the application of described lean principles: it allows for smooth flow due to small lots pulled by kanban signals and frequent supplies within constant cycles.

The Toyota Production System (TPS) has been widely used to pursue zero-inventory levels. In practice, however, this goal is difficult to achieve (WOMACK et al., 2007). In that sense, complementary inventory models must be developed. Among them, Mao et al. (2020) presented the concept of progress-lane (P-LANE), and demonstrated its application in a Toyota manufacturing plant in Tianjin, China. According to the authors, the P-LANE is a mode that stores the parts needed for one day manufacture's volume, as a terminal buffer to reduce the total cost and improve transportation efficiency. It converts the irregular and non-quantitative outbound logistics of collecting parts from suppliers into a regular and quantitative supply to the production line.

However, to keep the pace of the external and internal logistics, another TPS concept is necessary – the heijunka production. Heijunka, or leveled production, is a key principle in Lean Manufacturing, aiming at reducing waste and improving efficiency by smoothing out production schedules to ensureonsistent flow of goods. According to Huttmeir et al. (2009), the objective of heijunka is to avoid peaks and valleys in the production schedule.

McLachlin (1997) listed as some elements of the heijunka method: (i) there is a fixed and leveling production schedule; (ii) the same mix of end items of families are produced each day and, if possible, each hour or time slot used (matching daily demand rates); (iii) there is a reduction in upstream inventory swings and possible reactions to changes in the production plan; and (iv) there is little or no expediting.

Huttmeir et al. (2009) states that this approach is particularly synergistic with the milk run pickup system, which entails a scheduled delivery and collection route that minimizes transportation costs while maximizing efficiency. By implementing heijunka, companies can optimize their inventory levels and production rates, ensuring that materials are produced and delivered in a synchronized manner. The integration of these methodologies enhances overall operational efficiency, minimizes stockouts and overproduction, and ultimately contributes to a more agile and responsive supply chain.

2.2. Vehicle Routing Problem (VRP)

The milk run pickup and delivery system can be treated as a vehicle routing problem (VRP). The vehicle routing problem was initially proposed by Dantzig and Ramser (1959) for solving a fuel distribution problem. It presented the first mathematical resolution and algorithmic formulation for the VRP. Since then, the number of works related to VRP has grown significantly.

The VRP can be defined as a problem for designing optimal delivery or collection routes from one or several depots or suppliers to several geographically scattered cities or customers, subject to side constraints (LAPORTE, 1992). The problem is commonly represented using a graph $G = (V, A)$ where $V = \{1, \dots, n\}$ is the set of vertices representing the depots or suppliers and A is the set of arcs. With every arc $(i, j), i \neq j$ is associated with a non-negative distance matrix $C = (c_{ij})$. Depending on the problem context, c_{ij} can be interpreted as a travel cost or a travel time. If C is symmetrical, it is convenient to replace A by a set E of undirected edges. Also, for a reason of simplicity, it is assumed that all the vehicles are identical and have the same capacity Q.The VRP consists of designing a set of least-cost vehicle routes in a way that:

(i) Each depot or supplier in $V\backslash\{1\}$ is visited exactly once by exactly one vehicle.
(ii) All vehicle routes start and end at the depot.
(iii) Some side constraints are satisfied.

The most common side conditions include, as examples (LAPORTE, 1992):

(i) Capacity restrictions: a non-negative weight (or demand) d_i is attached to each depot or supplier $i > 1$ and the sum of weights of any vehicle route may not exceed the vehicle's capacity Q.
(ii) The number of depots or suppliers in a route is bound above by q (this is a special case of (i) with $d_i = 1$ for all $i > 1$ and $D = q$).
(iii) Total time restrictions: the length of any route may not exceed the prescribed bound L; this length is made up of arc travel times c_{ij} and stopping times δ_i at each depot or supplier i on the route (the stopping times can also be considered as service time for supplier attendance).
(iv) Time windows: the depot or supplier i must be visited within the time interval $[a_i, b_i]$ and waiting is allowed at the depot or supplier i.

Vehicle routing plays a significant role in the fields of physical distribution and logistics. Because of that, many variants of VRP were developed and studied such as those presented by Cordeau, et al. (2007), Lin et al. (2014) and Toth and Vigo (2002). Bodin (1975), in his work sought to taxonomically classify vehicle routing and sequencing problems. Later, Eksioglu et al. (2009) proposed a new taxonomic classification, considering several variations of the VRP studied since then. The work was updated by Braekers et al. (2016) and, finally, by Tan and Yeh (2021).

Among the variants of VRP, stands out the CVRP (capacitated vehicle routing problem), VRPTW (vehicle routing problem with time windows), PVRP (periodic vehicle routing problem), PDP (pickup and delivery problem), SVRP (stochastic vehicle routing problem), VRPB (vehicle routing problem with backhauls) and, more recently, the GVRP (green vehicle routing problem), which also considers environmental performance criteria in its formulation, and the combination of its formulations for the development of more adapted to real problems, called RVRP (rich vehicle routing problem).

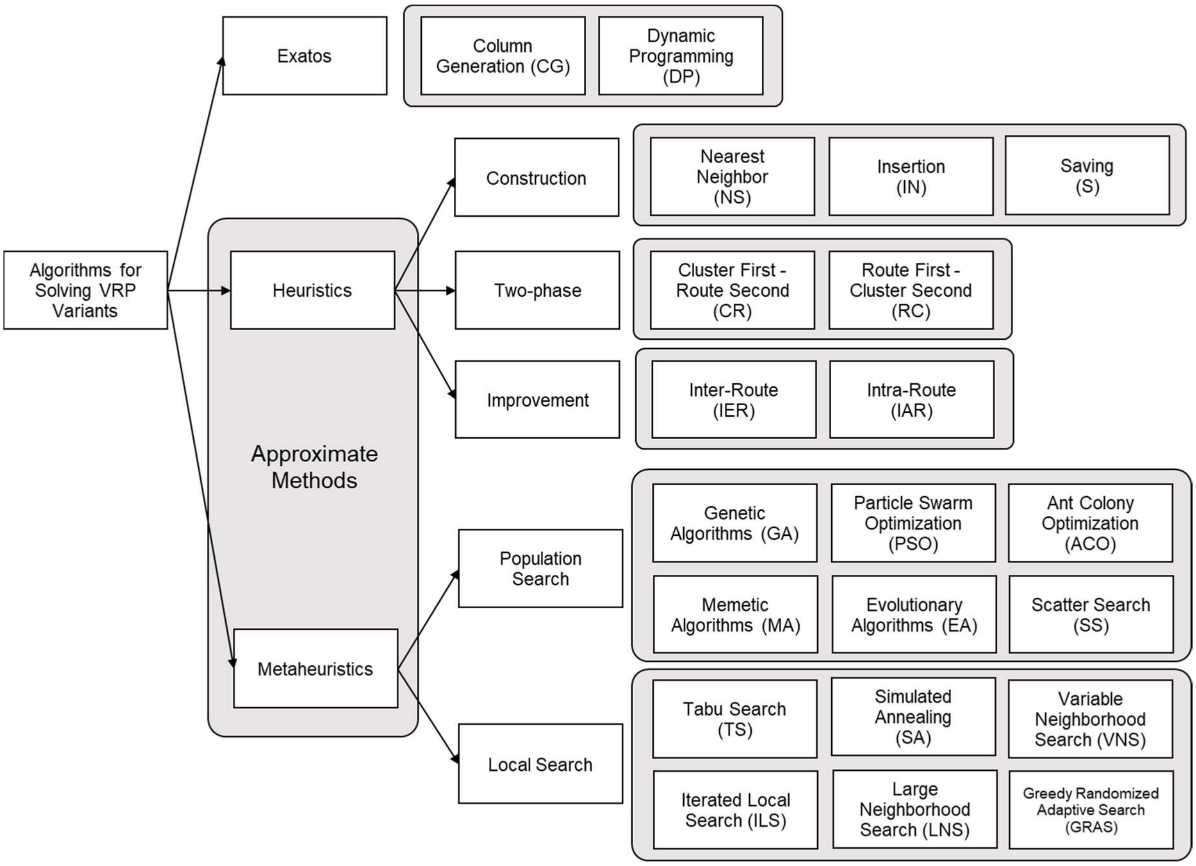
When decisions about inventory levels are added, there is still variation in the IRP (inventory routing problem) (CAMPBELL et al., 1998). As it is a complex problem, involving several strategic decisions in the supply chain, VRP models can also be considered in a multi-objective way (JOZEFOWIEZ et al., 2008), considering the optimization of different performance criteria, with the analysis of the trade-offs between them.

Due to the difficulty of being solved exactly, the VRP is considered an NP-hard problem (SADJADI et al., 2009). As it is a combinatorial optimization problem, the difficulty of finding an optimal solution increases exponentially according to the number of collection and/or delivery points, among other decision structures. Thus, in addition to exact methods, the VRP is commonly solved using heuristic or metaheuristic methods.

Heuristic methods are algorithms used to find a solution to problems whose exact solution is difficult to find (BRAYSY and GENDREAU, 2005a). However, heuristic solutions may result in local solutions, as they do not consider all possible solutions in a feasible space. For this reason, metaheuristics have been developed that combine, in addition to heuristic algorithms, perturbations in the system to avoid local optimality to the detriment of global optimal values (BRAYSY and GENDREAU, 2005b).

Figure 3 presents a diagram adapted from Braekers, et al. (2016) and Lin et al. (2014) that classifies the main methods of solving the VRP and their variations, divided into exact, heuristic, and metaheuristic methods.

Figure 3 – Classifications of VRP solving algorithms and some of their variations.



Source: adapted from Braekers, et al. (2016) and Lin et al. (2014).

As for the exact methods, the authors subdivide the ways to solve VRP problems between column generation and programming methods. For the heuristics, the methods are divided between (i) construction, which considers the nearest neighbor, insertion, and savings; (ii) two phases, divided into cluster-route and route-cluster; and (iii) improvement, inter-route, and intra-route. The metaheuristics are divided into (i) population search, which are genetic algorithms, particle swarm optimization, ant colony optimization, memetic algorithms, evolutionary algorithms, and dispersed search; and (ii) local search, divided into tabu search, simulated annealing, variable neighbor search, iterative local search, expanded neighbor search, and adaptive randomized greedy search.

There are also hybrid methods, which combine more than one of the different methods presented for solving the VRP. Some examples of the combination of metaheuristics can be found in Kaboudani et al. (2020) and Rajabi-Bahaabadi et al. (2021). Despite this work not focusing on the resolution methods of the VRP, we have considered them for the SLR classification.

About the application of the VRP to milk run pickup and delivery systems in these kinds of problems, each route considers the truck capacity that cannot exceed the volume of parts collected from more than one supplier. If the earliest arrival date and a deadline are also associated with each route, the version of vehicle routing problem with time windows (VRPTW), proposed by Solomon (1986), can be seen as a starting point for modeling JIT milk-run routing problems.

The traditional vehicle routing problems such as the capacitated vehicle routing problem (CVRP) and the vehicle routing problem with time windows (VRPTW), do not consider the inventory aspects that should be considered in the lead logistics environment. For these considerations, the well-known inventory routing problem (IRP), initially formalized by Dror et al. (1985), can be used.

The IRP combines the inventory and vehicle-routing problem but usually is used for the study of finished goods distribution rather than the supply of parts. Overall, the standard problems of transportation logistics are not directly applicable to the problem of parts supplied in inbound logistics, according to Boysen et al. (2015). In this sense, the following paragraphs summarize the literature approach tailored to the inbound logistics problems of the automotive industry.

Chuah and Yingling (2005) reported a study applied to an automotive plant in the USA threatening a special JIT supply routing problem, where suppliers were allocated in a tour, which is repeated daily. Supply and inventory costs were jointly considered at the objective function, while the given average demand needed to be satisfied and storage space at OEM was limited. The authors presented the concept of common frequency routing (CFR) since only suppliers with the same delivery frequency can be joined on one route.

Ohlmann et al. (2008) also developed their study in the automotive industry motivated by the problem of inbound logistics routing problem. The authors proposed the general frequency routing problem (GFR), complementing the below presented CFR. In GRF, suppliers with different delivery frequencies are allowed to be combined on the same route. In this case, the solution to the problem is more difficult to find, but the combined routes can result in more effective truck utilization.

Du et al. (2007) presented a dispatching system that dynamically integrates newly arriving tour requests from suppliers to OEM plants and from OEM to customers for milk-run deliveries into existing routes. The model presented by Sadjadi et al. (2009) addressed JIT milk-runs in the automotive industry as an extension of the VRPTW. Additionally, the authors included in the optimization model the inventory holding costs at the OEM plant and the minimum safety stock levels for satisfying demand.

In the past few years, research on the usage of milk-run to reduce transportation costs and vehicle waste in manufacturing has been rather sparse and mainly focused on the theoretical aspect. Recent publications present the milk run topic related to other important lean elements and optimization criteria. Mao et al. (2020), for example, introduce the concept of *progress-lane* (P-LANE) as a buffer between the external and internal logistics in their optimization model, called *milk run routing problem with progress lane* (MRPPL).

More recent papers also include the environmental aspects in their VRP application for lean logistics. Quan et al. (2021), for example, included the environmental aspects by measuring the CO_2 emission rate for the milk run route optimization.

2.3. Systematic Literature Review (SLR)

Several authors present studies on the application of VRP in the context of supply chain management (KONSTANTAKOPOULOS et al., 2022; MEYER and AMBERG, 2018; MOR and SPERANZA, 2020), but a smaller amount is found when looking for the application of VRP in the context of lean logistics.

For this reason, this section intends to answer the following research question: how do the authors apply the vehicle routing problem to the milk run system? Its main objectives are: (i) to identify the mathematical models, the resolution methods used, and the variations of the vehicle routing problem applied to the context of lean logistics and (ii) what gaps and research opportunities exist in this area.

To answer these questions, the methodology of systematic literature review (SLR) was used, through three steps proposed by Tranfield et al. (2003): research planning, development, and presentation of results, following the PRISMA procedure (PAGE et al., 2021).

2.3.1. Literature Review Protocol

For SLR quality assurance, it is necessary to prepare a transparent, complete, and accurate account of why the review was done, what was done, and what was found. The Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) statement published in 2009 is a reporting guideline developed to address poor reporting of systematic reviews. The statement comprised a checklist of 27 items recommended for systematic review reporting, and it was updated in 2020 (PAGE et al., 2021). The present SLR was conducted over three phases, proposed by Tranfield et al. (2003) and illustrated in Table 1: planning, conducting, and reporting.

Table 1 – SLR development phases.

Phase	Activity	Information	Method/tool
1. Planning	Search strategy formulation	Construct definition, keyword analysis, search string definition, and database selection	VOSviewer
2. Conducting	Articles Search	Articles research in selected databases	*Scopus* *Web of Science*
	Articles Selection	Definition of the filters and criteria for exclusion: Filter 1 – double-blinded analysis Filter 2 – conflict analysis Filter 3 – full read	Rayyan
3. Reporting	Bibliometric Analysis	Publication analysis by source, year, and country	Excel/VOSviewer
	Content Analysis	Identification of the VRP variant and mathematical method used to solve the VRP applied to the milk run system	Content classification table

Source: elaborated by the author.

Planning consists of the initial SLR phase. Research strategy definition, including the construction formulation, keyword analysis, search string definition, and database selection takes place during this phase. VOSviewer software (VAN ECK and WALTMAN, 2010) was used as a tool for construct definition and keyword identification during exploratory research.

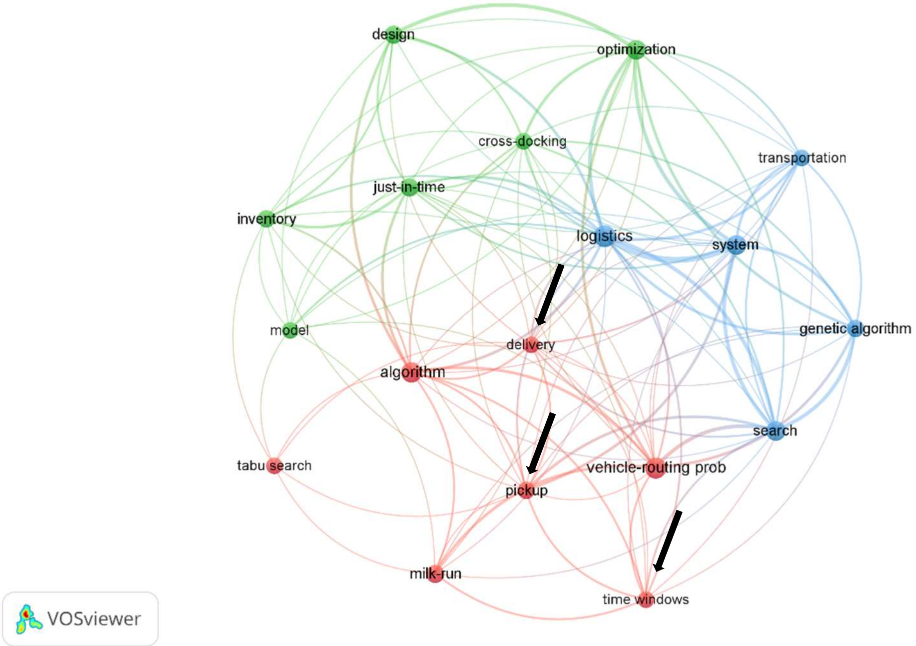
The conducting phase is the second step of an SLR. During this process, all collected material will be filtered and selected for SLR inclusion. The exclusion criteria and filter definition provide a comprehensive and unbiased search, which differs the systematic review from the traditional one (TRANFIELD et al., 2003). Rayyan web application (OUZANNI et al., 2016) was used in this phase for sample filtering and screening through a double-blinded process.

The last phase of reporting will be separated into two different topics. The first one provides a descriptive analysis of the collected data. The second one aims to link the findings across the identified contributions to answering the research question. For the reporting step, VOSviewer was also used for the bibliometric analysis of the selected sample data. For detailed content analysis, a vehicle routing problem taxonomic classification, developed by Eksioglu et al. (2009) and updated by Braekers et al. (2016) was used.

For the literature review construct, based on the research questions, it was defined two different clusters for *a priori* articles search. The first one was related to Lean Logistics and the second one to VRP and its variants. Initially, the terms “lean logistics”, “just in time” and “milk run” were related to the first cluster, and “vehicle routing problem”, “inventory routing problem”, “routing” and “inventory” were related to the second one were included. The Inventory Routing Problem (IRP) was included due to its strong relationship with the lean logistics concept, i.e., the trade-offs between routing frequencies and inventory levels.

Thereafter, an exploratory survey was conducted to find the most representative words in the found articles. Then, a keyword-based bibliometric analysis on the initial sample was performed for a better understanding of the relation between the two mentioned clusters. Figure 4 shows the keywords network created by VOSviewer software (VAN ECK and WALTMAN, 2010).

Figure 4 – Bibliometric analysis of keyword co-occurrence in the exploratory search.



Source: elaborated by the author.

The search string was defined by combining the synonyms of the keywords, as shown in Table 2. It’s important to mention that the terms “time windows”, “pick up” and “delivery” were added to the search string after the initial keyword bibliometric analysis was done in VOSviewer.

Table 2 – SRL search string definition.

Cluster	Search string
Milk run	(“*milk-run*” OR “*milk run*” OR *milkrun* OR “*just-in-time*” OR *JIT* OR “*just in time*” OR *lean*)
VRP	AND (“*vehicle routing*” OR “*routing problem*” OR *VRP* OR “*inventory routing*” OR *IRP* OR “*time window**” OR “*pickup*” OR “*pick-up*” OR “*delivery*”)

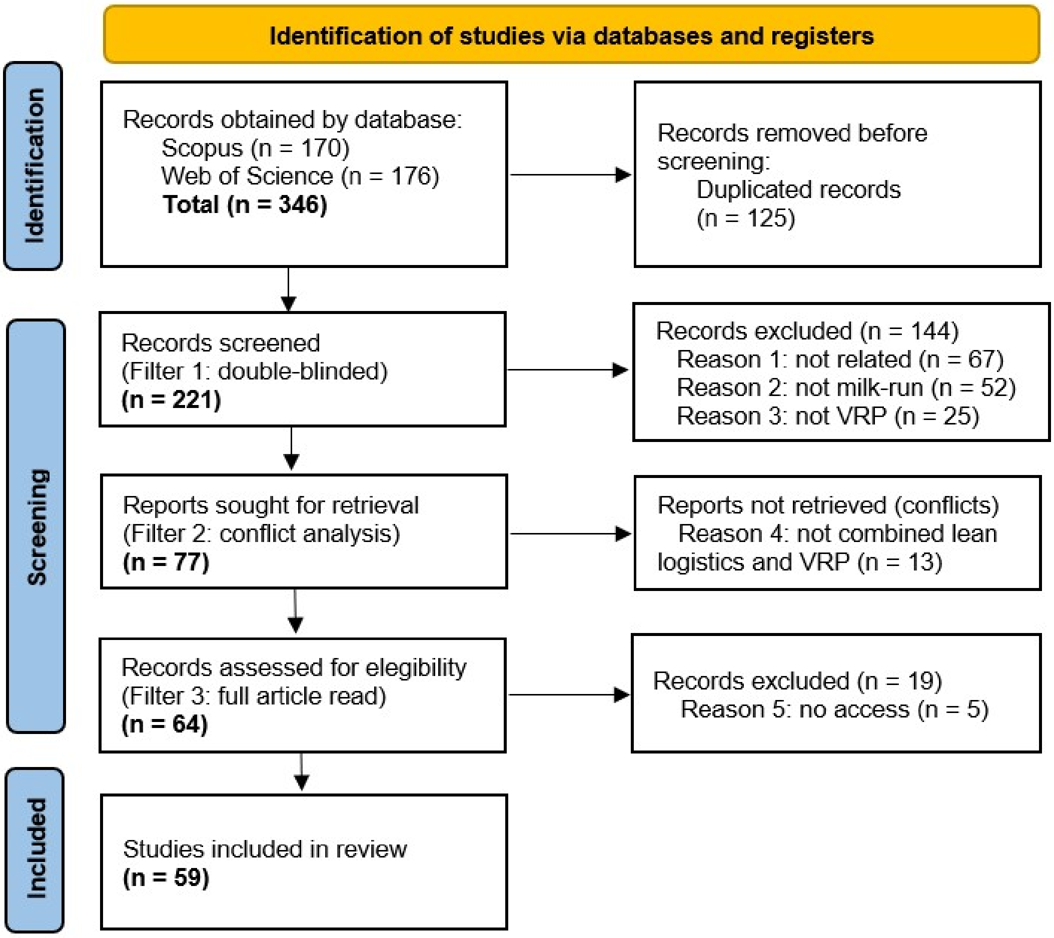
Source: elaborated by the author.

The databases “Scopus” and “Web of Science” were used for article research using the defined search string due to their relevance to the field of research. Peer-reviewed articles were included for analysis by filtering “articles” and “review articles”. No filter for idiom and publication year was applied.

Data collection was done on 30 April 2022, resulting in 170 documents from “Scopus” and 176 from “Web of Science”, after applying filters. A total of 346 documents were exported in *BibTex* format for further analysis. For the screening process, the Rayyan web application was used (OUZZANI et al., 2016). The first process was to remove duplicated files from both mentioned bases’ results. Of the total of 346 articles, 125 duplicates were removed, resulting in 221 documents for screening. Afterward, screening applied inclusion criteria related to the article’s theme alignment and scope. Articles not related to VRP, and its variants were removed, as those were not applied to the Lean Logistics context. Reviews, surveys, and meta research studies were also excluded. At this stage, a double-blind selection was performed, by analyzing the title, abstract, and keywords. After blinded classification, Rayyan indicates the conflicts, which need to be resolved by both peers. As a result of the screening process, a total of 64 articles were selected for the full reading stage.

Figure 5 shows the results of the screening process and the filtering criteria. In the first step, the identification of the total number of articles results from the 346 joined documents from both selected databases. From this total, 125 duplicated files were excluded. The remaining 221 were selected for the second step of the screening process. During this process, a double-blind selection was done by reading the articles’ titles, keywords, and abstracts. The first filter applied eliminated 67 files not related to the research question (as authors didn’t specify the journals as a research filter, many articles not related to lean logistics and VRP were found by the defined search string); 52 articles were excluded due to not relation with lean logistics cluster and 25 due to not relation with routing problems, resulting in an exclusion of 144 files after this first filter.

Figure 5 – PRISMA's flowchart for articles selection with filtering process and exclusion criteria.



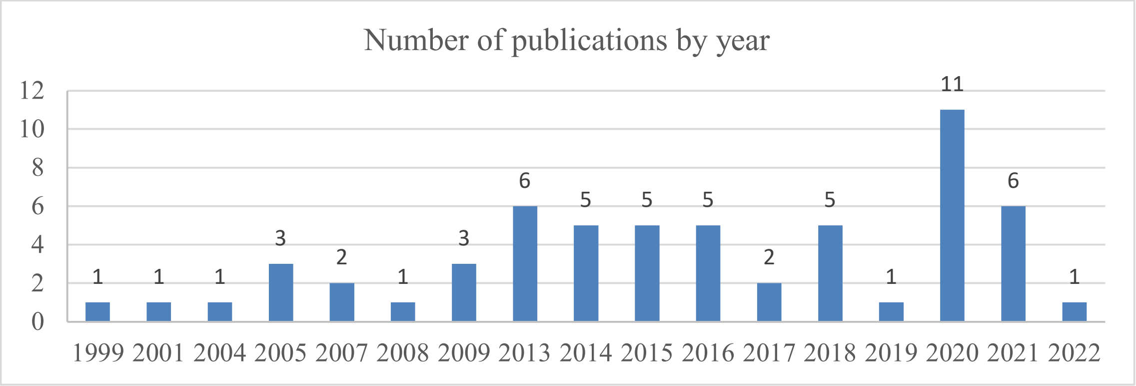
Source: elaborated by the author.

The second filter was applied to solve conflicts generated by the double-blind analysis. After the exclusion of 144 articles, 77 remained for the conflict analysis. From the total, 13 files were excluded because they were not completely related to lean logistics or vehicle routing problem clusters. Some papers, for example, dealt with logistics problems (i.e., distribution problems) but without considering a routing problem. These cases were excluded since this research intends to focus only on the application of routing problems to lean logistics environments. After the exclusion of 13 files on the second filter, 64 articles were selected for the next filter.

On the third filter, during the eligibility phase, 59 articles were selected for full-read analysis. Of the total, 5 articles couldn't be accessed. For a better understanding of the selected content, an initial bibliometric analysis was conducted over the 59 articles that resulted from the screening process. The results of the bibliometric analysis are reported on the following topic.

In this section, some statistical bibliometric data from the sample obtained from the applied research protocol is presented. Graph 1 summarizes the number of publications by year. The first publication about the VRP application in Lean Logistics was issued in 1999. Since that, there has been an increasing number of publications, especially in the period between 2013 and 2016, with the maximum number of publications in 2020.

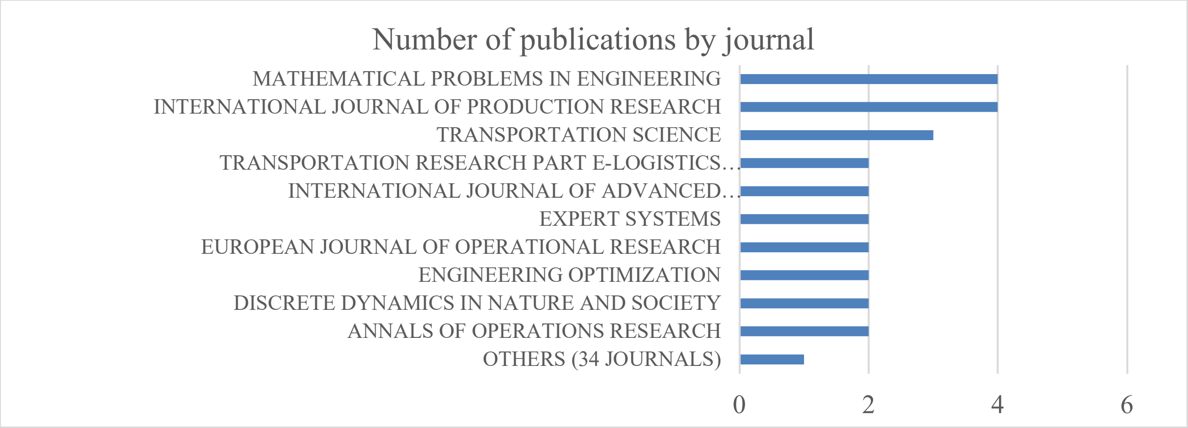
Graph 1 – Number of publications by year.



Source: elaborated by the author.

Graph 2 shows the number of articles published by journal. The most representative journals found in the sample are *"Mathematical Problems in Engineering"* and *"International Journal of Production Research"* with 4 publications, representing 7% of the total, each, followed by *"Transportation Science"* with 3 publications, representing 5% of the total. The journals "*Transportation Research Part E: Logistics and Transportation Review*", "*International Journal of Advanced Manufacturing Technology*", "*Expert Systems*", "*European Journal of Operational Research*", "*Engineering Optimization*", "*Discrete Dynamics in Nature and Society*" and "*Annals of Operations Research*", have two occurrences each one in the sample, and there are other 34 different journals with one occurrence each one. It can be noticed that analyzed publications are very spread among different journals.

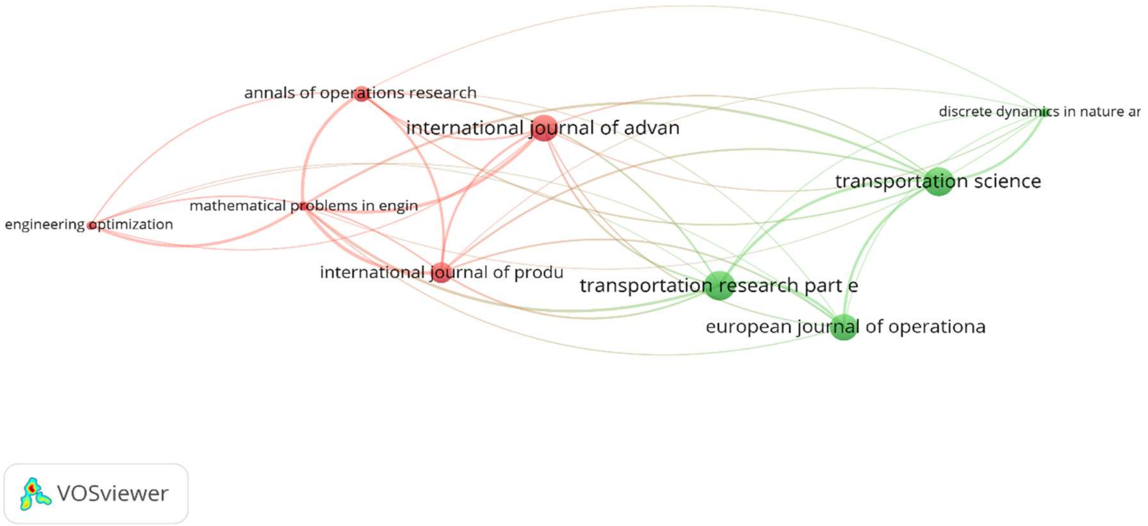
Graph 2 – Number of publications by journal.



Source: elaborated by the author.

Figure 6 shows a bibliographic coupling analysis done in VOSviewer software. The bibliographic coupling was done by source, considering at least two publications of the same source. The number of bibliographic coupling links between two documents equals the number of pairs of cited references in the same two documents that have the same match key (VAN ECK; WALTMAN, 2010). The circle’s size indicates the importance of each journal measured by the number of citations among analyzed publications.

Figure 6 – Bibliographic coupling analysis by source.



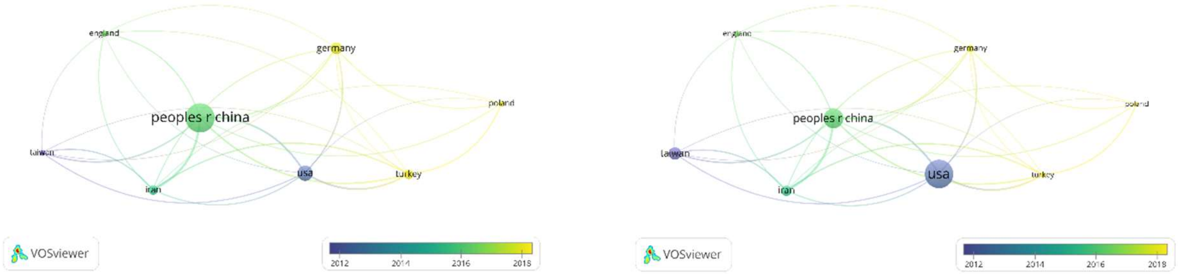
Source: elaborated by the author.

The most relevant sources, according to the bibliographic coupling analysis, are “*Transportation Research Part E: Logistics and Transportation Review*”, “*Transportation*

Science", "*International Journal of Manufacturing Technology*", "*European Journal of Operational Research*" and "*International Journal of Production Research*", considering the number of citations.

An additional analysis was done in VOSviewer considering the citations by source (country) among analyzed publications. Figure 7 shows the result considering (a) the number of documents by source and (b) the number of citations by source. The result demonstrates that China and USA are the most expressive countries. Despite China having more publications, the USA corresponds to a higher number of citations. It was also considered the year of publication, demonstrated by the color scale below the diagrams. The most recent publications were issued by Germany, Poland, and Turkey.

Figure 7 – Number of citations by country.

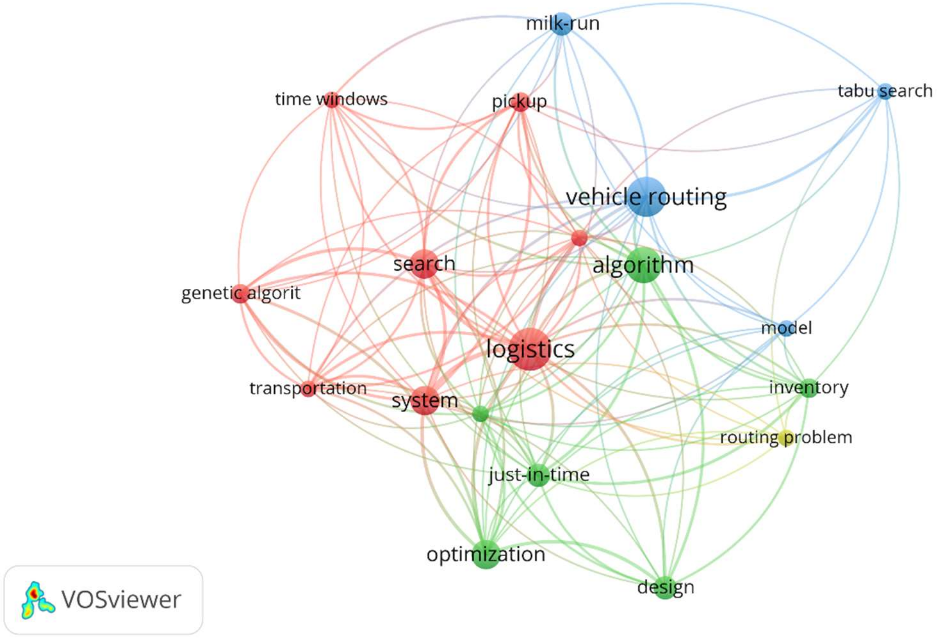


(a) considering the number of documents (b) considering the number of citations

Source: elaborated by the author.

Finally, Figure 8 presents an analysis of the co-occurrence of the article's keywords, considering at least 5 occurrences as a minimum number of keywords in the sample done in VOSviewer. A total of 19 keywords were found with the mentioned criteria. The most cited of them are "logistics", "vehicle routing", "algorithm", "optimization", "search" and "system".

Figure 8 – Co-occurrence of keywords.



Source: elaborated by the author.

The software suggested 4 clusters defined by different colors in Figure 8. "Vehicle routing" and "logistics" terms are linked with all the clusters. In the red cluster, some relevant terms are "transportation", "time windows", "pick up" and "genetic algorithm". In the blue one, some relevant terms are "just-in-time", "inventory" and "optimization".

After concluding the bibliometric analysis of the 59 articles resulting from the SLR, the next section presents a detailed content analysis.

2.3.2. Detailed Analysis of the Literature Review

In this section, the content analysis is presented after the collection of bibliometric data from the sample analyzed. In the content analysis stage, the data from the articles are compared and discussed to identify the methods used by the authors to apply the vehicle routing problem and its variants in the context of the milk run system, as well as to identify gaps and possibly new research opportunities.

Although this is not the focus of this thesis, we first analyzed the solution methods of the selected articles. Also, we analyzed the variations of the VRP problems applied to the lean logistics environment. Based on the classification of the resolution methods presented in Figure 3, Table 3 is presented, considering the mathematical model and solution approach for VRP variants applied to the milk run system.

Table 3 – Mathematical model, solution method and VRP variant applied to milk run systems.

			Resolution Method						
				Heuristics			Metaheuristics		
Article #	Authors	Mathematical model	Exact	Construction	Two-phase	Improvement	Population Search	Local Search	Variation of the VRP
1	Chuah and Yingling (2005)	MIP	CG						SVRP
2	Satoglu and Sipahioglu (2018)	MIP	CG						IRP
3	Ma and Sun (2013)	MIP	DP						VRPTW
4	Bocewicz et al. (2021a)	FMILP	DP						CVRP
5	Cakir et al. (2022)	FMILP	DP						VRPTW
6	Bertazzi et al. (2019)	MIP	DP						IRP
7	Bocewicz et al. (2019)	MIP	DP						IRP
8	Bocewicz et al. (2021b)	MIP	DP						VRPTW
9	Grzegorz et al. (2020)	MIP	DP						PDP
10	Hein and Almeder (2016)	MIP	DP						SVRP
11	Meyer and Amberg (2017)	MIP	DP						PVRP
12	Volling et al. (2013)	MIP	DP						SVRP
13	Gschwind et al. (2020)	MIP	DP						VRPTW
14	Zammori et al. (2016)	MISP	DP						SVRP
15	Adriano et al. (2020)	SIMULATION	DP						DVRP
16	Chang et al. (2009)	SIMULATION	DP						DVRP
17	Du et al. (2007)	SIMULATION	DP						DVRP
18	Emde et al. (2018)	SIMULATION	DP						DVRP
19	Euchi et al. (2015)	SIMULATION	DP						DVRP
20	Lee and Prabhu (2016)	SIMULATION	DP						DVRP
21	Novaes et al. (2015)	SIMULATION	DP						DVRP
22	Simik et al. (2020)	SIMULATION	DP						DVRP
23	Kim et al. (2005)	SIMULATION	DP						DVRP
24	Villarreal et al. (2016)	SIMULATION	DP						DVRP
25	Du et al. (2005)	SIMULATION	DP						DVRP
26	Güner et al. (2017)	SIMULATION	DP						DVRP
27	Staab et al. (2016)	SIMULATION	DP						DVRP
28	Lee et al. (2014)	SIMULATION	DP						DVRP
29	Kopecek and Pinte (2015)	SIMULATION	DP						DVRP
30	Vaidyanathan et al. (1999)	NLIP		NS					CVRP
31	You and Jiao (2014)	IP		S					CVRP
32	Yi et al. (2007)	MIP		S					CVRP
33	Lin et al. (2015b)	MIP			CR				MDVRP
34	Satoglu and Sahin (2013)	MINLP			RC				VRPSPD
35	Chen et al. (2020)	MINLP					ACO		IRP
36	Quan et al. (2021)	MIP					ACO		VRPTW
37	Yang and Liu (2013)	MIP					ACO		IRP
38	Chen and Sarker (2014)	MINLP					EA		DVRP
39	Ranjbaran et al. (2019)	MIP					EA		PDP
40	Wu et al. (2018)	ROP					GA	ILS	SVRP
41	Kocaoglu et al. (2020)	MIP					GA	SA	CVRP
42	Mao et al. (2019)	MIP					GA		PVRP
43	Sadjadi et al. (2009)	MIP					GA		IRP
44	Peng et al. (2020)	MIP					GA		IRP
45	Hasani and Zegordi (2020)	MINLP					MA		IRP
46	Hosseini et al. (2014)	MIP					SS		CVRP
47	Lin et al. (2015a)	MIP						AS/GRAS	PVRP
48	Tarantilis and Kiranoudis (2001)	IP						GRAS	CVRP
49	Emde and Schneider (2018)	MIP						LNS	CVRP
50	Zhou and Zu (2020)	MIP						NS/SA	IRP
51	Hosseini et al. (2014)	IP						SA	CVRP
52	Zhang et al. (2021)	ROP						SA	CVRP
53	Liao et al. (2017)	IP						TS	PVRP
54	Jiang et al. (2010)	MIP						TS	IRP
55	Mu and Eglese (2013)	MIP						TS	CVRP
56	Ohlmann et al. (2008)	MIP						TS	SVRP
57	Zhang et al. (2013)	MISP						TS	SVRP
58	Chang et al. (2004)	MISP						TS	SVRP
59	Karakostas et al. (2020)	MIP						VNS	IRP

Source: elaborated by the author.

Table 4 presents a statistical analysis of the mathematical models used in the articles. The mixed integer programming (MIP) method is the most representative of the sample. Of the total of 59 articles evaluated, it is applied in 28, representing 47% of the total. The second most used method is computational simulation, applied in 15% of the cases, representing 25% of the total.

Table 4 – Statistical analysis of mathematical models applied.

Mathematical model	Description of the model	Occurrences	Percentage	Accumulated percentage
MIP	Mixed Integer Programming	28	47%	47%
SIMULATION	Computational Simulation	15	25%	73%
IP	Integer Programming	4	7%	80%
MINLP	Mixed Integer Non-linear Programming	4	7%	86%
MISP	Mixed Integer Stochastic Programming	3	5%	92%
FMILP	Fuzzy Mixed Integer Linear Programming	2	3%	95%
ROP	Robust Optimization	2	3%	98%
NLIP	Non-linear Integer Programming	1	2%	100%

Source: elaborated by the author.

Mixed integer programming (MIP), mixed integer nonlinear programming (MINLP), and integer programming (IP), which are used to solve deterministic problems, represent 61% of the total. The presence of models for solving stochastic or dynamic problems, such as computational simulation, mixed stochastic integer programming (MISP), robust optimization (ROP), and mixed integer fuzzy programming (FMIP) corresponds to 33% of the total sample. Another finding concerns non-linear mathematical models. Combined, the mixed integer nonlinear programming (MINLP) models, and nonlinear integer programming (NLIP), represent 9% of the total sample.

Table 5 presents a statistical analysis of the solution methods used in the articles. Considering the three methods presented in Figure 1, it is observed that in the analyzed sample the exact and metaheuristic methods are the most used for solving vehicle routing problems and their variants applied to the milk-run system, with a total of 58 occurrences, representing 98% of the total.

Table 5 – Statistical analysis of resolution methods applied.

Method	Subgroup	Algorithm	Occurence	Percentage	Total by subgroup	Percentage by subgroup	Total by method	Percentage by method
Exact		DP-SIM	15	25%	29	49%	29	49%
		DP	12	20%				
		CG	2	3%				
Heuristics	Construction	NS	2	3%	4	7%	6	10%
		S	2	3%				
	Two-phase	CR	1	2%	2	3%		
		RC	1	2%				
Metaheuristics	Local Search	TS	6	10%	17	29%	29	49%
		SA	5	8%				
		GRAS	2	3%				
		ILS	2	3%				
		LNS	1	2%				
		VNS	1	2%				
	Population Search	GA	5	8%	12	20%		
		ACO	3	5%				
		EA	2	3%				
		MA	1	2%				
		SS	1	2%				

Source: elaborated by the author.

The sum of the percentages presented is greater than 100% since some articles present more than one method for solving the problem, also known in the literature as a hybrid solution. Of the total of 59 articles, 5 present hybrid solutions, or 8% of the total.

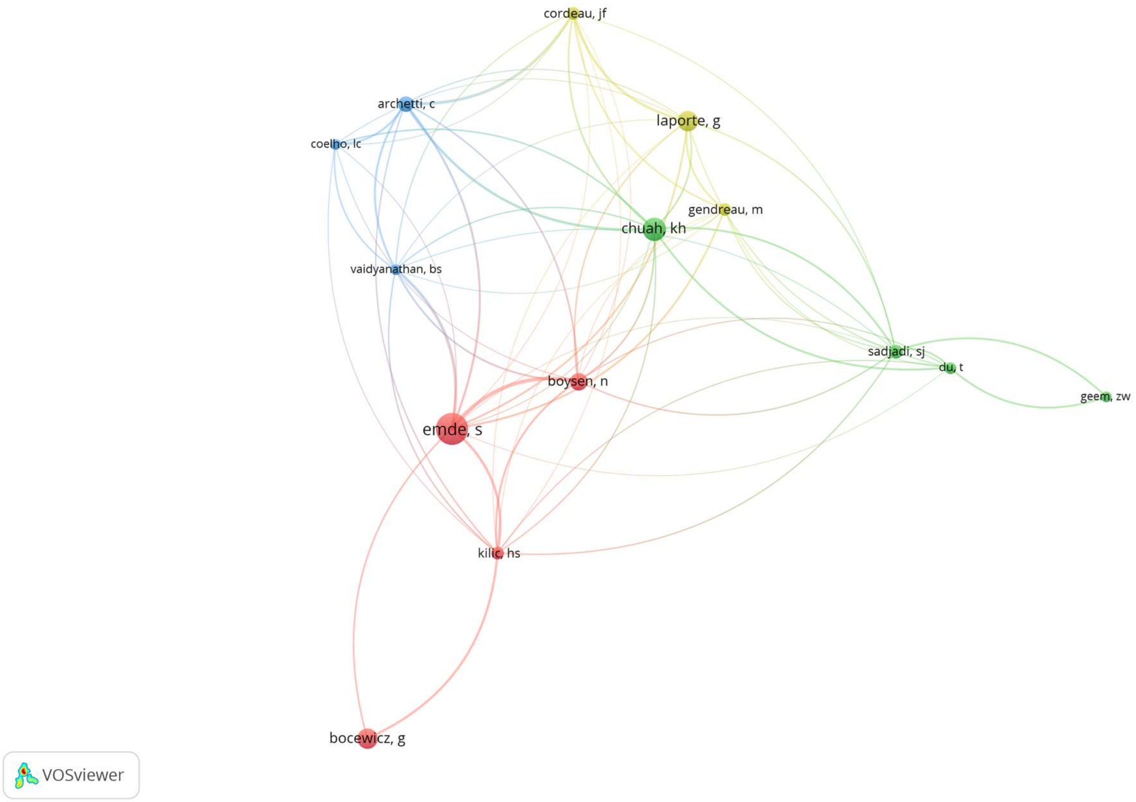
Among the exact methods, the computational simulation (DP-SIM) and dynamic programming algorithm (DP) stand out, being applied in 45% of the total sample, while the column generation (GC) algorithm is applied in 3%. Among the classic heuristics, the subgroup of construction heuristics is applied in 7% of the total sample, equally divided between the nearest neighbor search (NS) and economies (S), while the two-phase one is applied in only 2 articles, representing 3% of the total, one occurrence of the cluster-route algorithm (CR) and another of the cluster-route (RC).

Among the metaheuristics, the most representative method of the sample, the subgroup of local searches stands out, corresponding to 29% of the total. In this subgroup, the tabu search algorithm (TS) is the most used, applied in 6 articles of the total sample, corresponding to 10% of the total. Next are the simulated annealing (SA) algorithms, with 8% of the total, and adaptive randomized greedy search (GRAS), with 3% of the total. Three other algorithms of this subgroup, iterative local search (ILS), local neighborhood search (LNS), and variable neighbor search (VNS), are less used to solve milk-run problems.

In the subgroup of population searches, still considering the same method, the algorithm with the highest occurrence is the genetic one (GA), applied in 5 articles, corresponding to 8% of the total occurrences. Then, the ant colony optimization algorithm is applied in 3 articles, representing 5% of the total, and the evolutionary algorithm in 2 articles, or 3% of the total sample. Finally, the memetic and scattered search algorithms are applied to 1 article each, representing 2% of the total.

In Figure 9, an analysis of co-citation by cited authors was created using VOSviewer. The software considers the linkage of the authors cited at least 10 times in the sample analyzed. A total of 14 authors' linkage was created. The two most cited authors in the sample, represented by the circle's size are "Emde, S" and "Chuah, KH". The diagram also suggested four clusters illustrated by different colors. The clusters and the linkage between them will be useful for the detailed analysis presented in the following subsections.

Figure 9 – Co-citation analysis by cited authors, considering at least 10 citations.

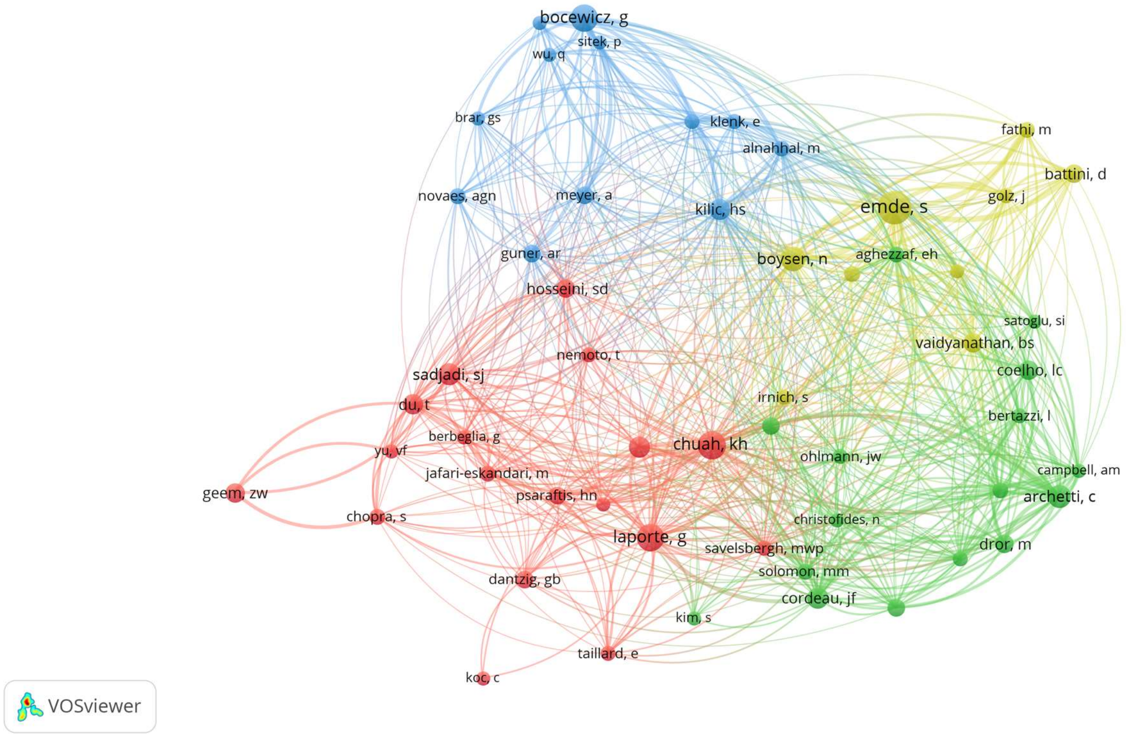


Source: elaborated by the author.

Figure 10 also presents an analysis of co-citation by cited authors but now considering the authors that were mentioned at least 5 times in the analyzed sample. The total number of authors cited now increases to 55, and the same four clusters were presented by the software. Increasing the number of authors cited will be helpful for the clustering classification analysis.

It can be noticed that “Emde, S” and “Chuah, KH” now changed from other clusters, considering a greater number of authors to be correlated.

Figure 10 – Co-citation analysis by cited authors, considering at least 5 citations.



Source: elaborated by the author.

Many of the authors shown in Figure 10 are their own sample articles’ authors. Other authors like “Cordeau, JF”, “Campbell, AM”, “Coelho, LC” and “Laporte, G”, are also very mentioned in the sample due to their contributions to the VRP and its variants formulations and resolution methods.

Secondly, the articles were classified according to the logistics context of the milk run system application. The result can be seen in Figure 11. Most of the authors applied the VRP to solve “inbound” logistics problems. In this context, the problem consists of collecting or picking up materials from many suppliers to feed a manufacturing plant or depot. The second major group is related to “in-plant” logistics. In this case, the VRP is applied to move parts from one or many depots to feed many consumption points in an assembly line.

Figure 11 – Logistics context of milk run system application of the SLR articles.

Inbound	**Outbound**	**In-plant**
Adriano et al. (2020)	Euchi et al. (2015)	Bocewicz et al. (2021a)
Bertazzi et al. (2019)	Hosseini et al. (2014)	Bocewicz et al. (2019)
Cakir et al. (2022)	Hosseini et al. (2014)	Bocewicz et al. (2021b)
Chang et al. (2009)	Kocaoglu et al. (2020)	Emde et al. (2018)
Chen et al. (2020)	Lee and Prabhu (2016)	Grzegorz et al. (2020)
Chen and Sarker (2014)	Ma and Sun (2013)	Hasani and Zegordi (2020)
Du et al. (2007)	Vaidyanathan et al. (1999)	Satoglu and Sipahioglu (2018)
Hein and Almeder (2016)	Yang and Liu (2013)	Simik et al. (2020)
Jiang et al. (2010)	You and Jiao (2014)	Zhang et al. (2021)
Liao et al. (2017)	Zhang et al. (2013)	Zhou and Zu (2020)
Lin et al. (2015a)	Chang et al. (2004)	Peng et al. (2020)
Mao et al. (2019)	Villarreal et al. (2016)	Emde and Schneider (2018)
Meyer and Amberg (2017)	Du et al. (2005)	Satoglu and Sahin (2013)
Mu and Eglese (2013)		Güner et al. (2017)
Novaes et al. (2015)		Staab et al. (2016)
Ohlmann et al. (2008)		Zammori et al. (2016)
Quan et al. (2021)		Kopecek and Pinte (2015)
Ranjbaran et al. (2019)		
Sadjadi et al. (2009)		
Volling et al. (2013)		
Wu et al. (2018)		
Yi et al. (2007)		
Tarantilis and Kiranoudis (2001)		
Kim et al. (2005)		
Chuah and Yingling (2005)		
Lee et al. (2014)		
Cross-docking		
Karakostas et al. (2020)		
Gschwind et al. (2020)		
Lin et al. (2015b)		

Source: elaborated by the author.

“Outbound” logistics consists of the application of the VRP to distribute goods from one or many depots to many customers. “Cross-docking” logistics, applied by only two authors, is related to the application of VRP for both operations of collecting parts from suppliers to the manufacturing plant or depot and distributing the final goods to customers.

Comparing the logistics context of the SLR articles shown in Figure 11 with the co-citation analysis done in VOSviewer software, it can be noticed that those authors that applied the VRP to solve “inbound” logistics problems are found predominantly in green and blue clusters of Figure 10; those ones that used for “outbound” problems in clusters red and yellow and, those ones that used for “in-plant” logistics in clusters yellow and blue.

2.3.3. Classifications of the Problems

The following sections will shed light on the basic VRP components, generally presented as classification methods by some authors (IRNICH et al., 2014; TOTH and VIGO, 2014). Mainly classifications concerned the transportation requests and how they can be performed, the fleet of vehicles and its characteristics, the related costs and profits, and the feasibility of the routes.

In this sense, the next subsections are divided into each topic of VRP classification, which are: the network characteristics, the transportation requests, the intra-route constraints (also called local constraints), the fleet characteristics, the inter-routes constraints (also called global constraints), and the objectives of the models.

According to the *network* characteristics, the VRP can be classified into two different types of problems. If the problem considers the service points as the vertices of a graph (customers, suppliers, depots) the problem is called Node Routing Problem (NRP). In case the model considers the services to be performed along street segments, for example, also called connections or links, the problem is named the Arc Routing Problem (ARP).

There are many specific applications for the arc routing problem like salt greeting and snow plowing or removal. Other examples are mail delivery, garbage collection, and meter reading. Even a mixture of tasks on vertices and arcs or edges is possible, leading to the General Routing Problem (GRP).

Despite several applications of the ARP, in the articles analyzed in this SLR all the models consider the Node Routing Problem (NRP) extension of the VRP. It means that all plants, depots, customers, and suppliers are allocated on the arc nodes. Also, all the models present a direct network. That could be expected since the Lean Logistics and milk-run system considers the service of pickup and delivery between suppliers, manufacturers, depots, and customers. Even in the case of problems that consider the in-plant logistics context, the models consider servicing many nodes or consumption points to deliver parts from one or many depots to the assembly line.

Regarding the origin and destination, in the sample analyzed most of the problems consider a single origin, 93% of the total, and only 7% considered multiple origins. On the other hand, only 3% of the articles consider it a single destination, and 97% of them consider multiple depots, customers, or suppliers. The problems that consider multiple origins also consider multiple depots, and they are related to “cross-dock” or “outbound” (distribution) logistics operations.

In relation to *delivery and collection* matters, the traditional VRP proposed by Dantzig and Ramser (1959) considered the fuel distribution problem from one depot to many customers. The counterpart for delivery of goods to customers is collection from customers or suppliers. Collections can be also called pickups. Problems with collection are also defined as a many-to-one VRP whilst problems with distribution as one-to-many VRP. Some variants of the VRP consider both collection and distribution (pickup and delivery) of goods occurring together in a route.

In the sample analyzed, 92% of the articles’ models require simultaneous pickup and deliveries at the nodes. The remaining 8% request either linehaul or backhaul service at the nodes, but not both. The milk run system is well known for its characteristics of collecting materials from suppliers and delivering empty returnable packages in combined routes. Even in the “in-plant” logistics context, the milk run system consists of delivering parts to consumption points in the assembly line and collecting empty returnable packages for refilling. From the 8% of the models that don´t request simultaneous deliveries and pickups, many of them are related to “cross-dock” or “outbound” logistics contexts.

One of the variations of the VRP that considers simultaneous pickups and deliveries is the Vehicle Routing Problem with Simultaneous Pickups and Deliveries (VRPSPD). In this variation, the vehicle capacity constraints ensure that no vehicle is overloaded at any point. A feasible route exists if neither the total amount to be collected nor that to be delivered exceeds the vehicle capacity. In the sample analyzed, the VRPSPD is used in just one article (SATOGLU and SIPAHIOGLU, 2018).

When observed by the point-to-point transportation approach, in the case of pickup-and-delivery problems, the VRP’s transportation requests consist of point-to-point transportation. Each transportation request consists of the movement of goods between two locations, one considering materials pickup, and another one considering its delivery. This variant of the VRP is called the Pickup-and-Delivery Problem (PDP). The PDP is used by two authors in the analyzed sample (BOCEWICZ et al., 2019; RANJBARAN et al., 2020). The first considers the variation applied to the “in-plant” logistics context and the second for the “inbound”.

As regards Repeated Supply, in the context of material supply, the customers may require repeated provision of goods. Considering a planning horizon, stock levels will determine the delivery frequencies. The Periodic VRP (PVRP) is a variant that considers repeatedly supply and has two planning levels. At the first level, a visiting pattern for each customer must be chosen from a feasible set. For the second one, a VRP must be solved for each day, considering the customers to be visited and their demand. In the sample analyzed, four authors used the PVRP to solve their problems (LIAO et al., 2017; LIN et al., 2015; MAO et al., 2020; MEYER and AMBERG, 2018). All of them applied this variation to solve "inbound" logistics problems.

Another variant used to treat the repeated supply is the Inventory Routing Problem (IRP). The main difference of this variation is that no custom orders are considered in an IRP. Instead, the deliveries should be determined according to the stock level of customers so that no stock out occurs. In this study, the IRP was the second variant more applied, found in 11 articles (19% of the total). Bocewicz et al. (2019), Goodarzi and Zegordi (2020), Satoglu and Sipahoglu (2018), Peng et al. (2020) and Zhou and Zhu (2020) applied the IRP variation to solve "in-plant" logistics problems, Bertazzi et al. (2020), Chen et al. (2020), Jiang et al. (2010) and Sadjadi et al. (2009) to solve "inbound" logistics problems, Yang and Liu (2013) to solve an "outbound" logistics problem, and Karakostas et al. (2020) to solve a "cross-dock" logistics problem.

Mentioning non-split and split Services, if the service tasks are performed by a single vehicle in the VRP, it is considered a non-split problem. However, if the demand for the service exceeds the vehicle capacity, more than one visit will be required, and split service is allowed. Splitting services into several smaller service requests can also produce significant cost savings, for example, when combining pickups from more than one supplier or deliveries to more than one customer.

In the sample analyzed, 78% of the articles consider non-split services, while the remaining 22% consider split services. It´s important to mention that despite most of the articles considering non-split problems (in the same route) the models can consider more than one route servicing the same supplier or customer. It means that instead of considering splitting services within the same route, generally, the models consider that more than one route is required, defined as pickup or delivery frequency.

Regarding dynamic and stochastic Routing, other important variants of the VRP, the Dynamic VRP (DVRP) and the Stochastic VRP (SVRP) arise with the consideration of uncertainty and variability of system conditions. A problem is considered dynamic when some information about the system is available during the service operation (online information). The problem is considered stochastic if some of the system's conditions are uncertain, and the uncertainty is described by a probability distribution.

In the sample analyzed, the DVRP is the most applied variation to solve lean logistics VRP problems, presented in 16 articles, representing 27% of the total. According to its characteristics, the DVRP is mainly solved by computational simulation methods, as shown in Table 5. Adriano et al. (2020), Chang et al. (2009), Chen and Sarker (2014), Du et al. (2007), Kim et al. (2005), Lee at al. (2014) and Novaes et al. (2015) applied the DVRP to solve "inbound" logistics problems. Emde et al. (2018), Güner et al. (2017), Kopecek and Pinte (2015), Simic et al. (2021) and Staab et al. (2016) to solve "in-plant" logistics problems and Du et al. (2005), Euchi et al. (2015), Lee and Prabhu (2016) and Villarreal et al. (2016) to solve "outbound" logistics problems.

Regarding the SVRP, it was used in 8 articles, corresponding to 13,5% of the total amount. Chuah and Yingling (2005), Hein and Almeder (2016), Ohlmann et al. (2008), Volling et al. (2013) and Wu et al. (2018) applied the SVRP to solve "inbound" logistics problems. Chang et al. (2004) and Zhang et al. (2013) to solve "outbound" logistics problems and Zammori et al. (2016) to solve an "in-plant" logistic problem.

The *intra-route constraints*, also called local constraints, will determine whether a route is feasible or not. The local constraints are related to vehicle loading, route length, reuse of the vehicles, time schedules, and the combination of these items occurring in practice.

Loading constraints are related to capacity constraints. The Capacitated VRP (CVRP) is one of the first variations of the VRP, including an overall bound on a resource that is consumed at every node reached by the vehicles.

Several capacity constraints can be included in the VRP, considering the weight, space, and volume. More complex loading constraints occur when both the shipments and the cargo are described either by 2-dimensional or 3-dimensional quantities. For each additional capacity constraint, their corresponding coefficients q_i and Q must be added to the model.

In the case of the articles analyzed in this study, a total of 11 items, or 19% of the sample, applied the CVRP variant to solve lean logistics VRP problems. Hosseini et al. (2014a), Hosseini et al. (2014b), Kodagu et al. (2020), Vaidyanathan et al. (1999) and You and Jiao (2014) applied the CVRP to solve "outbound" logistics problems, Mu and Eglese (2013), Tarantilis and Kiranoudis (2001) and Yi (2007) to solve "inbound" logistics problems, and Bocewicz et al. (2021), Emde and Schneider (2018) and Zhang et al. (2021) to solve "in-plant" logistics problems.

From the total, 58 articles consider capacitated vehicle constraints, and only one considers incapacitated vehicles. Furthermore, all the articles consider the capacity constraints related to weight or volume. No cases of 2-dimensional or 3-dimensional problems are found in the application of VRP for lean logistics context.

The route length, another type of local constraint, results from the definition of bounds that limit the consumption of a resource consumed on edges or arcs. Not only can the spatial distance be modeled by this restriction, but also constraints considering route duration (with travel times t_{ij}), routing costs, or the number of connections with certain property, with indicators $t_{ij} \in \{0,1\}$.

Environmental questions have been raised in recent years and included in the VRP variants. There are three articles in the analyzed sample that consider fuel consumption or CO^2 emissions as resources restrictions: Emde and Schneider (2018), Karakostas et al. (2020) and Quan et al. (2021).

Regarding the multiple use of vehicles, generally, the standard assumption for many variants of the VRP is that each vehicle performs one route over the planning horizon T. In the VRP with Multiple use of Vehicles (VRPM), the vehicles can perform several routes within the planning horizon. Considering the routing durations as $T_1, T_2, \dots, T_p$, a vehicle can perform $T_1 + T_2 + \cdots + T_p \leq T$ within a planning horizon if no additional restriction is included (for example, driver's working time limitations).

In the analyzed sample, no article has used VRPM variation for lean logistics VRP application. However, as already mentioned before, the use of pickup or delivery frequency can threaten the multiple use of vehicles during the planning horizon in an implicit way. Of the total of 59 articles, 14 of them, or 24%, consider the frequency depending on pickups or deliveries. Most of them are applied to solve "inbound" logistics problems. It can be noticed that the frequency pickup is a characteristic of lean logistics milk-run system, as demonstrated by Chuah and Yingling (2005), Mao et al. (2020), Meyer and Amberg (2018) and Ohlmann et al. (2008).

In relation to time windows and scheduling aspects, some variants of the VRP are those related to scheduling, for example, considering travel, service, and waiting times through time-window constraints. In the VRP with Time-windows (VRPTW) a travel time t_{ij} for each arc $(i, j) \in A$ and a time window $[a_i, b_i]$ for each vertex $i \in V$ are given.

There are some variations of time-widows' constraints. In the VRP with Soft Time-windows (VRPSTW), a convex function $p_i: [a_i, b_i] \rightarrow \mathbb{R}$ is used to generate solutions in which service is provided at a time close to the minimum of p_i. In the case of strict time windows, a penalty cost is considered in case of early or late arrivals at the vertex. The function of lateness can also be considered in the objective function and will be discussed in subtopic 4.3.6.

From the sample SLR articles, 17 of them consider soft time-windows constraints, representing 29% of the total, 19 articles consider strict time-windows constraints, representing 32% of the total, and another 23 do not consider time-windows constraints. The summary of time-windows constraints based on the logistics context of the VRP problem is presented in Table 6.

Table 6 – Time-windows constraints based on each logistics context.

Logistics Context	Soft Time-windows	Strict Time-windows	Time-windows not considered
IN PLANT		10	7
INBOUND	9	8	9
INBOUND/OUTBOUND	1	1	1
OUTBOUND	7		6
Total	17	19	23

Source: elaborated by the author.

From Table 6 it can be noticed that for “in-plant” logistics problems, time-windows restrictions are considered strict time-windows, or they are not considered. In the second case, the VRP variants used are DVRP or IRP. In the case of DVRP, usually, models are solved by computational simulation method. For the IRP, time-windows constraints are not considered since the model formulation is based on stock levels.

In the case of “inbound” logistics problems, time-windows constraints are well distributed considering the three mentioned types. It’s also important to mention that in case time windows are not considered, the VRP variants applied to solve the problems are DVRP, IRP, or SVRP. The same condition happens for “cross-dock” logistics problems (inbound/outbound).

For the “outbound” logistics problems, time-windows constraints are considered soft time-windows, or they are not considered. In case of distribution problems, the most applied variant of VRP is the CVRP.

Fleet characteristics related to the local constraints are considered when different kinds of vehicles are used in a heterogeneous fleet. Also, they can be used when the vehicles can leave for more than one origin point or depot.

Regarding multiple depot VRP, if a homogeneous fleet is considered, but the vehicles start and end their routes at different depots, the resulting problem is called Multiple Depot VRP (MDVRP). There is just one article in the RLS sample that considers the MDVRP (Lin et al., 2015). The authors considered multiple depots to solve a “cross-docking” logistics problem.

When the multiple depots model is considered with a heterogeneous fleet, the index k is set for depot p_k where $k \epsilon K$ set of different vehicles.

Concerning heterogeneous or mixed fleet VRP, the family of heterogeneous or mixed fleet VRP (HFVRP) considers more than one kind of vehicle that can differ in capacity, variable and fixed costs, speeds, and the nodes they can access. The fleet K is then divided into $|P|$ subsets of homogeneous vehicles $K = K^1 \cup K^2 \cup \ldots \cup K^{|P|}$, also called vehicle types. All the vehicles $k \in K^P$ from the pth type $(p = 1, \ldots, |P|)$ are characterized by a capacity $Q_k = Q^P$, variable routing costs $c_{ijk} = c_{ij}^P$, fixed costs $FC_k = FC^P$, and the subset $N_k = N^P \subseteq N$ of accessible nodes, and individual travel times $t_{ijk} = t_{ij}^P$ that can replace the standard travel times t_{ij}.

Among the RLS articles, one of them considers just one vehicle in the problem, 40 of them a limited number of vehicles, and the remaining 18 an unlimited number of vehicles composing the fleet. In addition, almost all the articles, 58 of them, consider capacitated vehicles, while just one considers non-capacitated vehicles. Regarding the fleet groups, 47 articles consider a similar type of vehicle, 10 consider heterogeneous fleet, and 2 consider load-specific constrained vehicles.

The *inter-route constraints* are also called global constraints, and their feasibility solution relies on how the routes (and their schedules) are combined. There are three main examples of how routes can be combined through global constraints. The first one considers balancing constraints that can result from fair considerations, e.g., the difference between maximum and minimum route duration in a solution should not be exceeded by a threshold.

The second example consists of inter-route resource constraints which occur when different vehicles compete for globally limited resources. And the third one is related to synchronization issues. The schedule of different vehicles in this issue must be coordinated. There can be settled tasks, operations, movements, loadings, or resource synchronizations among different routes.

In the analyzed sample, there were some examples of inter-route constraints. Kocaoglu et al. (2020) considers the driver workload restriction as an inter-route constraint in their model for an “outbound” milk run system, according to the first example mentioned above. Chuah and Yingling (2005) and Ohlmann et al. (2008) considered the third example mentioned. In their “inbound” milk run models, the vehicles with different pickup frequencies cannot attend the same supplier.

In terms of *solution objectives*, generally, the basic VRP problems such as the CVRP aim for cost minimization objectives. However, there are many other goals to be considered in the VRP problems. In this subsection, we classified the 59 SLR articles according to their objective function. There are three main classifications according to the model objectives, that are: single objective optimization, hierarchical objectives, and multi-criteria optimization.

Which matters to single objective optimization, in addition to cost minimization, the objective functions of VRP problems can also be related to route durations, distance minimization, and time of finishing routes (function of lateness). In VRP problems in which heterogeneous fleets are considered, the objective function can also be related to vehicle fleet definition and selection. Customer satisfaction can be also observed as a goal for VRP problems, as well as transportation tariffs and drivers' working hours. According to Irnich et al. (2014), a recently introduced and interesting area of research that incorporates energy consumption and pollutant emission control in the routing context.

Table 7 consolidates the objective function of single objective analyzed articles. It´s important to mention that single objective models correspond to 86% of the total papers, 51 from the total of 59. The main goal identified is related to the function of cost, with 27 occurrences, that correspond to more than 50% of single objective optimization goals. The cost-dependent objectives can be related to the transportation costs (fixed and variable) and inventory costs.

Table 7 – Single objective goals of the analyzed articles.

Objective	Occurrences	Percentage
Function of cost	27	53%
Travel time dependent	10	20%
Distance dependent	6	12%
Function of lateness	4	8%
Vahicle dependent	2	4%
Other	2	4%
Total	51	100%

Source: elaborated by the author.

The following most important objective is related to travel time, with 10 occurrences, representing 20% of single objective goals. Travel time is an important variable for the Lean Logistics context since one of its aims is to reduce WIP stock levels. There are also objective functions related to travel distance (12%), function of lateness (8%), and vehicle dependent (4%).

Two articles used other objective functions not mentioned above in their models. (Chen et al., 2020) developed a model for vehicle routing type selection. The objective function will determine if the better route should be done through milk-run or direct shipment models. (Chen and Sarker, 2014) considered customer satisfaction in their objective function.

Energy consumption and pollutant emission were not considered as a goal in the objective function of analyzed single objective models, although it was considered as a restriction in the model presented by Emde et al. (2018), Karakostas et al. (2020) and Quan et al. (2021).

Regarding the hierarchical objectives, in general, route length, duration, and completion time are conflicting objectives. When fleet decision is added, the number or type of vehicles used also conflicts with these goals. Because of that, hierarchical objectives can be formulated to solve this issue. In the first stage, fleet goals are prioritized, and then the ones related to route length, duration, and completion time, for example.

In the sample analyzed, 4 articles consider hierarchical objectives, corresponding to 7% of the total. Lin et al. (2015) considered a two-stage optimization model that determines the number of vehicles in the first decision and then the better route of each vehicle in the second one. Chang et al. (2004) applied an SVRP in which the objective function is to minimize the total cost of the first-stage solution and the expected recourse cost of the second-stage solution. Emde and Schneider (2018) also used a hierarchical optimization that minimizes first the number of vehicles, then the total duration of all routes. Gschwind et al. (2020) proposed a model in which the objective function priority to the minimization of the fleet in the first stage and as a secondary objective minimizes the total weighted flow time.

Furthermore, related to the multi-criteria optimization, another approach for solving VRP models considering conflict decision goals is the multi-criteria optimization. In this case, both conflict objectives are considered in the objective function and can be weighed by the decision maker. In the sample analyzed, 4 articles consider multi-criteria objectives, corresponding to 7% of the total.

Çakir et al. (2022) proposed a multi-criteria VRP decision model that minimizes the total distance and the total time-window costs (function of lateness). Mu and Eglese (2013) apart from minimizing total travel distance, also considered in their multi-criteria decision model to minimize the delay to the services, the overtime for the drivers, and the deviation from the original plan. You and Jiao (2014) used a multi-criteria objective model in which the goals are the smallest total distance together with transportation costs. Zhang et al. (2013) considered three different goals in the multi-criteria objective function. The first one is fixed vehicle employment costs, the second is mean travel time, and the third is the weighted expected earliness, tardiness, and excess route duration.

In the case of hierarchical and multi-criteria objectives, no author has included energy consumption and pollutant emission – environment considerations – in their models, neither in the objective function nor as a restriction in the model.

2.3.4. SLR Final Considerations and Research Opportunities

The SLR methodology provided valuable insights for the next chapter and model formulation of the case study. SLR was conducted through structured research and a descriptive bibliometric and content analysis of the final articles selected for detailed analysis. Table 8 summarizes the result of the content analysis presented in topic 2.3.3, also considering the logistics proposal of the milk-run system – (i) inbound, (ii) outbound, (iii) in-plant, and (iv) cross-dock.

From the summary, it is possible to observe that (i) inbound transportation is mainly formulated using *mixed integer programming* and *simulation* methods. Also, metaheuristics are used for solving this type of model. The most used variations of the VRP are DVRP, SVRP, IRP, and PVRP. Information characteristics are mainly deterministic, and the fleet characteristics are homogeneous. In terms of time-windows restrictions, there is an equal distribution between soft and strict time-windows constraints and models in which time-windows were not considered. Almost all the articles included in this group applied a single objective for model solving, and most of them considered a function of cost or distance dependence.

From the bibliometric and content analysis, it is also possible to affirm that the VRP models used for solving inbound lean logistics problems have some similar characteristics such as inventory dependence, justified by the use of IRP structures, considering a function of stock in the suppliers or the main plant or depot; periodic pickups, that can be considered as frequency dependent, implying the use of PVRP structures. Also, the time-windows restriction can be included or not in this type of problem.

The (ii) outbound logistics models are also formulated using *mixed integer programming* and *simulation* methods and solved by metaheuristics. In this case, the most common variants of the VRP are the CVRP, DVRP, and SVRP. The information characteristics are also deterministic, and the fleet is homogeneous. In outbound logistics transportation, there is no occurrence of strict time-windows constraints. The objective functions are mainly single ones, considering mainly the travel time.

In the case of outbound systems, the structure of the routes is different from inbound ones. In the first case, predominantly the routes can be variable, and the set of customers or depots visited. Also, the function of time (time-windows restrictions) is not so severe since there is no big impact of delaying supply, which can be solved by the travel time dependence of the objective function. The CVRP is mainly used since there are no other important restrictions above this in the outbound supply models.

In the case of (iii) in-plant logistics problems, the formulations are developed using *mixed integer programming* and *simulation*. In this case, the main resolution method used is the exact method. The most applied VRP variants are the IRP, DVRP, and CVRP. The models are mainly deterministic, and all the articles analyzed in this case used homogeneous fleets. In the case of in-plant logistics problems, a special characteristic is the predominant usage of strict time-windows constraints. Objective functions mainly treat a single objective related to cost, travel time, or vehicle utilization.

As mentioned, the in-plant logistics problems in the lean environment had a special characteristic of strict time-windows constraints. These structures are used because there are frequent supply routes necessary to feed the production line with small and constant volumes. Since the stock is reduced in this environment, strict time-windows constraints ensure no lack of parts in this milk-run feed system, which also uses the IRP as a structure to consider the inventory levels at supply points and depots. In this type of problem, the vehicles, generally tow motors with the same capacity are used to supply the production lines, which is why fleets are predominantly homogeneous.

Finally, the (iv) cross-dock logistics problems have few occurrences in the lean environment application of the VRP, which results in few and sparse data for a detailed analysis of its characteristics. Of the total of 59 articles, only 3 used the cross-dock, which is a combination of more than one scenario presented in this topic.

Another important finding to be noticed is that for both cases, there is a considerable incidence of DVRP structures to solve more recent problems that considered transportation traffic characteristics, online vehicle tracking, and the use of simulation methods for solving.

Most of the authors considered single objective models for solving the application of VRP in the lean logistics environment. As lean logistics have some special trade-off characteristics such as pickup frequency, supply timing, inventory levels, and capacity, some multi-criteria structures should be considered for a better understanding of these trade-offs and their impacts on inventory and cost. Another point to be raised is that few articles considered the environmental impact of vehicle emissions considering the frequent pickups and deliveries performed by milk run systems. The pollutant emissions considerations should be also considered in the trade-off analysis since it is an increasingly relevant topic in VRP literature.

Table 8 – The summary of SLR content analysis.

	Inbound			Outbound			In-plant			Cross-dock		
Mathematical model	MIP	14	54%	MIP	4	31%	MIP	7	41%	MIP	3	100%
	SIMULATION	6	23%	SIMULATION	4	31%	SIMULATION	5	29%			
	IP	2	8%	IP	2	15%	MINLP	2	12%			
	MINLP	2	8%	MISP	2	15%	FMILP	1	6%			
	FMILP	1	4%	NLIP	1	8%	MISP	1	6%			
	ROP	1	4%				ROP	1	6%			
Resolution method	Exact	6	23%	Exact	1	8%	Exact	6	35%	Exact	1	33%
	Heuristics	1	4%	Heuristics	2	15%	Heuristics	1	6%	Heuristics	1	33%
	Metaheuristics	14	54%	Metaheuristics	7	54%	Metaheuristics	5	29%	Metaheuristics	1	33%
Variation of the VRP	DVRP	7	27%	CVRP	5	38%	IRP	5	29%	IRP	1	33%
	SVRP	5	19%	DVRP	4	31%	DVRP	5	29%	MDVRP	1	33%
	IRP	4	15%	SVRP	2	15%	CVRP	3	18%	VRPTW	1	33%
	PVRP	4	15%	IRP	1	8%	PDP	1	6%			
	CVRP	3	12%	VRPTW	1	8%	SVRP	1	6%			
	VRPTW	2	8%				VRPSPD	1	6%			
	PDP	1	4%				VRPTW	1	6%			
Information characteristics	Deterministic	14	54%	Deterministic	7	54%	Deterministic	11	65%	Deterministic	3	100%
	Stochastic	5	19%	Stochastic	2	15%	Stochastic	1	6%			
	Dynamic	7	27%	Dynamic	4	31%	Dynamic	5	29%			
Fleet characteristics	Homogeneous	19	73%	Homogeneous	9	69%	Homogeneous	17	100%	Homogeneous	2	67%
	Heterogeneous	7	27%	Heterogeneous	4	31%			0%	Heterogeneous	1	33%
Time-windows constraints	Soft time-windows	9	35%	Soft time-windows	7	54%				Soft time-windows	1	33%
	Strict time-windowns	8	31%				Strict time-windowns	10	59%	Strict time-windowns	1	33%
	Not considered	9	35%	Not applicable	6	46%	Not applicable	7	41%	Not applicable	1	33%
Objective characteristics	Single objective	24	92%	Single objective	10	77%	Single objective	16	94%	Single objective	1	33%
				Hyerarchical objective	1	8%	Hyerarchical objective	1	6%	Hyerarchical objective	2	67%
	Multi-criteria objective	2	8%	Multi-criteria objective	2	15%						
Single objective goal	Function of cost	14	54%	Function of cost	2	15%	Function of cost	10	59%	Function of cost	1	33%
	Travel time dependent	2	8%	Travel time dependent	4	31%	Travel time dependent	4	24%			
	Distance dependent	4	15%	Distance dependent	2	15%						
	Function of lateness	2	8%	Function of lateness	2	15%						
							Vehicle dependent	2	12%			

Source: elaborated by the author.

3. PROBLEM STATEMENT

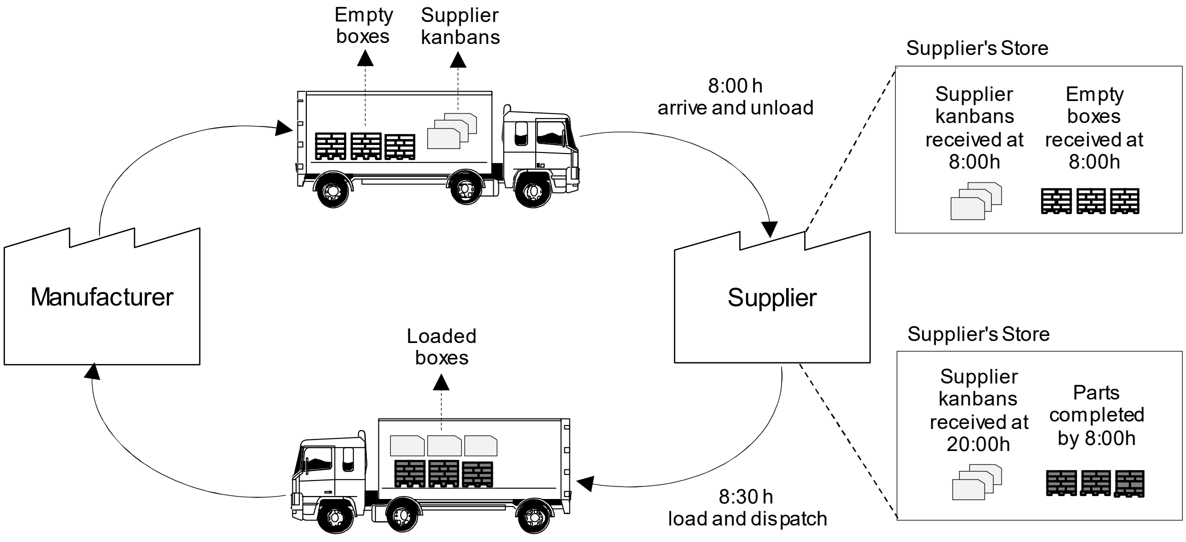
This chapter describes the milk run planning problem in the context of a lean manufacturing system. In Section 1, we provide contextualization by presenting an overview of the lean logistics process for supplying assembly lines. In Section 2, we introduce the case study based on an automotive OEM utilizing the milk run system for assembly line supply within this lean logistics framework.

3.1. Overall Description of Lean Logistics Milk-run System

In the Lean Logistics context, the milk-run system manages the logistics operation between an OEM (Original Equipment Manufacturer) and its suppliers. The system controls the sequence, timing, and frequency of materials picked up from the suppliers and delivered to the manufacturing plant. The general idea of this problem is to supply parts in regular, frequent, and small quantities.

The objective of using the milk-run system is to minimize transportation costs while making frequent deliveries of materials in small quantities. The milk-run system consists of consolidating many small shipments into full truckloads involving scheduled pickup routes that visit many suppliers. Besides, the benefit of consolidating the cargo between more than one supplier is the inventory reduction at the manufacturer's plant since the pickups can be spread throughout the day. The milk run flow is explained in Figure 12.

Figure 12 – Example of a milk run route.



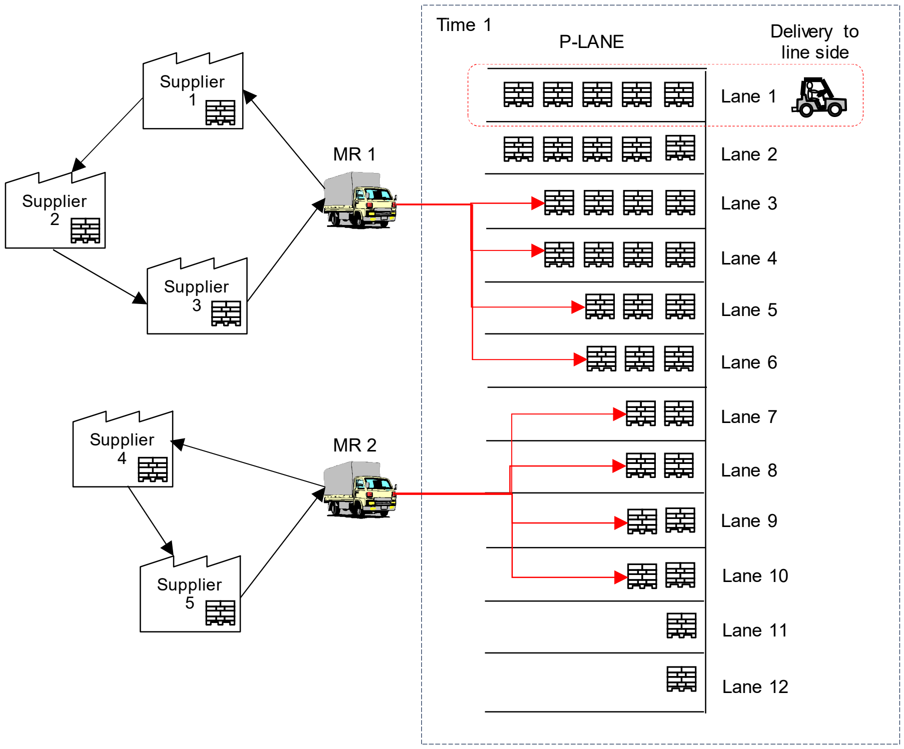
Source: adapted from (MONDEN, 2012).

The pickup frequency is two times per day, scheduled at 8:00h and 20:00h. It is considered that each truck loading at the supplier has enough parts to feed the manufacturer assembly line until the arrival of the next loading. In each return trip from the manufacturer plant to the supplier, empty boxes and the physical kanban cards are loaded. In the example, physical cards are shown to facilitate logical understanding, even if nowadays electronic information is utilized.

Usually, more than one supplier is served on the same milk-run route to improve transportation efficiency and to enable high pickup frequencies, as mentioned before, the lower the quantity of parts, the better for TPS. In general, as pointed out by Meyer (2015), the milk-run concept applied with a supplier kanban control is the result of the application of described lean principles: it allows for a smooth flow due to small lots pulled by kanban signals and frequent supplies within constant cycles.

According to Mao et al. (2020), the P-LANE is a stock area that stores the parts needed for one day manufacture's volume, as a terminal buffer to reduce the total cost and improve transportation efficiency. As shown in Figure 13, the P-LANE can be defined as a cross-dock process that connects the inbound and outbound logistics.

Figure 13 – Supply chain logistics process considering the progress lane function.



Source: adapted from Mao et al. (2020).

The P-LANE may contain several lanes, which are chosen depending on the produced volume per day, the available area in the assembly line, and the physical capacity of the P-LANE and can be considered as daily time supply slots. The parts collected from suppliers are assigned to the P-LANE after unloading according to the frequency of internal logistics.

Finally, the parts in each lane are taken to the assembly line by internal routes sequentially, following the production line-off progress. One lane can be considered as one daily slot time to be supplied to the line side. It converts the irregular and non-quantitative outbound logistics of collecting parts from suppliers into a regular and quantitative supply to the production line. The milk run routes can collect from suppliers several lanes in the same trip depending on the pickup frequency of that specific route. The parts are received at the P-LANE in the manufacturing plant and then sent to the line side at each defined time slot convenient for the internal logistics capacity.

According to the same authors, during the automobile manufacturing parts supply process, the P-LANE has three main effects: (i) makes the parts available with the production schedule – since it can prevent spillage and owing of parts inside the plant, thus reducing both the inventory costs and the shutdown risks; (ii) make the milk run more efficient, as the suppliers are allowed to be more flexible in determining the number of parts supplied per truck (compared with the direct shipment method); and (iii) realize high frequency and small volume supply of parts, making the progress difference between inbound and outbound logistics stable to ensure smooth production at the line side.

Two key conditions are necessary, according to Kotani (2007), to ensure a low WIP level and avoid high safety stock necessity when determining the order and delivery times: the lead time must be reduced as much as possible, and the intervals between two consecutive deliveries must be constant. Figure 14 illustrates that a constant cycle ensures not only levelled shipment sizes but also a lower WIP necessity since it is always the greatest time in between two consecutive deliveries, which will determine the number of kanban in the cycle.

Figure 14 – Inventory levels for a constant supply cycle time on the left and for varying cycle times on the right, assuming a constant demand over time.

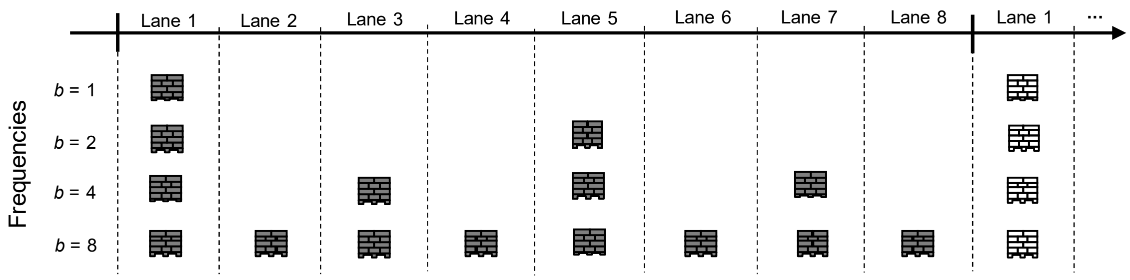
Source: adapted form Monden (2012).

From the example demonstrated in Figure 6, we can conclude that a constant supply cycle time can provide a more predictable and stable supply when compared to the varying cycle. In the Lean Logistics context, the constant cycle can match the component consumption due to the Heijunka method applied in Lean Logistics, explained in the next topic.

The heijunka method is related to the supply cycle mentioned in the above topic. For cycle length calculation, two important pieces of information are considered, the demand rate and physical dimensions of each part. Having different cycle times for each supplier, however, made consolidation between them on the same milk-run route more difficult, which is essential for realizing high frequencies at feasible transportation costs.

A study from a Toyota Manufacturing plant in the US, reported by Chuah and Yingling (2005), demonstrates that for a system with a cycle time of one day with a maximum delivery frequency of eight pickups only the frequency set {1, 2, 4, 8} is allowed. That means all inter-arrival times are multiple of the total working hours as shown in Figure 15.

Figure 15 – Pick-up pattern considering constant cycle times for the allowed frequencies of a Toyota Manufacturing plant in the USA.



Source: adapted from Chuah and Yingling (2005).

Figure 15 demonstrates that the frequency ratio will determine the number of receipts for a given part. In the illustrated example there are four possible frequencies, 1, 2, 4, and 8. If the frequency is 1, parts are received one time per day (in the figure, as an example, the pickup frequency $b = 1$ is allocated in *lane* 1). If $b = 2$, goods are received twice per day (in *lanes* 1 and 5, respecting the same time interval between the two consecutive receipts). If the frequency is $b = 8$, the maximum frequency in this case, the parts are received in every *lane* from 1 to 8 (considering that the P-LANE capacity is equal to 8 *lanes*).

According to the authors, the lower number of frequencies available increases the possibility of having more than one geographically close supplier with the same delivery frequency that can be joined on the same milk-run route. The policy combining only suppliers with common frequencies in the same milk-run route is called Common Frequency Routing (CFR) (CHUAH and YINGLING, 2005).

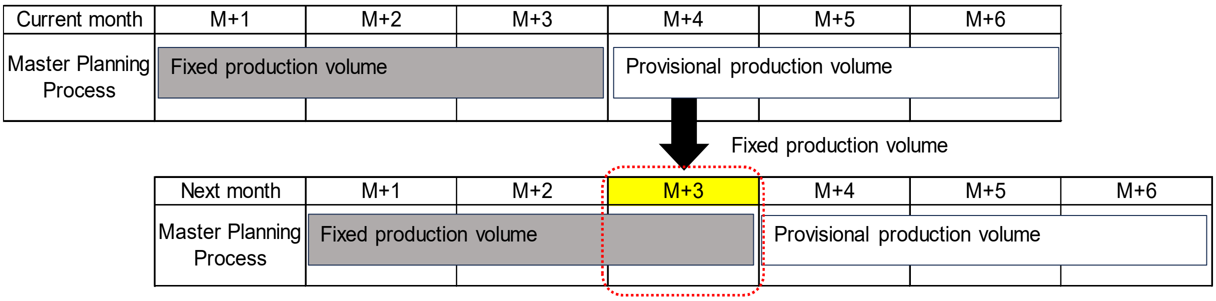
The milk run pickup system studied in this work, in terms of the pickup frequency can be defined as a Common Frequency Routing problem. The problem explanation and its specific characteristics are presented in the next topic.

3.2. Case Study: Lean Logistics Milk-run System

In this section, we describe in detail the process of milk run planning in the automobile industry in the context of the lean manufacturing system. The milk-run system planning process starts after the Master Production Planning (MPP) process. In the MPP process, the main goal is to balance the sales order and production capacity, avoiding overstock in the dealers and idle time in the production lines. In the case studied, the MPP process is executed once a month. In this process, the production volume is defined for the next six months, considering three months of fixed production volume and three months of provisional production volume, as illustrated by Figure 16.

This process takes place on a rolling planning horizon that links the mid-term Master Production Planning process with the short-term Operational Production Planning, where daily production volumes are fixed within the firming production month as highlighted in the Figure.

Figure 16 – Master production planning process.



Source: elaborated by the author.

After the total monthly volume is defined by each product model, the daily production volume is allocated by using the heijunka method for production leveling. Unlike the batch production process, lean manufacturing tries to distribute equally loaded distinct types of models among daily production volume. The result of leveling production volume is to balance the workload and keep the volume stable among planning periods. If otherwise, the production mix changes day by day, it will impact on the workload and the volume of components to be moved.

Production volume leveling is a crucial factor for milk-run coordination planning. Since the volume of components to be collected from each supplier is the same along the planning horizon (in the case studied one month), a delivery and pickup pattern can be defined for this period without considerable changes. As transportation planning and contracting are up to the manufacturer's responsibility, the production leveling volume allows the contractor to negotiate fixed routes during the planning period, achieving better negotiating costs.

In the case studied, the manufacturer logistics planning area subcontracts a 3PL logistics partner for executing the milk run routes. The milk run routing planning process is done manually considering the company's expertise in doing this process. As the route pattern follows the heijunka production volume, the main task during the planning period consists of evaluating the route's efficiency and the changes from the previous period. The planning starts two months before the production month (or one month after the production volume definition in the MPP process described in Figure 16).

Despite all the milk run planning and route schedules being defined by the manufacturer, the operation is executed by the third company, which has a non-limited fleet to attend to the contractor within the scheduled planning period. As explained before, the milk run route schedule is done two months before its execution. Because of that and knowing that the production and consequently pickup volumes will not change so much, the third company can revise its fleet, adapting to the new route schedule.

After the daily production plan is defined by each model, the product components' volume calculation proceeds based on the *bill of materials*, which contains the application of each component in each different model of the final product.

Each part is assigned to a supplier, defined by a supplier code, and identified with an unload dock code, which indicates the physical place it will be unloaded at the manufacturing plant. The indication of supplier code and unload dock code determines the loading and unloading place information for each component and will be used for milk-run routing planning. Also, each part has a specific package type.

The package types are standardized by the manufacturer and are their properties. The ownership of standard packages by the manufacturer provides volume efficiency improvement in external and internal logistics. This gain can be achieved since there are standard ways for package stacking, improving truckload efficiency, and in-plant logistics efficiency.

Each component is assigned to one standard box, and the boxes are palletized in a standard pallet. Both the boxes and the pallets are returnable packages used in the flow between the manufacturer and its suppliers. The logistics flow of returnable packages starts with the loading of empty packages in the milk-run routes at the manufacturer's dispatch docks. The packages are delivered to suppliers, and the truck is then loaded with assembly parts. As the production volume is calculated on a leveling pattern during the milk-run planning period, the volume of the empty package can be considered the same as the necessary load volume from suppliers.

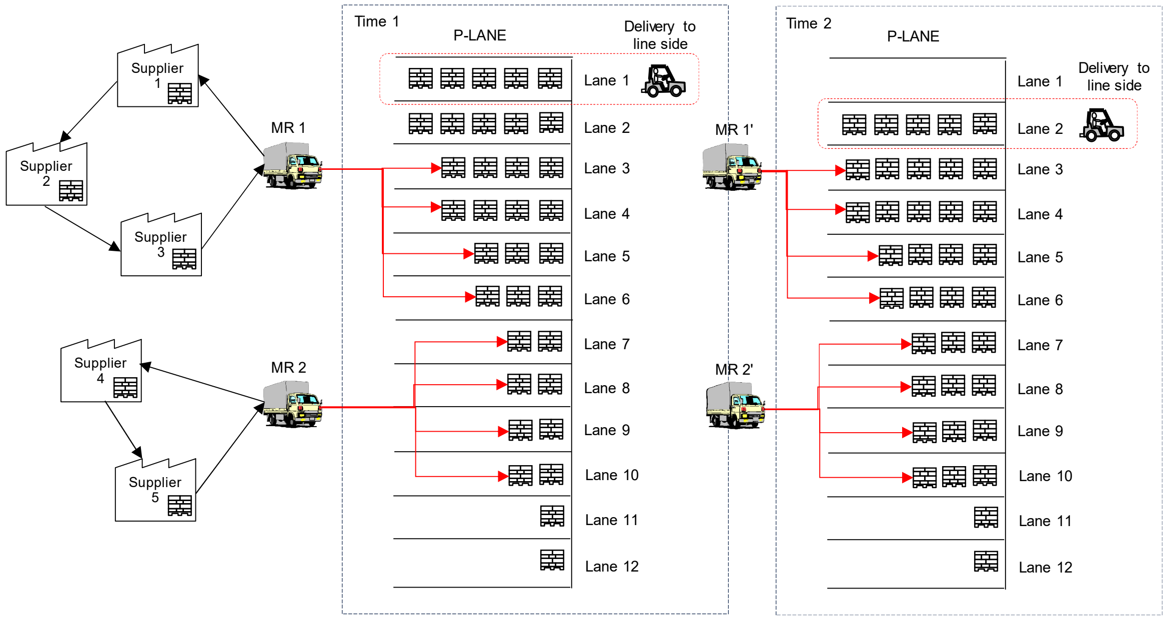
After knowing the quantity of parts to be provided by each supplier, and the package specification of each part, it is possible to start the monthly activity of milk-run planning. The planning process starts with the total number of pallets and the volume in cubic meters to be collected from each supplier. Each part has an internal routing inside the manufacturer's plant to reach the attachment point in the assembly line. The same supplier can supply parts for different internal routes. The internal route will therefore be used for pallet breakdown definition.

The leveled daily production volume is divided into production slots as a fraction of the total volume, called lanes. This division provides frequent and small quantities deliveries from parts receiving docks to the assembly line through internal milk runs attended by tow motors. Daily division into slots enables the manufacturer to reduce the parts stock aside from the production line, also reducing the area necessity in the assembly line site. The slot-breaking instruction is defined by a sequential order number, also used for pallet breakdown (the same pallet cannot have two different order numbers or different time slots).

Lean thinking pursues zero inventory. However, this goal is difficult to achieve in practice, and therefore other inventory models have been developed in lean manufacturing plants. The concept of production slots can be also called progress-lane (P-LANE). The P-LANE can be defined as a buffer to reduce the total transportation and handling costs and improve its efficiency. It works as an in-plant cross-dock system that connects the inbound logistics, from suppliers to the manufacturer, and in-plant logistics, from the PL to the production line.

If the supplier's pickup frequencies are increased, then the logistics efficiency and cost would be impacted. In this sense, the P-LANE can convert the regular but non-quantitative inbound logistics into regular and quantitative supplies to the line side, according to the production slots (*lanes*). As shown in Figure 17, the P-LANE is a physical area inside the plant between the truck unload docks and the production line. It has a layout division into the total quantity of slots defined for the manufacturer. Each slot or lane is identified with a sequence order number. For each part's purchase order, there is an identification of the date that parts will be used, and the sequence they must be sent to the line side, which implies where it should be allocated in the P-LANE area.

Figure 17 – Supply chain logistics process considering the progress lane function.



Source: adapted from Mao et al. (2020).

The number of slots on the P-LANE is defined according to the production hours, for example, if the manufacturer works with 2 shifts of 8 hours each, the number of slots will be 16. But it’s not a rule and the total number can vary depending on the capacity volume of the internal milk-runs’ tow motors or the physical space restriction in the line side. The greater the number of slots, the more fractioned will be the volume of the parts to be delivered in each slot, reducing the inventory level at the line side.

Table 9 illustrates an example of the necessity of each part by supplier along the *heijunka* distribution in P-LANE, considering 12 lanes. We can observe that the total daily demand for each part and supplier is equally distributed by the total number of lanes of the P-LANE, and the quantity of parts by lane is also distributed throughout the day. This information will be the initial input data for the milk run planning process.

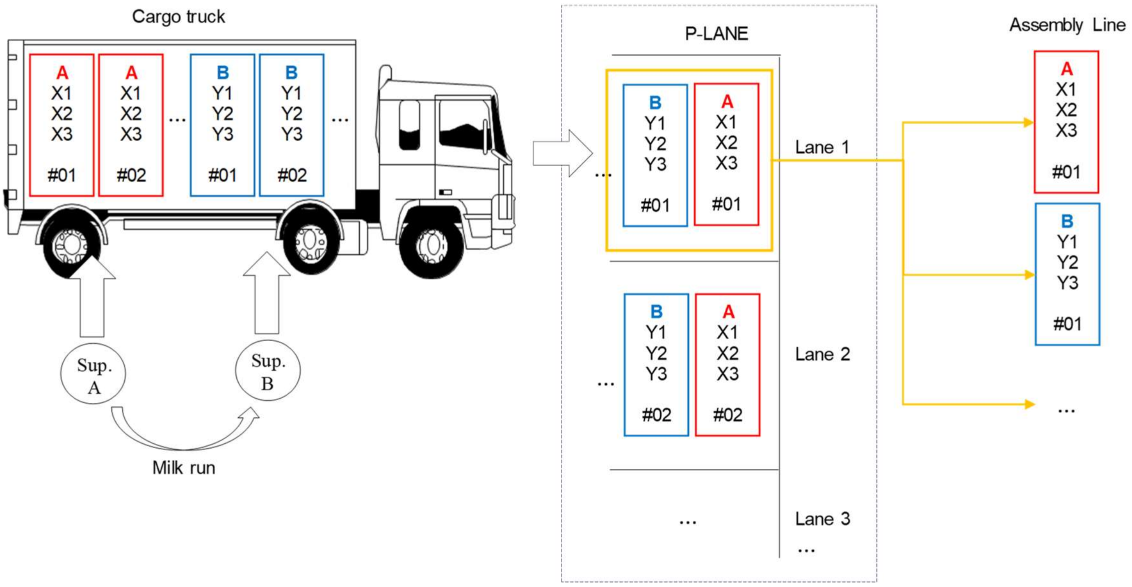
Table 9 - Necessity of parts by lane for each supplier.

		Lane of the P-Lane												
Supplier	Parts	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10	p11	p12	Total
A	X1	1	1	1	1	1	1	1	1	1	1	1	1	12
	X2	1	1	1	1	1	1	1	1	1	1	1	1	12
	X3	2	2	2	2	2	2	2	2	2	2	2	2	24
B	Y1	1	1	1	1	1	1	1	1	1	1	1	1	12
	Y2	3	3	3	3	3	3	3	3	3	3	3	3	36
	Y3	2	2	2	2	2	2	2	2	2	2	2	2	24
C	Z1	2	2	2	2	2	2	2	2	2	2	2	2	24
	Z2	5	5	5	5	5	5	5	5	5	5	5	5	60
Total		17	17	17	17	17	17	17	17	17	17	17	17	204

Source: elaborated by the author.

Different purchase order slots can be collected by the same milk-run truck from different suppliers. According to Figure 18, as an example, parts X1, X2, and X3 are collected from supplier A, and parts Y1, Y2, and Y3 are collected from supplier B. Both are collected by the same truck, with different order sequences #01 and #02 (indicating the lane number or sequence in the P-LANE). After unloading at the manufacturer plant, they are allocated according to the order sequence in the P-LANE area and are transferred to the assembly line according to the production progress.

Figure 18 – The logic of parts pick-up at suppliers and delivery to the assembly line.



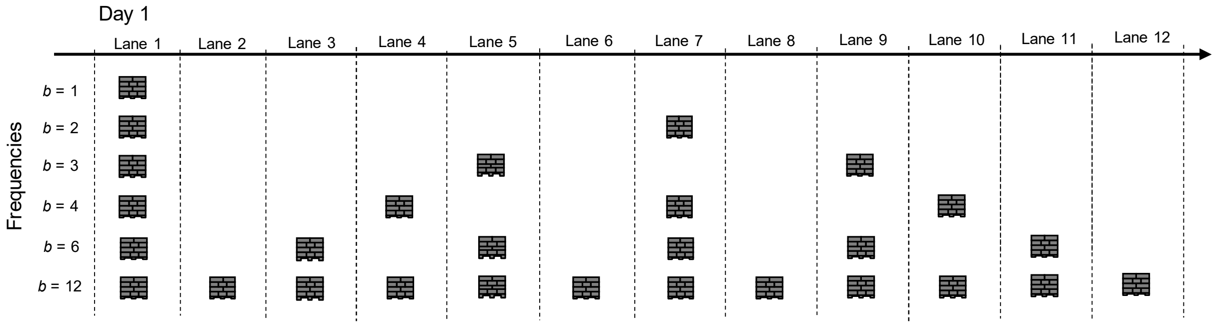
Source: adapted from Mao et al. (2020).

After the calculation of the CNQ and the total volume of pallets needed for each supplier to be sent to the production line on each internal route, the role of the internal logistics specialist is to define the better-receiving frequency according to pallet breakdown. The key factors for breaking down the pallets for the same supplier are (i) sequencing order number, (ii) unloading dock code, and (iii) internal delivery route.

The frequency is then calculated based on the total daily volume generated by each breakdown group. For example, if some small consumption parts are calculated as 1 cubic meter per day, it means only one pallet will be received for that group, and the frequency will be defined as $b = 1$, and the pallet will be received in just one slot or *lane*. Another example, If the total volume is 6 cubic meters (rounded up by 6 pallets), then the receiving frequency will be allocated as $b = 6$ (if this frequency corresponds to one of the P-LANE available frequencies), and the total amount will be received in a heijunka way in 6 slots on a day. If the number of lanes of the P-LANE is different from the example, the volume needs to be calculated considering the better pallet efficiency (based on the internal logistics and considering that one pallet is equivalent to one cubic meter).

In summary, the total volume collected from each supplier is estimated in the number of cubic meters or its equivalent quantity of pallets, and then the number of pallets by lane will be calculated considering the better fit between the total number of pallets needed and the available receiving frequencies, what is demonstrated in Figure 19. Considering the *heijunka* concept, when the total capacity of P-LANE is 12 lanes, the possible configuration of the receiving frequency is demonstrated in Figure 19. The possible frequencies considered in this case are $b = \{1, 2, 3, 4, 6, 12\}$ it means for each frequency b the total daily volume can be equally divided (*heijunka*) into the 12 production slots of a day (*lanes*).

Figure 19 – Receiving pattern example for a P-LANE of 12 lanes.



Source: elaborated by the author.

In the example shown in Figure 19, we demonstrate the possibilities of pallets receiving frequency allocation in a 12-*lane* P-LANE. If the volume collected from one supplier is low (less or equal to one cubic meter) the receiving frequency in the P-LANE will be $b = 1$, one pallet by day and this receiving is enough to cover one day of production. If the volume is less or equal to two, the frequency will be $b = 2$, two pallets per day, and one receiving is enough to cover half a day of production, which means the period of resupply should be in each 6 *lanes* for the mentioned example. This will be used for the demand calculation for each supplier and each lane of the P-LANE.

The receiving frequency will directly impact the milk run pickup frequency, according to the demand generated for each supplier. Higher receiving frequencies (because of the demand volume) will imply higher milk run pickups. Pursuing the zero-stock goal of lean logistics, the ideal condition would be a higher pickup frequency for all suppliers, reducing the physical inventory necessity at the manufacturing plant.

According to the number of lanes in the P-LANE, a determined number of pickup frequencies are available for each route. In Table 10 the possible pickup frequencies are presented according to the number of lanes in the P-LANE.

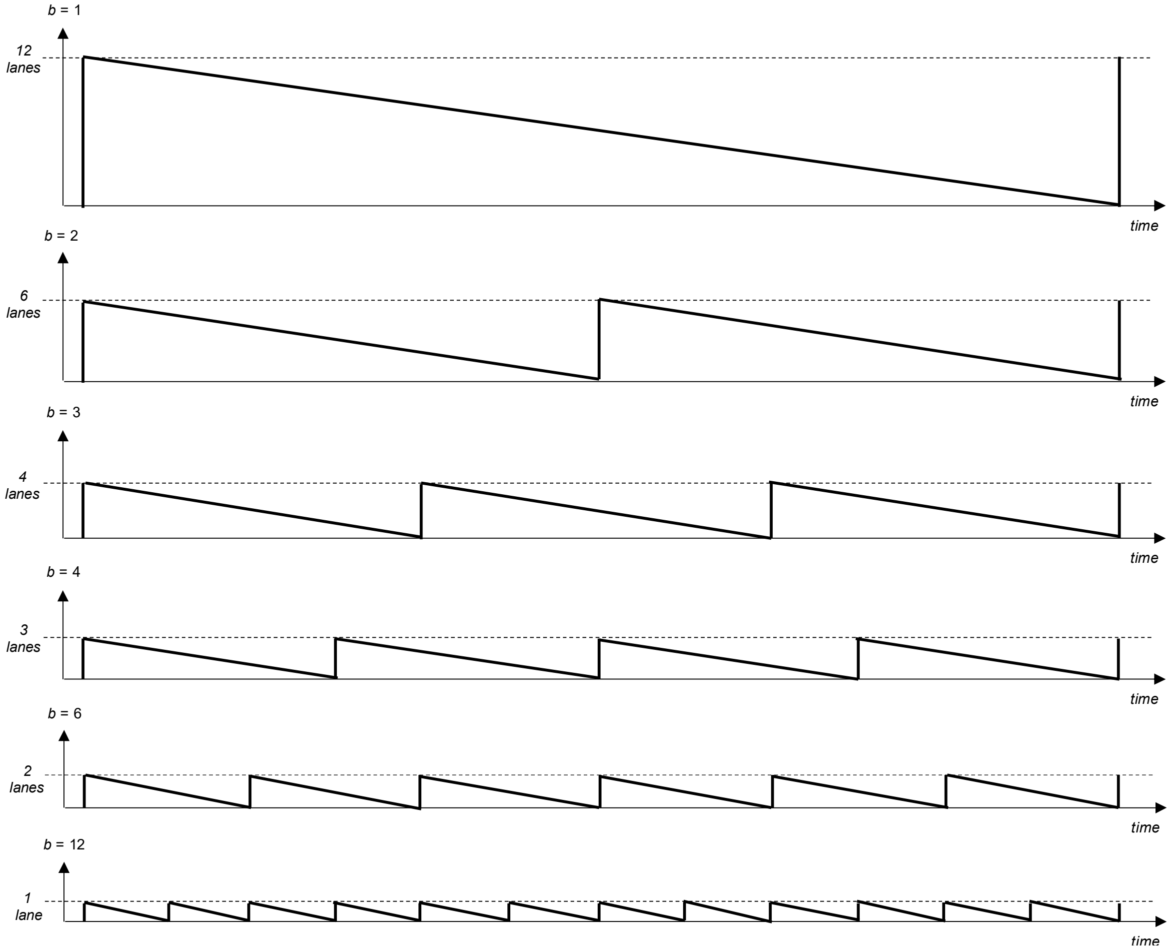
Table 10 – available pickup frequencies for each P-LANE volume.

Number of lanes on the P-LANE	Possible pickup frequencies
4 lanes	s = 1, 2, 4
6 lanes	s = 1, 2, 3, 6
8 lanes	s = 1, 2, 4, 8
12 lanes	s = 1, 2, 3, 4, 6, 12
16 lanes	s = 1, 2, 4, 8, 16
24 lanes	s = 1, 2, 3, 4, 6, 8, 12, 24
36 lanes	s = 1, 2, 3, 4, 6, 9, 12, 18, 36
48 lanes	s = 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

Source: elaborated by the author.

Figure 20 illustrates the stock behavior at P-LANE for each receiving frequency according to the replenishment point. The frequency $b = 1$ indicates one receiving per day and the stock represents 12 *lanes* or a daily total demand. Frequency $b = 12$, on the opposite side, indicates 12 receiving per day, and the stock of each frequency represents just one lane.

Figure 20 – Receiving pattern for a P-LANE of 12 lanes.



Source: elaborated by the author.

It can be observed that there is a trade-off between the milk run pickup frequency and the inventory level at the manufacturer’s plant. If the frequency is lower, the inventory level increases. On the other hand, if the pickup frequency is higher, the inventory level decreases. The pickup frequency increase, on the other hand, could impact the transportation cost, depending on the possibility of consolidating the cargo between more than one supplier to optimize the vehicle’s capacity.

After frequency is assigned, milk run route planners will try to configure the best combination of suppliers, aiming to achieve good truck efficiency. There are some heuristic methods to combine suppliers and determine routes, but all the calculation is done manually. The segmentation of suppliers by region (defined by the main road used for the pickups) is one of the methods used to combine more than one supplier in the same route. If more than one supplier is in the same region, normally defined by the main access road, it would be a candidate for the same route.

It is important to mention that the process of monthly route planning is done manually by the milk run specialist and after that, the information is uploaded to the parts procurement system for call ordering generation, and information sharing with the suppliers. The planning process takes from 3 to 5 days to be completed, including the negotiation of the number of pickups and pickup schedules with the suppliers. Some changes can happen during the month, for example, to improve the route efficiency of the supplier allocation, but it is not common. If necessary, just one or two routes are changed to reallocate that specific supplier, but there is no replanning process.

The milk run route starts from the manufacturer plant, with the process of empty package loading to the trucks. Each truck has a route schedule that defines the sequence of suppliers to be visited, and the timing scheduled for each one. In the supplier plant, the empty packages are unloaded and replaced by the prepared cargo which the call order was released one day before. After finishing the pickup in all suppliers scheduled to that route, the driver returns to the manufacturer plant for cargo unloading.

The cargo is unloaded and sequenced in the P-LANE according to the purchase order sequence information (production date and lane to be allocated). Each lane has due time to be sent to the line side by an internal logistics route. If the truck is delayed, it compromises the inventory level at the line side and may cause parts shortage and production line stop. Because of that, the due timing of each lane should be also considered in the route planning process.

Earlier receiving is allowed despite it will generate an inventory level increase in the P-LANE. The higher the pickup frequency of each route, the lower the inventory level at P-LANE (in-plant stock) according to the image shown in Figure 20. The milk run pickup process is repeated daily for the considered fixed volume month. The route pickup frequency and schedule will not change, since the production volume is leveled.

The milk run planning process described in this section can be basically resumed into two main activities that are:

(i) Calculation of the demand of pallets from each supplier in each lane of the P-LANE, considering the available receiving pattern based on the number of lanes and heijunka volume.

(ii) Based on the demand of each supplier for each lane, define the milk run routes by combining the suppliers and defining the route's pickup frequency and schedule.

For the first one, the lean logistics concept based on the production leveling can provide some heuristics for an assertive calculation based on MS Office Excel® formulas. For the second, meanwhile, the manual calculation is more complicated. From the total planning process, more than 75% of the timing is used for the route’s definition and scheduling, which means more than three days. Of course, that are some lean directions, but the local supply chain structure of each region can bring considerations that cannot be generalized, for example, the average distance between suppliers and the manufacturer, the step of cargo consolidation and cross-docking between the suppliers and the manufacturer, the logistics infrastructure and costs, and so one.

The result is an attempt and error method used to define the milk run routes with an average efficiency consideration of 70% of the truck’s load. Because of that, we intend to propose a mathematical model to solve this second step of route definition and schedule, aiming for a faster and more efficient solution. For that, we first adapted a lean logistics milk run VRP found in Mao et al. (2020) to our problem context. The model selected for solving the case study problem is the Milk-run Routing Problem with Progress Lane (MRPPL). Due to the computational timing and its complex structure of indexes and binary variables, we proposed a new model. The two models will be explained and compared in the next section.

In summary, for the problem modeling considering the lean manufacturing performance criteria, the following assumptions were considered:

- Given a group of suppliers and the total quantity of pallets that should be collected to meet the demand of one production day, the model should decide the number of routes to be created, and which suppliers are allocated to each route.

- Each supplier should be allocated to one route, and it is not allowed that more than one route attends the same supplier.
- In addition to the number of routes definition, the model should decide the pickup frequency for each route. The pickup frequency indicates the number of travels each vehicle should realize in one day to meet the total demand of the manufacturer.
- There is a due timing for receiving the pallets at the manufacturer plant given by the sequence of lanes in the P-LANE. The earliness arrival of the goods in the plant caused a higher stock volume, and there is a penalty cost associated with it. On the other hand, the tardiness of arrival should impact the production line and have a penalty cost.
- The demand for pallets for one production day is divided by the number of lanes in the P-LANE. The sequence of lanes in the P-LANE should be respected, from the lower to the higher, following the FIFO (first-in first-out) supply rule.
- The number of pallets picked up on each trip should respect the heijunka or leveling rule of lean logistics. In this sense, the pickup frequency is directly related to the number of lanes in the P-LANE, and the number of possible trips to be executed will depend on the number of lanes as shown in Table 10 and Figure 20.

Considering the typology of already existing Vehicle Routing Problems, the proposed model should present the characteristics of a Vehicle Routing Problem with Time Windows (VRPTW) since it have the P-LANE due arrival timing consideration combined with an Inventory Routing Problem (IRP) as inventory levels at plant should be contemplated. It is also important to mention that the vehicles' capacity should be considered to define the total number of trips for each route to meet the daily demand of each supplier. The fleet is homogeneous. And there is a periodic cycle repeated for each route according to the assigned frequency.

4. MATHEMATICAL MODELLING

This chapter introduces two MIP formulations for the Milk-run Routing Problem with Progress Lane (MRPPL). The first is an adaptation of the model presented by Mao et al. (2020), identified in the SLR, since it was the most similar model found in the literature with the case study problem characteristics. For better representing the real problem, some adaptations to the model were necessary and will be explained in section 4.1.

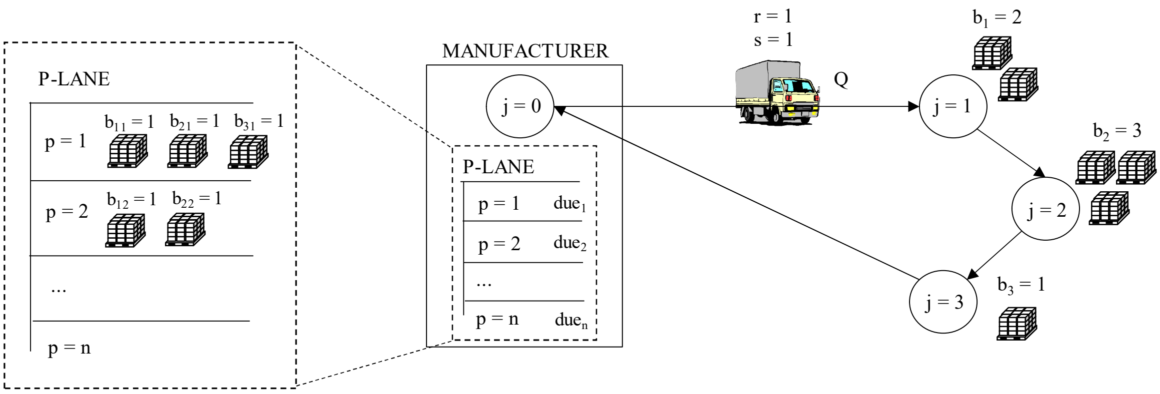
Due to the high number of indices and binary variables in the **adapted model**, resulting in high computation times for obtaining optimal solutions, we have also proposed a new model, which relies on different types of variables and constraints. The details of the proposed model are presented in Section 4.2.

4.1. Milk-run Routing Problem with Progress Lane – route-based (MRPPL-RB)

The MRPPL-RB is a MIP that can be described as un undirected complete graph $\mathrm{G} = (\mathrm{V}, \mathrm{A})$, where $\mathrm{V} = \{0,1, \ldots, \mathrm{n}\}$ is the vertex set and A is the arc set. The vertices $j = 1, \ldots, n$ is the set of suppliers and the vertex 0 corresponds to the manufacturer. Supposing that there are n suppliers, if one route is considered for each supplier, then the set $\mathrm{R} = \{1, \ldots, \mathrm{r}\}$ of routes can be defined. In addition, $\mathrm{P} = \{1, \ldots, \mathrm{p}\}$ is the set of lanes in the P-LANE, and $\mathrm{F} = \{1, \ldots, \lceil \sum_{\mathrm{j}} \mathrm{b}_{\mathrm{j}} / \mathrm{Q} \rceil\}$ is the set of visiting frequencies for each route r, considering b_j as the total demand needed for one day of production from supplier $j \in V \backslash \{0\}$ and Q as the vehicles capacity, considering an homogeneous fleet. The parameter b_{jp} corresponds to the demand needed from supplier $j \in V \backslash \{0\}$ in the lane p of the P-LANE. The total demand b_j for each supplier can be calculated as $b_j = \sum_{pEP} b_{jp}$.

In the below paragraphs, we better explain the model, its sets, indexes, parameters, variables, and the relationship between them. Figure 21 represents a sample image of the milk run P-LANE problem.

Figure 21 – The milk run routing problem with progress lane (P-LANE).



Source: elaborated by the author.

In the example demonstrated there are three suppliers, $j = 1, j = 2$, and $j = 3$ attended in the route $r = 1$, with the trip of frequency $s = 1$, and truck capacity Q. The manufacturer is represented by $j = 0$. The P-LANE area in the manufacturer's plant is represented as a physical storage area composed of a determined number of lanes $p = 1, \dots, n$, which one composed by a certain necessary quantity of parts from each supplier b_{jp} to cover a fixed time slot production range. The total amount of parts that each supplier needs to dispatch in a day is represented by the sum of parts needed from lanes 1 to n, represented by b_j. To avoid any parts shortage at the line side, each pickup of each route has a due date timing for P-LANE arrival, represented by due_p.

The model is represented by the index below, parameters, and variables:

Index and Sets

$j, k \in$ V	vertices of the problem, including the manufacturer and the suppliers
$r \in$ R	routes
$s, t \in$ F	frequencies
$p, g \in$ P	lanes in the P-LANE

Parameters

d_{jk}	distance between two vertices j and k
ϕ	transportation cost per unit distance
c_{jk}	transportation cost between two vertices j and k, where $c_{jk} = \phi * d_{jk}$
ω	fixed transportation cost by route
tt_{jk}	travel time between two vertices j and k

b_{jp}	the number of pallets needed from supplier $j \epsilon V\backslash\{0\}$ on P-LANE $p \in P$
b_j	the number of pallets needed from supplier j in manufacturers' one day production, where $b_j = \sum_{p \in P} b_{jp}$
η_{js}	the *average* number of pallets needed from supplier j per visit if supplier j is visited s times, where $\eta_{js} = b_j/s$
due_p	the due date timing of parts arrival at lane p in the P-LANE;
θ	earliness penalty cost per unit time
ψ	tardiness penalty cost per unit time
L_j	the loading time per volume for the parts collected from supplier j;
Q	vehicles' capacity
M	large number

Variables

x_r	is equal to 1 if route $r \epsilon R$ is used to collect parts from suppliers and 0 otherwise
y_{jr}	is equal to 1 if supplier j is assigned to route r and 0 otherwise
z_{jkr}	is equal to 1 if the arc (j, k) is visited by route r and 0 otherwise
u_{rs}	is equal to 1 if the frequency of route r is s and 0 otherwise
A_{rs}	is the arrival time of the sth visit from the manufacturer for route r
σ_{jrs}	is equal to 1 if supplier j is assigned to route r ($y_{jr} = 1$) and the frequency of route r is s ($u_{rs} = 1$) and 0 otherwise
δ_{jkrs}	is equal to 1 if the arc (j, k) is visited by route r ($z_{jkr} = 1$) and the frequency of route r is s ($u_{rs} = 1$) and 0 otherwise
ξ_{jrstp}	is equal to 1 if supplier j is assigned to route r ($y_{jr} = 1$) and the frequency of route r is s ($u_{rs} = 1$), or ($\sigma_{jrs} = 1$), and the tth visit of route r meet the demand needed from supplier j on lane p of the P-LANE and 0 otherwise
F_{jp}	is the finish time when the parts collected from supplier j meet the demand on P-LANE p
E_{jp}	is the earliness of supplier j on P-LANE p, where $E_{jp} = \max\{0, due_p - F_{jp}\}$
T_{jp}	is the tardiness of supplier j on P-LANE p, where $T_{jp} = \max\{0, F_{jp} - due_p\}$
ϑ_{jr}	positive variable to MTZ formulation

Optimization Model

The objective function (1) consists of minimizing total costs, including variable and fixed transportation costs in the first and second terms, and the penalty earliness and tardiness cost in the third and last terms.

$$min \sum_{j \in V} \sum_{k \in V} \sum_{r \in R} \sum_{s \in F} \phi s d_{jk} \delta_{jkrs} + \sum_{r \epsilon R} \sum_{s \epsilon F} \omega s u_{rs} + \sum_{j \in V\backslash\{0\}} \sum_{p \epsilon P} \theta E_{jp} + \sum_{j \in V\backslash\{0\}} \sum_{p \epsilon P} \psi T_{jp} \quad (1)$$

Constraints (2) ensure that route $r + 1$ can be opened only if route r has been used. The variable x_r is a binary variable that determines if route r will be used ($x_r = 1$) or not ($x_r = 0$). Since the value of $r + 1$ route should be lower or equal to r, it can only assume value $x_{r+1} = 1$ if $x_r = 1$, meaning that route $r + 1$ can only be opened if route r was already opened. Equation (2) also determines the quantity of routes necessary to meet the demand for all suppliers.

$$x_{r+1} \leq x_r , \quad \forall\, r \in R; \tag{2}$$

Constraints (3) ensure that each supplier can be visited by only one route. The variable y_{jr} determines if the supplier $j \in V\backslash\{0\}$ is assigned to the route r ($y_{jr} = 1$) or not ($y_{jr} = 0$). For all the routes open, the sum of variable y_{jr} can only be equal to 1, which implies that each supplier j can only be attended by one route r.

$$\sum_{r \in R} y_{jr} = 1 , \quad \forall\, j \in V\backslash\{0\}; \tag{3}$$

Constraints (4) specify that there must be at least one supplier allocated to an open route. The equation ensures that, since one new route has been opened, at least one supplier should be assigned to it.

$$\sum_{j \in V\backslash\{0\}} y_{jr} \geq x_r , \quad \forall\, r \in R; \tag{4}$$

Constraints (5) and (6) establish the relationship between variables y and z, which also ensure that the flow in each route is conserved. The variable z_{jkr} indicates if the arc $(j, k) \in V$ is visited by the route r ($z_{jkr} = 1$) or not ($z_{jkr} = 0$). It ensures that, if supplier j is assigned to route r ($y_{jr} = 1$) and the arc j, k is visited ($z_{jkr} = 1$), the supplier k should be also attended by the same route.

$$\sum_{k \in V} z_{jkr} = y_{jr} , \quad \forall\, j \in \mathrm{V}\backslash\{0\}; r \in R; \tag{5}$$

$$\sum_{j \in V} z_{jkr} = y_{kr}, \quad \forall\, k \in \mathrm{V} \setminus \{0\}; r \in R; \tag{6}$$

Inequalities (7) and (8) are MTZ (Miller-Tucker-Zemlin) subtour elimination constraints, well known in VRP literature (MILLER et al., 1960).

$$\vartheta_{jr} - \vartheta_{kr} + |V| z_{jkr} \leq |\mathrm{V}| - 1, \quad \forall\, j, k \in \mathrm{V}; r \in R; \tag{7}$$

$$\vartheta_{jr} \leq |\mathrm{V}| - 1, \quad \forall\, j \in \mathrm{V}; r \in R; \tag{8}$$

Constraints (9), (10), and (11) ensure that each route can have exactly one frequency and determine the frequency of each route once the assignment of suppliers to the routes is given. The variable u_{rs} determines the pickup frequency of the route r. The index s corresponds to the maximum available pickups for the route r. If $u_{14} = 1$, for example, it means that route 1 has 4 pickups. Since equation (9) sums up the variable u_{rs} for all frequencies in the set F, it ensures that each route r can have only one sequence s from F assigned to it. Constraints (10) and (11) also ensure that the daily cargo b_j collected by route r from supplier j does not exceed the vehicle's capacity Q given a number of travels $s \in F$ determined for route r.

$$\sum_{s \in F} u_{rs} = x_r, \quad \forall\, r \in R; \tag{9}$$

$$\sum_{j \in V \setminus \{0\}} b_j y_{jr} \leq \sum_{s \in F} sQu_{rs}, \quad \forall\, r \in R; \tag{10}$$

$$\sum_{j \in V \setminus \{0\}} b_j y_{jr} \geq \sum_{s \in F} [(s-1)Q + 1] u_{rs}, \quad \forall\, r \in R; \tag{11}$$

In constraints (10) the total daily volume for the supplier j, b_j, if that supplier is attended by route r ($y_{jr} = 1$) should be less or equal to the vehicles' capacity Q, multiplied by the number of trips realized by route r given the frequency s, if that frequency is assigned to that route (u_{rs} = 1). Equation (11), otherwise, ensures that the total number of trips given a frequency s should be executed since the daily demand from supplier j, b_j, if that supplier is attended by route r ($y_{jr} = 1$) should be greater or equal to the penultimate pick-up trip multiplied by the vehicles' capacity $Q + 1$ if that frequency is assigned to route r (u_{rs} = 1).

Constraints (12), (13), (14), and (15) define the relationship between binary variables for the allocation of arcs to routes and frequencies (δ_{jkrs}) and binary variables for the allocation of suppliers to routes and frequencies (σ_{jrs}). Variable σ_{jrs} links the binary routing variable for supplier allocation (y_{jr}) with the frequency variable (u_{rs}). If the supplier j is assigned to route r and the route r is assigned to the pickup frequency s, then $\sigma_{jrs} = 1$ ($y_{jr} = 1$ and $u_{rs} = 1$), and $\sigma_{jrs} = 0$ if not. The variable σ_{jrs} identifies, then, which supplier is assigned in with route related to each pickup frequency. It is important to mention that each supplier can have only one assigned route, and each route can have only one pickup frequency from the allowed range according to the number of lanes on the P-LANE.

$$2\delta_{jkrs} \leq z_{jkr} + u_{rs}\,, \quad \forall\, j,k, \in V, r \in R, s \in F; \tag{12}$$

$$1 + \delta_{jkrs} \geq z_{jkr} + u_{rs}\,, \quad \forall\, j,k, \in V, r \in R, s \in F; \tag{13}$$

$$2\sigma_{jrs} \leq y_{jr} + u_{rs}\,, \quad \forall\, j \epsilon V\backslash\{0\}, r \in R, s \in F; \tag{14}$$

$$1 + \sigma_{jrs} \geq z_{jr} + u_{rs}\,, \quad \forall\, j \epsilon V\backslash\{0\}, r \in R, s \in F; \tag{15}$$

Constraints (16), (17), and (18) ensure the relationship between binary variables for the allocation of suppliers to routes and frequencies (σ_{jrs}) and variables to determine which visit on each route and frequency fulfills the demand of each supplier in each lane (ξ_{jrstp}). The variable δ_{jkrs}, like the variable σ_{jrs}, creates the relation between the variables z_{jkr} and u_{rs}. If the arc $(j,k) \in V$ is visited by the route r and the route r is assigned to pick up frequency s, then $\delta_{jkrs} = 1$ ($z_{jkr} = 1$ and $u_{rs} = 1$), and $\delta_{jkrs} = 0$ if not. The variable ξ_{jrstp}, creates the relation between variables y_{jr} and u_{rs}, and designates for which lane p of the P-LANE parts will be allocated. $\xi_{jrstp} = 1$ if the supplier j is assigned to route r, or $y_{jr} = 1$, the frequency of route r is s, or $u_{rs} = 1$, and the tth pickup of the route r meet the demand needed from supplier j on the lane p of the P-LANE.

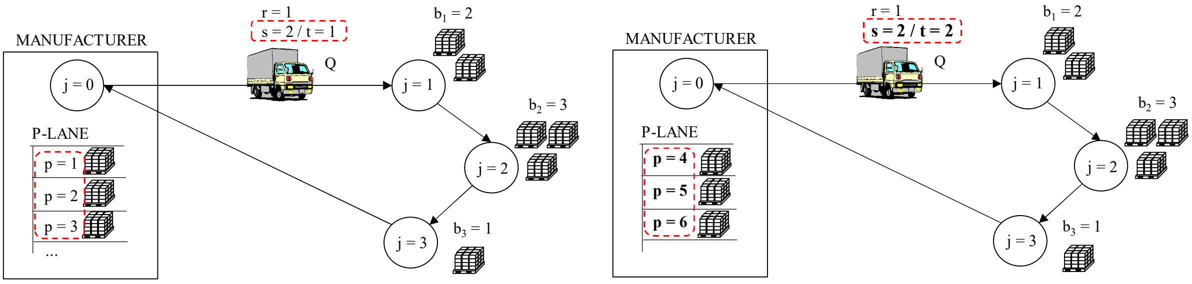
$$\sum_{t=1}^{s} t n_{js} \xi_{jrstp} \geq \sum_{k=1}^{p} b_{jp} \sigma_{jrs}\,, \quad \forall\, j \in V\backslash\{0\}, r \in R, s \in F, p \in P; \tag{16}$$

$$\sum_{t=1}^{s} [(t-1)n_{js} + 1] \xi_{jrstp} \leq \sum_{k=1}^{p} b_{jp} \sigma_{jrs}\,, \quad \forall\, j \in V\backslash\{0\}, r \in R, s \in F, p \in P; \tag{17}$$

$$\sum_{t=1}^{s} \xi_{jrstp} = \sigma_{jrs}\,, \quad \forall j \in V\backslash\{0\}, r \in R, s \in F, p \in P; \tag{18}$$

To illustrate the logic of constraints (16), (17), and (18), Figure 22 shows the two consecutive tours of the route $r = 1$ with pickup frequency $s = 2$ under an analysis of the variable ξ_{jrstp}. In the first pickup, variable ξ_{jrstp} can be write as ξ_{j121p} for each supplier demand needed in each lane of the P-LANE. In the second pickup, it will assume the value ξ_{j122p}. It can be noticed that the variable ξ_{jrstp} will show the image of the P-LANE for one day of production (if $\xi_{jrstp} = 1$, parts from supplier j will be used in the lane p for that pickup number of route r with pickup frequency s, and $\xi_{jrstp} = 0$ if not). In the example of the figure, the first pickup, $t = 1$, will collect parts necessary from lanes $p = 1, p = 2$ and $p = 3$, and the second one, $t = 2$, from lanes $p = 4, p = 5$ and $p = 6$.

Figure 22 – Two consecutive pickups of the same route.



Source: elaborated by the author.

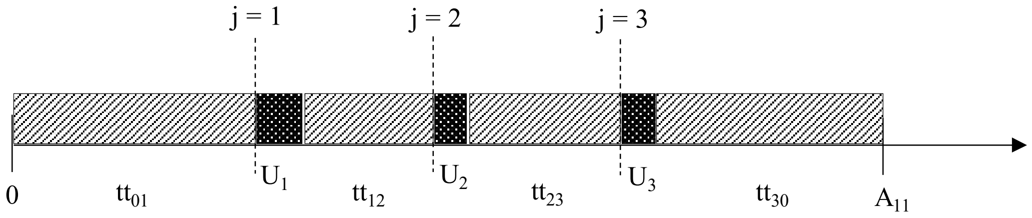
In this sense, variable ξ_{jrstp} linkages the average number of pallets n_{js} collected from supplier j considering it is collected by a route with frequency s with the demand b_{jp} of pallets from each supplier j in each lane p of the P-LANE trough constraints (16) and (17). The consideration of variable σ_{jrs}, that assigns each supplier j to a route r with pickup frequency s, will also ensure that the demand of all lanes p in the P-LANE will be attended through the constraint (18).

Constraints (19) gives the relationship between the departure and arrival time of each visit for each route.

$$A_{rs} \geq A_{rs-1} + \sum_{j \in V} \sum_{K \in V} tt_{jk} z_{jkr} + \sum_{j \in V\{0\}} \sum_{t \in F} U_j n_{jt} \sigma_{jrt} - M \sum_{t=0}^{s-1} u_{rt} \ , \quad \forall r \in R, s \in F; \tag{19}$$

Figure 23 illustrates the timeline resulting from constraints (19), assuming a route $r = 1$ with pickup frequency $s = 1$. The route timing starts when the vehicle leaves from the manufacturer $j = 0 \in V$. There are two different parameters considered during the pickup trip, the first one is the travel time parameter tt_{jk} that corresponds to the travel time between two vertices j and $k \in V$, and the second one is the service time parameter U_j, the loading timing per volume needed for the parts collected from supplier $j \in V\backslash\{0\}$. In the example, the milk run collects parts at three different suppliers and then returns to the manufacturer plant. The total lead time is calculated by the sum of travelling times and servicing times and is represented by the variable A_{rs}.

Figure 23 – Timeline of one route considering travel and service times.



Source: elaborated by the author.

If route r has more than one pickups $(s > 1)$, the timeline presented in Figure 23 will be incremented for each pickup s of route r, and the total lead time is calculated by considering the arrival timing of the last pickup A_{rs-1} plus the total lead time of the sth pickup.

Constraints (20) and (21) ensure that the completion time of all kinds of parts are coordinated with the whole P-LANE.

$$F_{jp} \geq A_{rt} + M(\xi_{jrstp} - 1), \quad \forall j \in V\backslash\{0\}, r \in R, s \in F, p \in P, t \in F; \tag{20}$$

$$F_{jp} \leq A_{rt} + M(1 - \xi_{jrstp}), \quad \forall j \in V\backslash\{0\}, r \in R, s \in F, p \in P, t \in F; \tag{21}$$

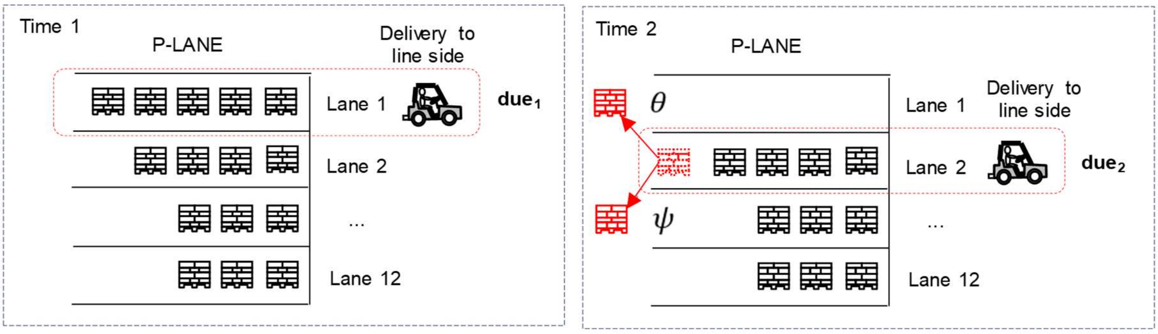
Constraints (22) and (23), utilizing the result of equations (20) and (21) define the earliness and tardiness timing arrival at lane p of the P-LANE.

$$E_{jp} \geq due_p - F_{jp} \ , \quad \forall j \in V\backslash\{0\}, p \in P; \tag{22}$$

$$T_{jp} \geq F_p - due_{jp} \ , \quad \forall j \in V\backslash\{0\}, p \in P; \tag{23}$$

For better understanding of constraints (22) and (23), Figure 24 illustrates the relationship between variables F_{jp}, the finish time when parts collected from supplier j meets the demand on P-LANE p, E_{jp}, if parts arrive earlier and T_{jp}, if parts arrive tardier than the due_P date of the P-LANE p, and the parameters θ, the earliness penalty cost per unit time of advancement, and ψ, the tardiness penalty cost per unit time of postponement.

Figure 24 – Earliness and tardiness penalty costs according to time arrival at P-LANE.



Source: elaborated by the author.

Following the manufacturer's production progress, each line has a specific due time to be delivered to the line side to avoid parts shortage. In this sense, for each lane p in the P-LANE, the is a due_p timing for parts arrival to avoid lack of parts to feed the production line. As a penalty, there are two variables E_{jp} and T_{jp} for measuring the earliness or tardiness of each receiving time at the P-LANE.

If there is some anticipation, variable E_{jp} will be used for calculating the penalty cost based on the unit cost per time advancement θ. On the other hand, if there is some delay, variable T_{jp} will be used for calculating the penalty cost based on the unit cost per time delay ψ. The next chapter presents the validation of the model considering a snipped scenario of the case studied.

And finally, constraint (24) establishes the binary variables.

$$x_r, y_{jr}, z_{jkr}, u_{rs}, \sigma_{jrs}, \delta_{jkrs}, \xi_{jrstp} \, , \ \in \{0,1\} \ , \quad \forall \ j,k \in V, r \in R, s \in F, p \in P. \tag{24}$$

4.2. Milk-run Routing Problem with Progress Lane – arc-based (MRPPL-AB)

We propose a model considering a reduced number of binary variables, motivated by potentially accelerating the solution process of instances of the real problem. The additional index, parameters, and variables are presented below:

Indices and Sets

$i, j, k \in C \subseteq V$	suppliers
$t \in \mathrm{T}$	travels

Parameters

TC	total transportation cost

Variables

χ_{ij}	binary variable that indicates if the route from node *i* to *j* is executed
v_{js}	binary variable that determines if frequency *s* is assigned to supplier *j*
γ_{jtp}	binary variable that indicates if the demand of lane *p* is attended by trip *t* for the supplier *j*
v_j	positive variable that indicates the vehicle cargo at supplier *j*
TC_{ij}	positive auxiliar variable for variable transportation cost calculation
w_{jt}	positive variable that indicated the start time of service at supplier *j*
ϖ_{jt}	positive variable that indicated the end time of service at supplier *j*
φ_{jt}	positive variable that indicated the start time when the vehicle dispatched from the manufacturer

Optimization Model

The objective function (25) consists of minimizing the total costs, including the fixed and variable transportation costs, represented by the variable TC_{ij}, and the penalty costs of earliness and tardiness arrivals at the P-LANE, represented by variables E_{jp} and T_{jp}.

$$min \sum_{i \in A} \sum_{j \in A} TC_{ij} + \sum_{j \in C} \sum_{p \in P} \theta E_{jp} + \sum_{j \in C} \sum_{p \in P} \psi T_{jp} \qquad (25)$$

The transportation cost variable TC_{ij} is calculated in the equations (26), (27), and (28). Constraint (26) define the variable cost by travel distance for each trip for all suppliers [$\forall(i,j) \in C$], since the variable χ_{ij} = 1, meaning that the route from supplier i to supplier j is executed. The cost is calculated by the number of trips executed, represented by the route frequency s and the variable cost per kilometer $\phi * c_{ij}$ if the frequency s is assigned to supplier j ($v_{js} = 1$). The equation is activated if the arc i,j is traveled ($\chi_{ij} = 1$). On the contrary ($\chi_{ij} = 0$), a big number defined by the parameter M disables the constraint. The same logic will be applied to constraints (27) and (28).

$$TC_{ij} \geq \sum_{s \in S} s\, \phi c_{ij} v_{js} - (1 - \chi_{ij})M, \qquad \forall (i,j) \in C; \tag{26}$$

Constraint (27) calculates the fixed cost for each trip made on a route with frequency s. The fixed cost is calculated when the vehicle departs from the manufacturer ($i = 0$) and considers the variable cost from the manufacturer to the first supplier attended by this route ($\chi_{0j} = 1$). In this case, the fixed transportation cost ω is summed with the travel cost from the manufacturer to the first supplier visited $\phi * c_{ij}$, considering the number of travels represented by s if the frequency s is assigned to supplier j.

$$TC_{0j} \geq \sum_{s \in S} s\, (\phi c_{0j} + \omega) v_{js} - (1 - \chi_{0j})M, \qquad \forall\, j \in C; \tag{27}$$

In the LMRP model, we consider two different nodes for the manufacturer, illustrated by $i = 0$ and $i = n + 1$, indicating respectively the start and end node of each route. Constraint (28) calculates the variable transportation cost of each route with frequency s at the moment that the vehicle returns to the manufacturer plant ($j = n + 1$).

$$TC_{i(n+1)} \geq \sum_{s \in S} s\, \phi c_{i(n+1)} v_{is} - (1 - \chi_{i(n+1)})M, \qquad \forall\, i \in C; \tag{28}$$

Constraints (29) and (30) correspond to vehicle flow constraints. Constraint (29) ensures that all the arcs (i,j) are attended by one route. Constraint (30) ensures that the number of arrivals with each supplier should be equal to the number of departures from them. Constraints

(29) and (30) also ensure the elimination of sub-routes, since there is just one routing attending each arc, since the sum for all nodes j, should be equal to $\chi_{ij} = 1$, and the number of routes arriving in one supplier k (χ_{ik}) should be equal to the number of routes departing from it (χ_{kj}).

$$\sum_{j \in A} \chi_{ij} = 1\,, \ \ \forall i \in C; \tag{29}$$

$$\sum_{i \in A} \chi_{ik} = \sum_{j \in A} \chi_{kj}\,, \ \ \forall k \in C; \tag{30}$$

Constraint (31) ensures that the demand of all the lanes p in the P-LANE is attended by the number of trips t for each supplier j, and the demand of lane p should be collected by only one trip t for supplier j.

$$\sum_{t \in T} \gamma_{jtp} = 1\,, \ \ \forall j \in C, \forall p \in P; \tag{31}$$

Constraint (32) determines that for each supplier j, only one pickup frequency s can be chosen.

$$\sum_{s \in S} v_{js} = 1\,, \ \ \forall j \in C; \tag{32}$$

Constraint (33) ensures that the total daily demand b_j for each supplier j is attended. The equation considers all the parts collected in all trips t for all lanes p from each supplier j. The sum of these quantities should be equal to the daily demand for that supplier.

$$\sum_{t \in T} \sum_{p \in P} b_{jp} \gamma_{jtp} = b_j\,, \ \ \forall j \in C; \tag{33}$$

Constraint (34) enforces the relation between the variables γ_{jtp} and v_{js}. If the pickup frequency s is defined for the supplier j, the number of the trips t executed by that route should be less or equal to the frequency s. For example, if some route has a pickup frequency $s = 4$, the number of trips t executed by that route should be $t \leq 4$. It means that it is not possible to have more trips t than the pickup frequency s chosen for the route r assigned to supplier j.

$$\sum_{t \in T} t\,\gamma_{jtp} \leq \sum_{s \in S} s\,v_{js}\,, \quad \forall j \in C, \forall p \in P; \tag{34}$$

Given a route r with frequency s for the supplier j, constraint (35) ensures that all the trips t in which $t \leq s$ should be executed. This constraint guarantees that the number of trips t traveled for the route s should be equal to the route frequency s.

$$\sum_{p \in P} \gamma_{jtp} \geq \sum_{\substack{s \in S \\ s \geq t}} v_{js}\,, \quad \forall j \in C, \forall t \in T; \tag{35}$$

Constraints (36) and (37) ensure that the suppliers on the same route ($\chi_{ij} = 1$) should have the same pickup frequency s assigned. Combined, they guarantee that if the arc (i,j) is attended by route r, both equations are activated, and it means that $v_{is} = v_{js}$. Both equations combined also work to propagate the frequency for all the suppliers assigned to the same route.

$$v_{js} \geq v_{is} - \left(1 - \chi_{ij}\right), \quad \forall (i,j) \in C, \forall s \in S; \tag{36}$$

$$v_{js} \leq v_{is} + \left(1 - \chi_{ij}\right), \quad \forall (i,j) \in C, \forall s \in S; \tag{37}$$

Constraints (38), (39), and (40) are vehicle load propagation and capacity restrictions. Constraint (38) accumulates the cargo collected from supplier j, represented by $\eta_{js} * v_{js}$, since the arc (i,j) is attended ($\chi_{ij} = 1$). It also considers the limitation of the vehicle's capacity Q. If the route χ_{ij} is executed, then the capacity Q will restore the cargo collected from the next supplier on the same route. Constraint (39) implies that the cargo collected from supplier i should be equal or greater than the average cargo collected from this supplier $\eta_{is} * v_{is}$, given the pickup frequency s. Constraint (40) ensures that the cargo v_i is not higher than the vehicle's capacity Q.

$$v_j \geq v_i + \sum_{s \in S} \eta_{js} v_{js} - Q\left(1 - \chi_{ij}\right), \quad \forall (i,j) \in A \mid i \neq 0, j \neq n+1; \tag{38}$$

$$v_i \geq \sum_{s \in S} \eta_{is} v_{is}, \quad \forall i \in A \mid i \neq 0; \tag{39}$$

$$v_i \leq Q, \quad \forall i \in A, i \neq 0; \tag{40}$$

Constraint (41) establishes the relationship between the variables γ_{jtp} and v_{js}. The total amount of parts collected from supplier *j* in each trip *t* to attend to the demand of the lane *p,* represented by $\gamma_{jtp} * b_{jp}$ should be less or equal to the average quantity η_{js} picked up by pickup frequency, given a frequency *s*.

$$\sum_{p \in P} b_{jp} \gamma_{jtp} \leq \sum_{s \in S} \eta_{js} v_{js}, \qquad \forall j \in A \mid j \neq 0, \forall t \in T; \tag{41}$$

Since the lanes in the P-LANE are sequentially enumerated according to their consumption necessity at the line side per each production slot, constraint (42) ensures that for each trip t, the lane *p* demand should be satisfied before the lane *p* + 1. This restriction ensures that the concept of FIFO (first in – first out) of lean manufacturing is respected.

$$\sum_{t \epsilon T} t\, \gamma_{jtp} \leq \sum_{t \epsilon T} t\, \gamma_{jtp+1}, \qquad \forall j \in C, \forall p \in P \mid p \neq |P|; \tag{42}$$

Constraints (43) and (44) ensure that the heijunka concept is respected. Given a pickup frequency s, the number of lanes collected from supplier j in each trip t should be the same. This concept optimizes the vehicle capacity usage since there is no variation in the same route r between the different trips t according to the pickup frequency s.

$$\sum_{p \in P} \gamma_{jtp} \leq \sum_{p \in P} \gamma_{j,t+1,p} + \sum_{\substack{s \in S \\ s \leq t}} v_{js} |P|, \qquad \forall j \in C, \forall t \in T \mid t \neq |T|; \tag{43}$$

$$\sum_{p \in P} \gamma_{jtp} \geq \sum_{p \in P} \gamma_{j,t+1,p} - \sum_{\substack{s \in S \\ s \leq t}} v_{js} |P|, \qquad \forall j \in C, \forall t \in T \mid t \neq |T|; \tag{44}$$

Constraints (45) to (53) are related to time constraints. Constraint (45) accumulates the time spent in node i on the tth trip, plus the travel time from the node i to the node j (tt_{ij}) on the same trip t, and the service timing $L_j * \eta_{js}$ on the node j, related with the cargo amount picked up at the supplier j with the pickup frequency s, if the arc (i, j) is attended by this route $(\chi_{ij} = 1)$. Variable w_{it} is used to indicate the service start timing of the trip t, since the start timing of this route could be different from 0.

$$w_{jt} \geq w_{it} + tt_{ij} + \sum_{s \in S} L_j \eta_{js} v_{js} - M(1 - \chi_{ij}), \quad \forall (i, j) \in A \mid i \neq n+1, j \neq 0, \forall t \in T; \tag{45}$$

Constraints (46) and (47) consider the end of service time when the last supplier i is attended on trip t and the vehicle returns to the manufacturer plant $n + 1$. Since the inequalities (46) and (47) should be satisfied according to bellow restrictions, the variable ϖ_{it} assumes the value of the service start timing w_{it}, plus the timing spent to collect all demand from supplier i, represented by the term $l_i * \eta_{is}$, if supplier i is assigned to the frequency s $(v_{is} = 1)$, and the travel time from the last supplier visited for that route to the manufacturer plant $tt_{i(n+1)}$. The inequalities are enabled if the trip from supplier i to the manufacturer $n + 1$ is realized $(\chi_{i(n+1)} = 1)$.

$$\varpi_{it} \geq w_{it} + \sum_{s \in S} l_i \eta_{is} v_{is} + tt_{i(n+1)} - M(1 - \chi_{i(n+1)}), \qquad \forall\, i \in C, \forall t \in T; \tag{46}$$

$$\varpi_{it} \leq w_{it} + \sum_{s \in S} l_i \eta_{is} v_{is} + tt_{i(n+1)} - M(1 - \chi_{i(n+1)}), \qquad \forall\, i \in C, \forall t \in T; \tag{47}$$

Constraints (48) and (49) determine the timing of the service start time when the vehicle dispatches from the manufacturer plant. The start timing is defined by variable φ_{it}, that will also be used in constraints (50) e (51). Since the initial travel timing of determined route could be different from 0, variable φ_{it} is used to determine the service start riming for the trip *t* of some route since the arc χ_{0i} is attended $(\chi_{0i} = 1)$. The inequations calculate the servicing start time defined by variable w_{it} subtracting the travel timing from the manufacturer 0 to the first supplier i visited tt_{0i}, if that route is executed $(\chi_{0i} = 1)$.

$$\varphi_{it} \geq w_{it} - tt_{0i} - M(1 - \chi_{0i}), \qquad \forall i \in C, \forall t \in T; \tag{48}$$

$$\varphi_{it} \leq w_{it} - tt_{0i} + M(1 - \chi_{0i}), \qquad \forall i \in C, \forall t \in T; \tag{49}$$

Constraints (50) and (51) propagate the initial service timing for all the suppliers and the number of trips allocated to the same route.

$$\varphi_{it} \geq \varphi_{it} - M\left(1 - \chi_{ij}\right), \qquad \forall i \in C, \forall t \in T; \tag{50}$$

$$\varphi_{it} \leq \varphi_{it} + M\left(1 - \chi_{ij}\right), \qquad \forall i \in C, \forall t \in T; \tag{51}$$

Constraint (52) calculates the earliness timing of arrival for the trip t to the P-LANE. It considers the end service timing of each travel ϖ_{jt}, compared to the due timing of each lane p in the P-LANE due_p if the demand of that lane is attended by trip t for supplier j ($\gamma_{jtp} = 1$).

$$E_{jp} \geq due_p - \varpi_{jt} - M\left(1 - \gamma_{jtp}\right), \qquad \forall j \in C, \forall p \in P, \forall t \in T; \tag{52}$$

Similarly, constraint (53) calculates the tardiness of arrival timing for the trip t to the P-LANE. It considers the end service timing of each travel ϖ_{jt}, compared to the due timing of each lane p in the P-LANE due_p if the demand of that lane is attended by trip t for supplier j ($\gamma_{jtp} = 1$).

$$T_{jp} \geq \varpi_{jt} - due_p - M\left(1 - \gamma_{jtp}\right), \qquad \forall j \in C, \forall p \in P, \forall t \in T; \tag{53}$$

Constraints (54) and (55) define the domain of the variables.

$$\chi_{ij}, \upsilon_{js}, \gamma_{jtp}, \ \in \{0,1\}\ , \quad \forall\ i,j\ \in A, s\ \in S, t\ \in T, p\ \in P. \tag{54}$$

$$\upsilon_j, TC_{ij}, w_{jt}, \varpi_{jt}, \varphi_{jt}, \ \geq\ 0\ , \quad \forall\ i,j\ \in A, t\ \in T. \tag{55}$$

Compared with the model adapted from Mao et al. (2020), the proposed model contains a lower quantity of binary variables and indices to be considered in its constraints. In this sense, the computation times to obtain an optimal solution is expected to be lower than with the

MRPPL. Aiming at the comparison of the computational efficiency of each model, a small representative scenario was created and will be presented in the first topic of the next chapter.

5. TESTS AND RESULTS

We present the results of computational experiments to verify the performance of the models introduced in the previous chapter. In Section 5.1, we describe a scenario based on fictitious data for the comparison of the computational performance between the models MRPPL-RB and MRPPL-AB. In Section 5.2 the best-performing model, namely the MRPPL-AB, is used to solve several variations of an instance from the real problem. The idea is to use the same scenario for solving different parameter values and evaluate the result, with a special analysis of the lean manufacturing performance criteria. Finally, in Section 5.3, the MRPPL-AB model is used to solve all instances provided by the company of the case study.

5.1. Test for MRPPL-RB and MRPPL-AB Validation and Comparison

In this section, we validate the mathematical model and solve it by considering fictitious data based on some information about the real problem. The validation process is an important step for analyzing the model application to the real problem, considering the processing timing and generated results.

The validation test was carried out using an Intel® Xeon® CPU E5-2630 v3 2.40GHz workstation with 32.0 GB RAM using the GAMS® software version 24.9.2 to implement the mathematical model and the general-purpose MIP solver of the IBM CPLEX Optimization Studio version 12.7.1.0.

5.1.1. Initial Test Data Presentation

As mentioned before, in this section, we present the results of tests with MRPPL-RB and MRPPL-AB, respectively, using data generated by information of the real problem. The purpose is to assess whether the models accurately represent the practical problem and to identify the data considerations required for each model's application.

For these initial tests we consider a sample of 12 vertices ($j = 1, \dots, 12$) where the first one ($j = 1$) represents the manufacturer and the other 11 ($j = 2, \dots, 12$) the suppliers. The P-LANE set is composed of 8 *lanes* ($p = 1, \dots., 8$). Also, we considered that a maximum of 8 routes ($r = 1, \dots, 8$) could be used, and 4 pickup frequencies allowed, considering the *heijunka* statement ($s = 1, 2, 4, 8$). Table 11 presents the distances c in kilometers between the suppliers and the manufacturer and the travel timing tt_{jk} in minutes between them, respectively. For simplification, since this test works with fictitious data, the travel timing tt_{jk} considers the same values of the distance tt_{jk}.

Table 11 – The distance d_{jk} in kilometers between two vertices.

Supplier	j1	j2	j3	j4	j5	j6	j7	j8	j9	j10	j11	j12
j1	0	37	87	81	53	5	8	112	98	73	74	74
j2	34	0	60	55	13	30	41	112	98	91	74	74
j3	87	60	0	10	49	86	95	129	114	112	96	96
j4	84	57	5	0	45	82	91	126	110	109	93	93
j5	56	11	51	45	0	54	43	119	105	98	82	81
j6	4	32	82	76	48	0	9	107	93	74	69	69
j7	8	41	91	85	57	9	0	116	91	66	62	62
j8	110	112	132	126	117	109	118	0	38	59	60	60
j9	97	99	123	118	103	95	88	40	0	20	31	31
j10	84	87	112	106	91	83	67	57	23	0	18	18
j11	72	74	96	91	79	70	66	57	28	19	0	0
j12	72	74	96	90	79	70	65	58	28	19	0	0

Source: elaborated by the author.

The fixed transportation cost per kilometer ϕ was considered as \$20 monetary units. The fixed transportation cost by opened route ω as \$2,500, the earliness penalty cost per minute θ is \$0.01, and the tardiness penalty cost per minute ψ is \$10,000. The difference between the earliness and tardiness cost, as explained in the case study description, is considered because if some delay happens, the risk of line stop is high, and then the second one is much more representative than the first one. The unloading time per volume of parts collected from supplier U_j is fixed in 15 minutes. The trucks' capacity Q was considered as 36 cubic meters (one cubic meter can be considered to the equivalent volume of one pallet) and the demand of each supplier for each lane b_{jp} in the number of pallets is presented in Table 12.

Table 12 – Demand in cubic meters for each supplier in each lane.

	p1	p2	p3	p4	p5	p6	p7	p8	Total
j2	2	2	3	2	2	3	2	2	18
j3	3	3	3	3	3	3	3	3	24
j4	3	3	3	3	3	3	3	3	24
j5	1	2	1	2	1	2	1	2	12
j6	7	8	7	8	7	8	7	8	60
j7	2	2	3	2	2	3	2	2	18
j8	1	2	1	2	1	2	1	2	12
j9	2	1	2	1	2	1	2	1	12
j10	6	6	6	6	6	6	6	6	48
j11	4	5	4	5	4	5	4	5	36
j12	8	7	8	7	8	7	8	7	60
Total	39	41	41	41	39	43	39	41	324

Source: elaborated by the author.

The due date timing of P-LANE arrival due_p is showed in Table 13. The due date for the first lane p_1 is considered after 720 minutes of the first route delivery timing. This timing sliding was considered for the model to have enough time to realize the milk run pickup routes after arrival at the manufacturer plant. After the first lane, the total production timing of 1440 minutes (24 hours) was divided into a slot of 8 $lanes$, and the additional time for each $lane$ is 180 minutes or 3 hours. Each means that each $lane$, when sent to the line side, corresponds to 3 production hours, or 1/8 of the production volume.

Table 13 – Due timing of P-LANE arrival.

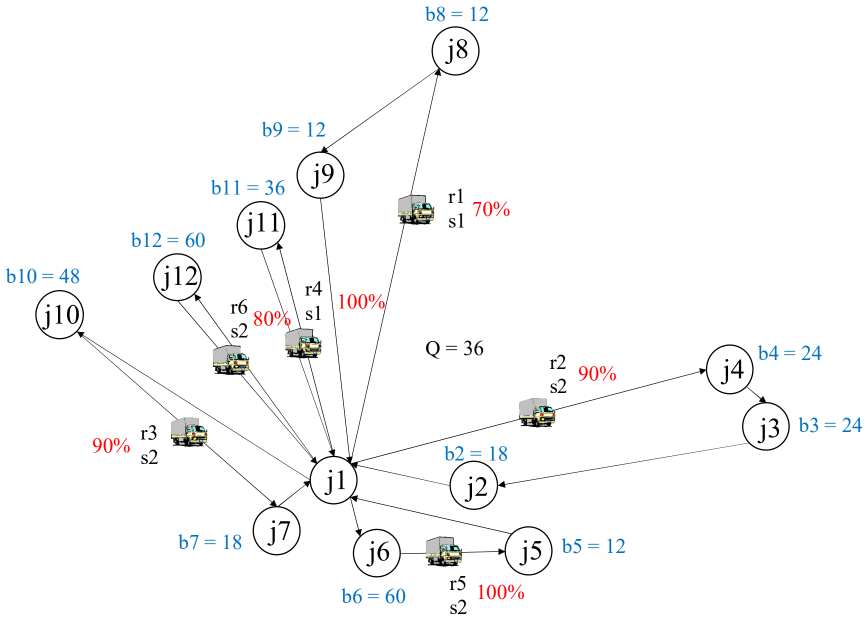
Lane	p1	p2	p3	p4	p5	p6	p7	p8
Timing	720	900	1080	1260	1440	1620	1800	1980

Source: elaborated by the author.

5.1.2. MRPPL-RB Validation and Results

The optimal solution obtained using the model MRPPL-RB with the data presented in the subsection before was found after 28,917.75 seconds. Figure 25 demonstrates a graphic result of the MRPPL-RB. For 11 suppliers attended ($j = 2, \dots, 12$), a total of 6 routes ($r = 1, \dots, 6$) were used. The figure demonstrates each route pickup sequence based on the result of variable z_{jkr}. There were two pickup frequencies observed ($s = 1, 2$). For routes 1 and 4, the frequency of 1 daily pickup was chosen, and for routes 2, 3, 5, and 6, the frequency of two pickups by day.

Figure 25 – The MRPPL-RB result with fictitious data.



Source: elaborated by the author.

Figure 25 also includes the daily demand from each supplier b_{jp} information and the truck capacity considering a homogeneous fleet Q. From Table 12 we can assume that for collecting the total demand of each supplier for each lane resulting in 324 pallets, a total of 9 trips (considering the vehicle's capacity) should be used. Considering the results of this test, with the assumption of each frequency route, a total of 10 trips was defined, resulting in a general truck efficiency of 90%. It can be judged as a good result compared to the 70% considered when manually calculated by the manufacturer. We also indicated in the figure the truckload efficiency of each route, demonstrating that only one route has 70% efficiency and the other ones above 80%.

The result demonstrates a feasible tradeoff between supplier location, demand, and truckload efficiency. For example, for supplier $j = 12$ with a high necessity volume, one dedicated route $r = 6$ was created with two pickups. Despite this supplier is near suppliers $j = 10$ and $j = 11$, all of them have a high demand, justifying the dedicated route for $j = 11$ and $j = 12$. Besides that, in the case of suppliers $j = 5$ and $j = 6$, the total demand of 72 pallets was combined to generate a two-pickup frequency $s = 2$ route $r = 5$ with 100% efficiency.

The result of variable ξ indicates the general overview of the inventory levels since the binary variable indicates if lane p is allocated in the tth pickup of the frequency s for route r and supplier j. If we combine the variable ξ with the necessary volume by lane for each supplier

b_{jp}, the result is shown in Table 14. In this table, we can see the number of pallets collected from each supplier in each pickup of each route and in which lane it will be allocated.

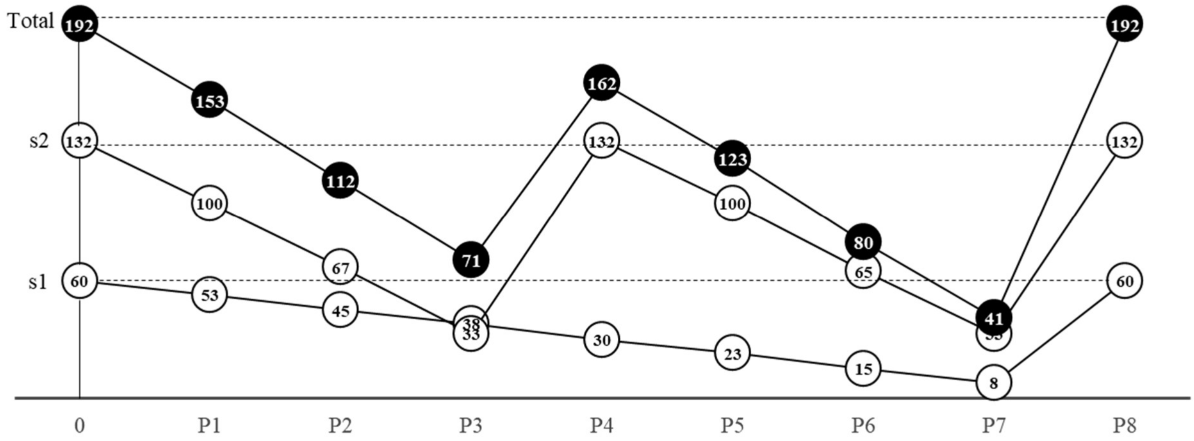
Table 14 – Necessary volume collected for each lane from each pickup, route, and supplier.

				Lane of the P-LANE								
Supplier	Route	Frequency	Pickup	p1	p2	p3	p4	p5	p6	p7	p8	Total
j8	r1	s1	s1	1	2	1	2	1	2	1	2	12
j9	r1	s1	s1	2	1	2	1	2	1	2	1	12
j2	r2	s2	s1	2	2	3	2					9
j3	r2	s2	s1	3	3	3	3					12
j4	r2	s2	s1	3	3	3	3					12
j2	r2	s2	s2					2	3	2	2	9
j3	r2	s2	s2					3	3	3	3	12
j4	r2	s2	s2					3	3	3	3	12
j10	r3	s2	s1	6	6	6	6					24
j7	r3	s2	s1	2	2	3	2					9
j10	r3	s2	s2					6	6	6	6	24
j7	r3	s2	s2					2	3	2	2	9
j11	r4	s1	s1	4	5	4	5	4	5	4	5	36
j5	r5	s2	s1	1	2	1	2					6
j6	r5	s2	s1	7	8	7	8					30
j5	r5	s2	s2					1	2	1	2	6
j6	r5	s2	s2					7	8	7	8	30
j12	r6	s2	s1	8	7	8	7					30
j12	r6	s2	s2					8	7	8	7	30

Source: elaborated by the author.

Based on the volume calculated in Table 14, we can plot the inventory level balance within one production day to estimate the stock volume in the P-LANE. The result of the stock level is demonstrated in Figure 26, where the inventory is divided by each pickup or receiving frequency $s = 1$ and $s = 2$ and consolidated as the total volume.

Figure 26 – Inventory level variation by lane in the quantity of pallets.



Source: elaborated by the author.

Since there are only two different pickup frequencies selected by the model, we can see that inventory levels varied in the periods “0” and “P4”. Based on the analysis of inventory levels presented in the figure, we can consider that, in terms of stock, the model doesn’t consider the minimum level frequencies ($s = 4, 8$). In this sense, the stock level in the P-LANE is higher considering low pickup frequencies ($s = 1, 2$).

The result of variable E is presented in Table 15. We already mentioned the trade-off between zero stock pursuit versus the pickup frequency of milk run routes. In this example we can see that, for example, if the pickup frequency is $s = 1$, we are anticipating the pickups of lanes $p = 2, \dots, 8$ to the same arrival of lane $p = 1$. The same for the pickup frequency $s = 2$, where the pickups of the lanes $p = 2, 3 ,4$ are anticipated to be the same arrival of lane $p = 1$, and the lanes $p = 6, 7, 8$ to the lane $p = 5$.

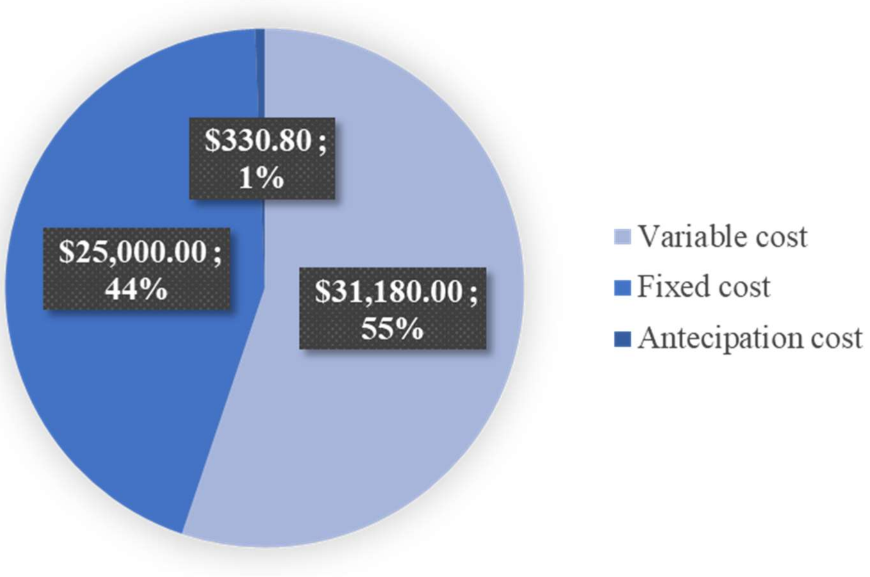
Table 15 – The result of variable E: earliness penalty cost.

	Lane of the P-LANE						
Supplier	p2	p3	p4	p5	p6	p7	p8
j2	200	380	560		180	360	520
j3	200	380	560		180	360	520
j4	200	380	560		180	360	520
j5	200	380	560		180	360	520
j6	200	380	560		180	360	520
j7	200	380	560		180	360	520
j8	200	380	560	740	920	1100	1260
j9	200	380	560	740	920	1100	1260
j10	200	380	560		180	360	520
j11	200	380	560	740	920	1100	1260
j12	200	380	560		180	360	520

Source: elaborated by the author.

Nonetheless, as the external logistics cost, as well as the line stop cost, are higher than the inventory costs, the earliness arrival is accepted, and that explains the result of variable E that considers an earlier arrival for almost all lanes' due date timings. The summary of the cost result breakdown is presented in Figure 27.

Figure 27 – The cost breakdown of the objective function.

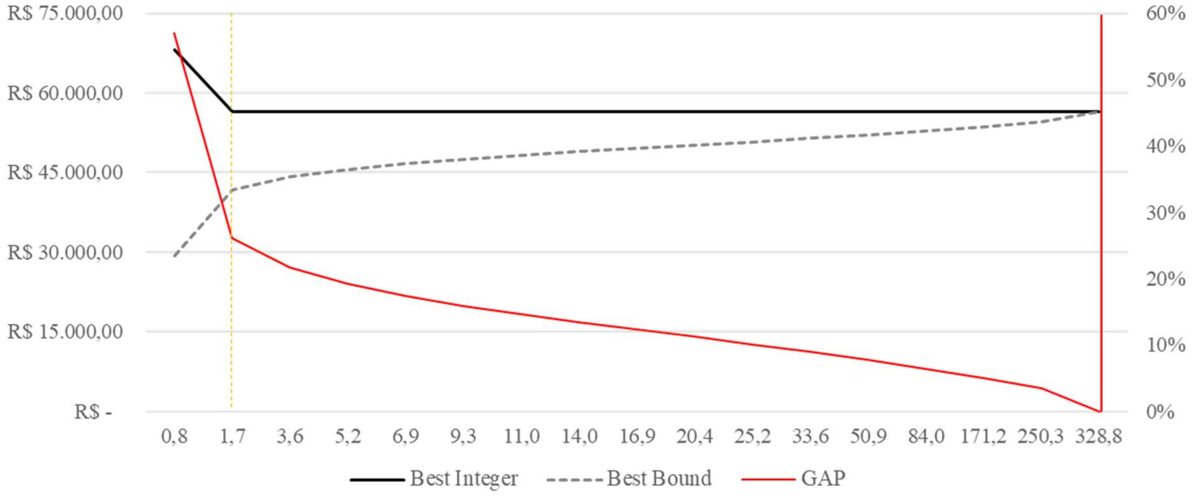


Source: elaborated by the author.

Figure 27 summarizes the cost for each portion of the objective function. The variable cost, which corresponds to the transportation costs, is the most representative cost, with a 55% portion of the total cost. The fixed cost is based on the number of trips and is referred to as 44% of the total cost. The less representative cost is the anticipation cost, already mentioned before, with a portion of 1%. Since the inventory level cost is difficult to calculate, we considered a representative cost for model running. Otherwise, this parameter needs to be tested to check its impact on the route frequency definition. As a result, the total amount of milk run logistics cost for one production day volume attendance was computed as $56,510.80.

The CPLEX resolution is analyzed in Figure 28. We can see that at the beginning of the resolution, there was a big GAP between the best bound and the integer bound, that started to decrease after 1,800 seconds with a GAP of almost 25%. The optimal result was achieved after 28,917.80 seconds of running (5,48 hours).

Figure 28 – Resolution GAP evaluation and computing time



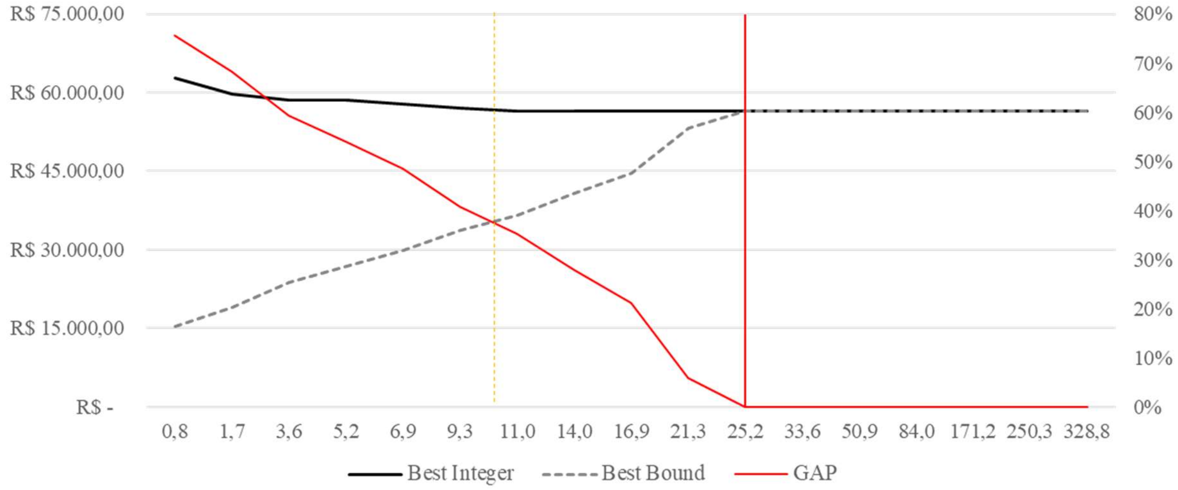
Source: elaborated by the author.

Another parameter to be analyzed is the due date timing of each P-LANE. We already mentioned that the earlier penalty costs are not so representative compared to the tardiness of arrival penalty costs, due to the line stop risk. But even considering a milk run running round step before the due date timing of each lane as shown in Table 13, more tests are necessary to define this parameter and its impact on the model. For example, according to the result shown in Table 15, if one route frequency is defined as $s = 1$, and considering the parameter due_p, in the first lane arrival the cost will be computed for all the other 7 lanes. As already mentioned, there is a trade-off between the zero-stock pursuit and the truckload efficiency improvement, that needs to be better analyzed by the number of routes considered, the timing of each route, and the due date timing of each arrival.

5.1.3. MRPPL-AB Validation and Results

The optimal solution obtained using model MRPPL-AB with the data presented in the subsection before was found after 1,509.38 seconds. The result was the same as MRPPL-RB, with the optimal cost of R$ 56,510.80. The main difference between the MRPPL-AB model and the MRPPL-RB is related to the resolution timing. While the MRPPL-RB takes around 5.5 hours to be solved, the proposed model MRPPL-AB found the optimal solution with 0% GAP after 1,509.38 seconds (25.2 minutes). The CPLEX solution timing for the proposed model is demonstrated in Figure 29.

Figure 29 – Resolution GAP evaluation and computing time.



Source: elaborated by the author.

According to the comparison regarding computation times, it can be noticed that the proposed model MRPPL-AB led to a better solution timing of 25.2 minutes, compared with 328.8 minutes of the MRPPL-RB model. Considering the same data, the solver could find the optimal solution in timing that represents 8% of the MRPPL-RB. This difference can be explained by the comparison of the number of variables between the two models.

The number of binary and integer variables are presented in Table 16:

Table 16 – Comparison of the number of variables for each model.

	MRPPL-RB	MRPPL-AB
Binary variables	x_r y_{jr} z_{jkr} u_{rs} σ_{jrs} δ_{jkrs} ξ_{jrstp}	x_{ij} v_{js} y_{jtp}
Positive variables	A_{rs} F_{jp} E_{jp} T_{jp} ϑ_{jr}	u_j TC_{ij} w_{jt} θ_{jt} y_{jt} E_{jp} T_{jp}
Quanity of Binary variables	75570	1105
Quantity of Positive variables	488	702

Source: elaborated by the author.

Table 16 shows that the number of binary variables in the MRPPL-RB model, which is 75,570 based on the data used, is higher than the 1,105 variables in the MRPPL-AB model, considering the same data. It can be explained due to the number of indexes attached to each binary variable. In the MRPPL-RB model there are four binary variables with three or more indexes attached to each, $z_{jkr}, \sigma_{jrs}, \delta_{jkrs}, \xi_{jrstp}$. The variable ξ_{jrstp} has 61,440 possible combinations considering the values attributed to the indexes of the fictitious data generated. On the other hand, the proposed model has only one variable with three indexes, which is the variable y_{jtp}.

Despite the number of positive variables is higher in the proposed model (MRPPL-AB), with a total of 702 compared with 488 from the MRPPL-RB, the number of binary variables, which make computational processing more complex, is much higher in the MRPPL-RB compared with the proposed model.

5.2. Is MRPPL-AB a Lean Model? An Analysis of Parameter Variation

The first step before applying the proposed model to the real problem was to test it through the variation of different parameters to better understand how the model behaves with different data sets concerning the lean logistics performance indicators.

Thus, based on an instance of the real problem, the MRPPL-AB model was used for solving it, followed by various tests involving adjustments to the following parameters: capacity (*Q*), penalty costs for earliness or tardiness (*P*), fixed transportation cost (*C*), and due time (*D*). The following subsections present the baseline scenario data and the results of these tests, with the naming convention defined as follows: each test name begins with the letter 'T,' followed by the test number and an additional letter that denotes the parameters. For example, the test using the baseline scenario is labeled 'T1B,' where 'T' stands for 'test,' '1' indicates the test number, and 'B' represents the parameters - in this case, 'B' indicates the ***baseline*** scenario data. Similarly, test 'T3Q' refers to test number 3 with a variation in capacity, and so on.

The main idea is to compare the results of different tests considering each parameter variation and between them and the baseline scenario. This comparison will allow us to predict what variation parameters can interfere with the variable results and how this affects the trade-offs of lean performance criteria.

5.2.1. Baseline Scenario Data

We use one of the instances provided by the company studied for the following tests. This instance will be used as the baseline scenario for parameter adjustments and the generation of test scenarios aiming at assessing the sensitivity of the model results to various parameters while identifying the most critical parameters for the problem.

The baseline scenario consists of a sample of 10 vertices ($j = 1, \ldots, 10$) where the first and the last ones ($j = 1$ and $j = 10$) correspond to the manufacturer plant at the beginning and end of each route, respectively. And the other 8 nodes ($j = 2, \ldots, 9$) are the suppliers. The P-LANE capacity is 8 lanes. It means the allowed pickup frequencies are: 1, 2, 4, and 8 (criteria to achieve the *heijunka* or leveled pickup between each trip in the same route). The distance d_{ij}

in kilometers and the travel time tt_{ij} in minutes from supplier i to supplier j are presented in the Table 17 and Table 18, respectively.

Table 17 – The distance in kilometers between suppliers.

Supplier	j1	j2	j3	j4	j5	j6	j7	j8	j9	j10
j1	0	95	26	37	87	81	53	5	8	0
j2	96	0	76	91	95	90	96	94	103	96
j3	21	74	0	27	65	60	32	19	28	21
j4	34	87	28	0	60	55	13	30	41	34
j5	87	90	67	60	0	10	49	86	95	87
j6	84	86	64	57	5	0	45	82	91	84
j7	56	94	35	11	51	45	0	54	43	56
j8	4	90	21	32	82	76	48	0	9	4
j9	8	99	30	41	91	85	57	9	0	8
j10	0	95	26	37	87	81	53	5	8	0

Source: elaborated by the author.

Table 18 – The time in minutes between suppliers.

Supplier	j1	j2	j3	j4	j5	j6	j7	j8	j9	j10
j1	0	82	23	32	75	70	46	5	7	0
j2	83	0	66	78	82	78	83	81	89	83
j3	18	64	0	24	56	52	28	17	24	18
j4	30	75	24	0	52	48	12	26	36	30
j5	75	78	58	52	0	9	42	74	82	75
j6	72	74	55	49	5	0	39	71	78	72
j7	48	81	30	10	44	39	0	47	37	48
j8	4	78	18	28	71	66	42	0	8	4
j9	7	85	26	36	78	73	49	8	0	7
j10	0	82	23	32	75	70	46	5	7	0

Source: elaborated by the author.

The variable transportation cost per kilometer ϕ provided by the company is \$20.00 monetary units. The fixed transportation cost per trip per route, ω was given is \$2,500.00 monetary units. The penalty costs by earliness or tardiness arrival at the P-LANE were also specified by the company, set at $\theta = \$1,00$ and $\psi = \$100,00$ monetary units per minute, respectively. The penalty for the delayed arrival is higher than that for early arrival because delays can impact the parts delivery to the line side, potentially leading to line stoppages due to parts shortages. Conversely, early arrival affects component stock volumes at the manufacturing plant.

The unloading time per volume of parts collected from supplier L_j is fixed at 15 minutes per pallet. A homogeneous fleet with a vehicle capacity Q = 36 cubic meters (where one cubic meter approximates the volume of one pallet) was considered. The demand of each supplier per lane b_{jp}, given in pallets, is presented in Table 19.

Table 19 – Demand in cubic meters for each supplier in each lane.

	Lane								
Supplier	p1	p2	p3	p4	p5	p6	p7	p8	Total (B_j)
j1	0	0	0	0	0	0	0	0	0
j2	1	1	1	1	1	1	1	1	8
j3	1	1	1	1	1	1	1	1	8
j4	1	1	1	1	1	1	1	1	8
j5	2	2	2	2	2	2	2	2	16
j6	1	1	1	1	1	1	1	1	8
j7	3	3	3	3	3	3	3	3	24
j8	7	7	7	7	7	7	7	7	56
j9	4	4	4	4	4	4	4	4	32
j10	0	0	0	0	0	0	0	0	0
Total	20	20	20	20	20	20	20	20	160

Source: elaborated by the author.

The due date timing of P-LANE arrival due_p is showed in Table 20. The due date for the first lane p_1 is considered after 720 minutes of the first route delivery timing. This timing sliding was considered for the model to have enough time to realize the milk run pickup routes after arrival at the manufacturer plant. After the first lane, the total production timing of 1,440 minutes (24 hours) was divided into a slot of 8 *lanes*, and the additional time for each *lane* is 180 minutes or 3 hours. Each means that each *lane*, when sent to the line side, corresponds to 3 production hours, or 1/8 of the manufacturer's production volume.

Table 20 – Due timing of P-LANE arrival.

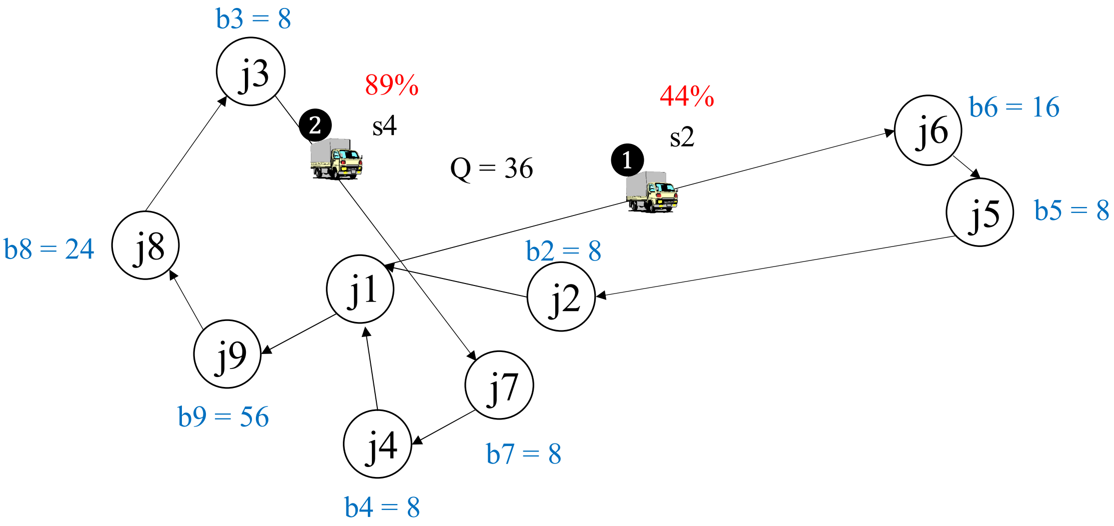
Lane	p1	p2	p3	p4	p5	p6	p7	p8
Timing	720	900	1080	1260	1440	1620	1800	1980

Source: elaborated by the author.

5.2.2. Results of the Baseline Scenario

The optimal solution was found after 334.91 seconds. Figure 30 demonstrates a graphic representation of the scenario T1B. The indication number of each supplier, from this figure on, will be used based on its number from $j1$ to $j48$, according to its representation in the real problem.

Figure 30 – The illustrative result of scenario T1B.



Source: elaborated by the author.

According to Figure *30*, there are two routes created to attend to the total of 8 suppliers. The first route has 3 suppliers allocated and a pick-up frequency equal to two. Each of the two trips has an efficiency of 44% considering the demand of each supplier for this route. The second route has 5 suppliers allocated to it and the pickup frequency is equal to 4, resulting in an efficiency of 89% for each trip. The total average vehicle's efficiency for the scenario T1B is 74%.

Scenario T1B optimization results in a total cost of $45,280.00 monetary units, which the breakdown is: (a) transportation cost $35,080.00, (b) earliness cost $10,200.00 and (c) tardiness cost $0. The transportation cost corresponds to 77% of the total cost, while the penalty earliness cost corresponds to the remainder 23%. There is no delay penalty cost. The results demonstrate that the model prioritizes attending the P-LANE according to or earlier to the due timing of each lane, generating the penalty of earliness arrivals. On the other hand, if the model opened more routes or increased the routes' frequency, the efficiency of each trip would be lower and the transportation cost higher.

The total distance in kilometers resulted in 1,014 Km and the total spent time to attend all suppliers was equal to 876 minutes (14.6 hours). Considering the two routes, a total of 6 trips were used to attend the 8 suppliers, divided into two different routes. These results will be used as a reference for the comparison of the same scenario, under the variation of a specific parameter, which will be explored in the next paragraphs.

5.2.3. Variation of the Vehicle's Capacity

The first parameter to be analyzed is the vehicle's capacity. The idea is to simulate the same scenario of T1B, varying only the parameter Q related to the vehicle's capacity. The result of the variables will be analyzed to evaluate the impact of parameter Q variation. The results evaluated are the quantity of routes used to attend all suppliers, the number of trips necessary to collect all pallets, the average efficiency of the trips, the total cost, the earliness cost, the total distance, the total time, and the process time for optimal result achieved.

Table 21 shows the comparison result of the mentioned variables. A total of 10 different capacities were evaluated, above the T1B reference scenario, demonstrated in the bellow table from scenario T1Q to T10Q. The vehicle's capacity varies from 10 cubic meters to 100 cubic meters (one cubic meter is equivalent to one pallet).

Table 21 – Vehicles capacity variation result.

Scenario	Q	Routes	Trips	Efficiency	Total Cost	θ Cost	Total Dist.	Total Time	Process Time
T1B	36	2	6	74%	45280	10200	1014	876	335
T1Q	10	4	20	80%	87440	5040	1620	1420	52
T2Q	12	3	14	62%	70640	7280	1374	1240	68
T3Q	20	3	10	69%	56000	10200	1030	912	295
T4Q	24	3	7	95%	50320	17600	761	664	317
T5Q	30	2	6	89%	45680	11680	950	830	198
T6Q	40	2	6	67%	45280	10200	1004	876	770
T7Q	48	3	5	67%	45120	20560	603	527	499
T8Q	50	3	5	64%	45120	20560	603	527	742
T9Q	60	3	5	53%	45120	20560	603	527	528
T10Q	100	3	5	32%	45120	20560	603	527	377

Source: elaborated by the author.

From the table we can assume that the number of routes does not change much according to the vehicle's capacity variation. Otherwise, the number of trips necessary to pick up all demand from suppliers reduces proportionally with the vehicle's capacity parameter, from a total of 20 trips with a capacity $Q = 10$ to 5 trips since the capacity surpass $Q = 48$ cubic meters. It was already expected since the total demand for pallets to be collected from suppliers of GROUP_A totalizes 136 pallets or cubic meters.

From T7Q to T10Q, it can also be noted that the variable's result does not have significant changes except for the average efficiency, since the number of trips is the same, and the vehicles' capacity is increased, and the optimization timing, which reduces according to the capacity increases.

The parameter variation of vehicles' capacity Q also indicates that the total cost reduces proportionally to the vehicle's capacity incrementation. It can be explained due to the reduction in the trips necessary to collect all demand from suppliers, reducing the transportation cost. On the other hand, the earliness penalty cost increases with an increase in the vehicles' capacity. Since the number of trips is reduced, it is more difficult to achieve the due timing of P-LANE arrival for each lane, impacting a stock increase at the manufacturer's plant.

The total distance and the total timing are also inversely proportional to the vehicle's capacity. The lower distance and time are related to the higher value of parameter Q. Concerning the vehicle's efficiency, the higher volumes are related to T5Q and T4Q scenarios, with the capacity of 30 and 24 cubic meters, respectively. Despite the efficiency being higher, the variables related to costs results are not the best ones.

Concluding this parameter variation batch, we can judge that the initial test scenario T1B has a good average result comparing the evaluated performance criteria when compared to the other results. The number of routes and trips created is reasonable, resulting in a lower earliness penalty cost and transportation cost, which also results in a better total cost, with average computational timing.

5.2.4. Variation of Penalty Costs

The second batch of tests are related to the penalty costs' parameters, the earliness θ and the tardiness ψ. The idea was to vary the θ and ψ values for evaluating the result of the model's decision variables and its impact on them. There are a total of 21 tests realized and compared with the T1B result.

From scenarios T1P to T6P, the θ parameter was increased and the ψ parameter was considered zero. From scenarios T7P to T11P, the ψ parameter was increased and the θ parameter was considered zero. From scenarios T12P to T15P, both θ and ψ parameters were increased with the same value. From scenarios T16P to T18P, ψ parameter was increased more than the θ parameter. And, finally, from scenarios T19P to T21P, θ parameter was increased more than the ψ parameter. The result is shown in Table 22.

Table 22 – Penalty costs variation result.

Scenario	θ	ψ	Routes	Trips	Efficiency	Total Cost	θ Cost	ψ Cost	Total θ	Total ψ	Total Dist.	Total Time	Process Time
T1B	1	100	2	6	74%	45280	10200	0	10200	0	1014	876	335
T1P	0	0	4	5	89%	22100	0	0	47600	48	480	418	28
T2P	1	0	4	5	89%	22100	0	0	0	33760	480	418	86
T3P	10	0	4	5	89%	22100	0	0	0	33760	480	418	78
T4P	100	0	4	5	89%	22100	0	0	0	33760	480	418	102
T5P	1000	0	4	5	89%	22100	0	0	0	33760	480	418	85
T6P	10000	0	4	5	89%	22100	0	0	0	33760	480	418	109
T7P	0	1	4	5	89%	22148	0	48	47600	48	480	418	148
T8P	0	10	4	5	89%	22580	0	480	47600	48	480	418	143
T9P	0	100	4	5	89%	23160	0	0	47464	0	533	463	127
T10P	0	1000	4	5	89%	23160	0	0	47464	0	533	463	118
T11P	0	10000	4	5	89%	23160	0	0	47464	0	533	463	95
T12P	10	10	1	8	55%	77120	0	0	0	0	2512	2184	49
T13P	100	100	1	8	55%	77120	0	0	0	0	2512	2184	13
T14P	1000	1000	1	8	55%	77120	0	0	0	0	2512	2184	5
T15P	10000	10000	1	8	55%	77120	0	0	0	0	2512	2184	4
T16P	10	100	1	8	55%	77120	0	0	0	0	2512	2184	24
T17P	10	1000	1	8	55%	77120	0	0	0	0	2512	2184	23
T18P	100	1000	1	8	55%	77120	0	0	0	0	2512	2184	10
T19P	100	10	1	8	55%	77120	0	0	0	0	2512	2184	29
T20P	1000	10	1	8	55%	77120	0	0	0	0	2512	2184	31
T21P	1000	100	1	8	55%	77120	0	0	0	0	2512	2184	9

Source: elaborated by the author.

We can see from T1P to T6P that since ψ parameter is equal to zero, it does not matter the value of θ parameter since it is close to zero. The result considering both parameters equal to zero (scenario T1P) has a total cost of $22,100.00 monetary units. Even though the penalty costs for this scenario are zero, it resulted in a θ timing accumulated in 47,600 minutes and a ψ timing accumulated in 48 minutes. For the other scenarios (from T2P to T6P) the results were the same, considering an accumulated ψ timing of 33,760 minutes, explained by the fact that θ parameter is higher in this group of scenarios.

Scenarios T7P to T11P have an opposite result, since now the ψ penalty cost is higher than the θ one. For T7P and T8P, 48 minutes of ψ penalty arrival were generated, and the result was the same observed in scenario T1P. It can be explained by the fact that only a few minutes of tardiness can be accepted since its cost is low, to avoid the extra cost of opening a new trip or increasing the already existing routes transportation costs. On the other hand, it is necessary to adequately define the ψ penalty cost, despite its difficult calculation, since the delayed arrivals could impact the line stop due to components shortage. From scenarios T9P to T11P, the result was the same, generating an accumulated θ timing of 47,464 minutes.

The remainder scenarios from T12P to T21P achieved all the same result, without generating earliness or tardiness arrivals, since the penalty costs are considerable for both parameters. These scenarios' optimal resolution created just one route with pickup frequency 8 to collect parts at the suppliers. It means the same route visit to all the 8 suppliers on 8 different trips, which incurs inefficiencies in the vehicle's capacity and higher values for the total distance and total time, impacting on the total cost also.

From the second batch of tests, we can see that the first scenario T1B has a feasible result related to the penalty costs for parameters θ and ψ. If they are the same, there is no weight increasing the penalty of tardiness arrival, increasing the risk of parts shortage and line stop. On the other hand, a zero cost for the parameter θ implies that any anticipation – and consequently stock accumulation at the manufacturer's plant – is acceptable. Even though total costs and processing time are higher for T1B scenario, it better fits the real problem lean criteria conditions that consider the route's creation with different pickup frequencies to attend the demand from a group of suppliers without generating excessive stock at the manufacturer plant and infeasible transportation costs.

5.2.5. Variation of Transportation Fixed Cost

The next evaluated parameter was the fixed transportation cost ω. The fixed transportation cost is calculated by each trip executed for one determined route and depends on the route pickup frequency. Table 23 demonstrates the results of the other variables when varying the fixed transportation cost, and its comparison with the scenario T1B.

Table 23 – Fixed transportation cost variation result.

Scenario	ω	Routes	Trips	Efficiency	Total Cost	θ Cost	ψ Cost	Total Dist.	Total Time	Process Time
T1B	2500	2	6	74%	45280	10200	0	1014	876	335
T1FC	2000	2	6	74%	42280	10200	0	1004	876	254
T2FC	1500	2	6	74%	39280	10200	0	1004	876	210
T3FC	1000	2	6	74%	36280	10200	0	1004	876	165
T4FC	500	2	6	74%	33280	10200	0	1004	876	100
T5FC	0	3	9	52%	30100	10200	0	995	869	55

Source: elaborated by the author.

As observed in the table, scenarios T1FC to T4FC, which the fixed cost decreases from \$2,000.00 to \$500.00 monetary units, results in the same number of routes, trips, the same vehicle's efficiency, and the same total distance and time. The only changeable result was the total cost, directly impacted by the fixed transportation cost.

The result of T5FC is different since the fixed transportation cost was considered as zero. In this case, a greater number of routes and trips were created to meet the demand of all suppliers. Despite the vehicle's efficiency being lower, the total distance and time spent, compared with the other scenarios, is almost the same.

From this result we can see that a reasonable fixed transportation cost should be considered to avoid an excessive number of routes and trips creation, while represents the real situation where the fixed transportation costs do impact on the final decision of the quantity of routes used to collect all demand from suppliers and their pickup frequency, that will impact in the total quantity of trips and vehicles used.

5.2.6. Variation of Due Timing of P-Lane Arrival

The last evaluation done was related to the P-Lane due arrival timing parameter due_p. For this analysis, a total of 8 scenarios were evaluated and compared with the T1B result. As already explained before, the due timing of arrival is considered since the P-Lane works like a buffer between the external milk-run logistics and the internal routes to supply parts for the assembly line. Respecting the concept of FIFO (first-in first-out), the lanes of the P-Lane should be sent to the line side in a consecutive sequence, from the lower to the higher.

Because of that, each lane in the P-Lane has a due timing, that is related to the timing it will be consumed in the line side. In this sense, the earliness arrival is allowed but incurs an increase in the components stock inside the P-Lane. On the other hand, the tardiness arrival is also allowed but has a higher penalty since it should impact parts shortage and line stop (if parts arrive later than the P-Lane sending time to the line side).

The difference for the parameter due_p definition ca be related with the starting usage time for the first lane, that will impact in the arrival timing of the first trip of each route, and the timing between two lanes, that can be impacted by the daily available production timing and the number of lanes.

Variating the parameter due_p will consequently impact the result of the decision variables of the model. Table 24 compares the results of them, considering 8 different scenarios from T1D to T8D.

Table 24 – Due timing for P-Lane arrival variation result.

Scenario	due	Routes	Trips	Efficiency	Total Cost	θ Cost	ψ Cost	Total Dist.	Total Time	Process Time
T1B	p1 720 p2 900 p3 1080 p4 1260 p5 1440 p6 1620 p7 1800 p8 1980	2	6	74%	45280	10200	0	1014	876	335
T1D	p1 360 p2 540 p3 720 p4 900 p5 1080 p6 1260 p7 1440 p8 1620	4	10	44%	59600	14400	0	1010	880	1176
T2D	p1 480 p2 660 p3 840 p4 1020 p5 1200 p6 1380 p7 1560 p8 1740	3	8	55%	51080	14400	0	1378	1202	643
T3D	p1 560 p2 740 p3 920 p4 1100 p5 1280 p6 1460 p7 1640 p8 1820	2	6	74%	45520	11520	0	950	830	263
T4D	p1 740 p2 920 p3 1100 p4 1280 p5 1460 p6 1640 p7 1820 p8 2000	2	6	74%	44040	12960	0	804	698	348
T5D	p1 740 p2 980 p3 1220 p4 1460 p5 1700 p6 1940 p7 2180 p8 2420	2	6	74%	48200	9600	0	1180	1032	355
T6D	p1 740 p2 1100 p3 1460 p4 1820 p5 2180 p6 2540 p7 2900 p8 3260	2	6	74%	53000	14400	0	1180	1032	189
T7D	p1 480 p2 600 p3 720 p4 840 p5 960 p6 1080 p7 1200 p8 1320	3	8	55%	46280	9600	0	1378	1202	687
T8D	p1 480 p2 630 p3 780 p4 930 p5 1080 p6 1230 p7 1380 p8 1530	3	8	55%	48680	12000	0	1378	1202	730

Source: elaborated by the author.

From the table we can conclude that due to the low penalty cost of earliness arrival, most of the scenarios have similar penalty costs, total costs, traveled distance, and time. The most impacted results observed were the number of routes and the number of trips. The earlier the timing of the first P-Lane arrival, the greater the number of trips necessary to collect all parts from suppliers, as is observed in the scenarios T7D and T8D. It can be explained because due to suppliers' distance and the route timing, it is necessary to reduce the number of suppliers on the same route or increase the route frequency for the vehicle to be able to arrive earlier at the manufacturing plant.

5.2.7. Summary of Findings: MRPPL-AB as a Lean Model

The results obtained from section 5.2.3 to 5.2.6 demonstrate the relationship between the parameters and variables of the MRPPL-AB model and evaluate the trade-offs regarding the performance criteria of lean logistics indicators. The summary of the results is presented in Table 25.

Table 25 – Summary parameters' variation tests.

Parameter	# of tests	Parameter variation	# of routes	# of travels	Efficiency	Transp. cost	Ear cost	Tar cost	Distance	Timing	Process timing
Q	10	↑	-	↓	↓	↓	↑	-	↓	↓	↑
ear tar	21	↑	↓	↑	↓	↑	-	-	↑	↑	↓
FC	5	↓	↑	↑	↓	↓	-	-	↓	↓	↓
due	8	↓	-	-	↓	↑	-	-	-	↑	↑

Source: elaborated by the author.

Analyzing the summary is possible to affirm that, regarding the parameter capacity Q, as the vehicles' capacity is increased, the number of travels, the vehicles' efficiency, and the total transportation costs, the total traveled distance, and the timing are decreased. On the other hand, the earliness cost is increased, which means that the stock in the plant is related to the vehicles' capacity Q.

The penalty costs parameters, otherwise, impact the number of travels, the transportation cost, and the timing spent to execute all travels. If the earliness cost is increased, representing the inventory cost at the manufacturer, the number of trips will also increase, since there will be a restricted timing for the routes' arrival at the plant. The same effect is observed when varying the parameter of tardiness cost penalty. Despite both earliness and tardiness demonstrating the same effect in the objective function, in the real world, the tardiness arrival has a worst effect since the parts shortage risk could become a line stop.

The fixed transportation cost FC is related to the cost of having one more vehicle available for routing planning. In this sense, decreasing the value of fixed transportation costs will impact the increase of the number of routes and travels created. Despite the fact that their number are higher, the total transportation cost will be lower, also impacting the vehicles' efficiency. The total travelling distance and timing also decrease.

About due timing parameter, since the timing difference between two consecutive lanes receiving is reduced, the vehicles' efficiency is also reduced, and the transportation costs are increased. It makes sense since the number of trips, and consequently vehicles used to collect total demand from all suppliers should also be increased.

Section 5.2 allows us to demonstrate that MRPPL-AB attends to the lean logistics performance criteria as long as its parameters are correctly defined. For example, the setting up of earliness and tardiness penalty costs (and consequently inventory and line stop risk) will directly impact the routing frequency. The same logic is used for the vehicles' capacity, the fixed transportation cost and the due timing for receiving parts at the P-LANE.

As a conclusion, there is a trade-off between the number of routes and vehicles used, the number of trips, related to the route pick-up frequency definition and the inventory levels at the manufacturer plan. Since the lean logistics goals aim at the reduction of inventory levels, at the same time as the transportation waste reduction, the model presents a good fit for calculating these goals, while the correct parameters are being inputted.

5.3. MRPPL-AB Application to the Real Problem

In this section we present the results obtained for the real problem analyzed in the case studied using the formulation MRPPL-AB. In the real problem, the company has a set of 47 suppliers and the manufacturer's plant (that is represented by the first $j = 1$ and the last $j = n + 1$ nodes visited). The vehicle should depart from the manufacturer's plant, loaded with empty packages, to execute one route. After collecting the pallets from the suppliers, the vehicle returns to the plant. The total daily demand collected from suppliers is 752 pallets.

The number of suppliers attended on each route is not defined, and it can have only one supplier. In the real problem, the 47 suppliers are separated into 7 different groups, according to their location and the nearby road network. The separation of the suppliers into the groups is show in Table 26.

Table 26 – Suppliers' group identification.

Group name	Quantity of suppliers	Suppliers' identification (node)
GROUP_A	8	j2, j4, j5, j6, j7, j8, j9, j10
GROUP_B	5	j20, j24, j25, j26, j27
GROUP_C	9	j15, j16, j17, j18, j19, j21, j22, j23, j44
GROUP_D	5	j28, j29, j30, j31, j32
GROUP_E	6	j41, j42, j43, j46, j47, j48
GROUP_F	6	j3, j33, j34, j35, j36, j40
GROUP_G	8	j11, j12, j13, j14, j37, j38, j39, j45
Total	47	

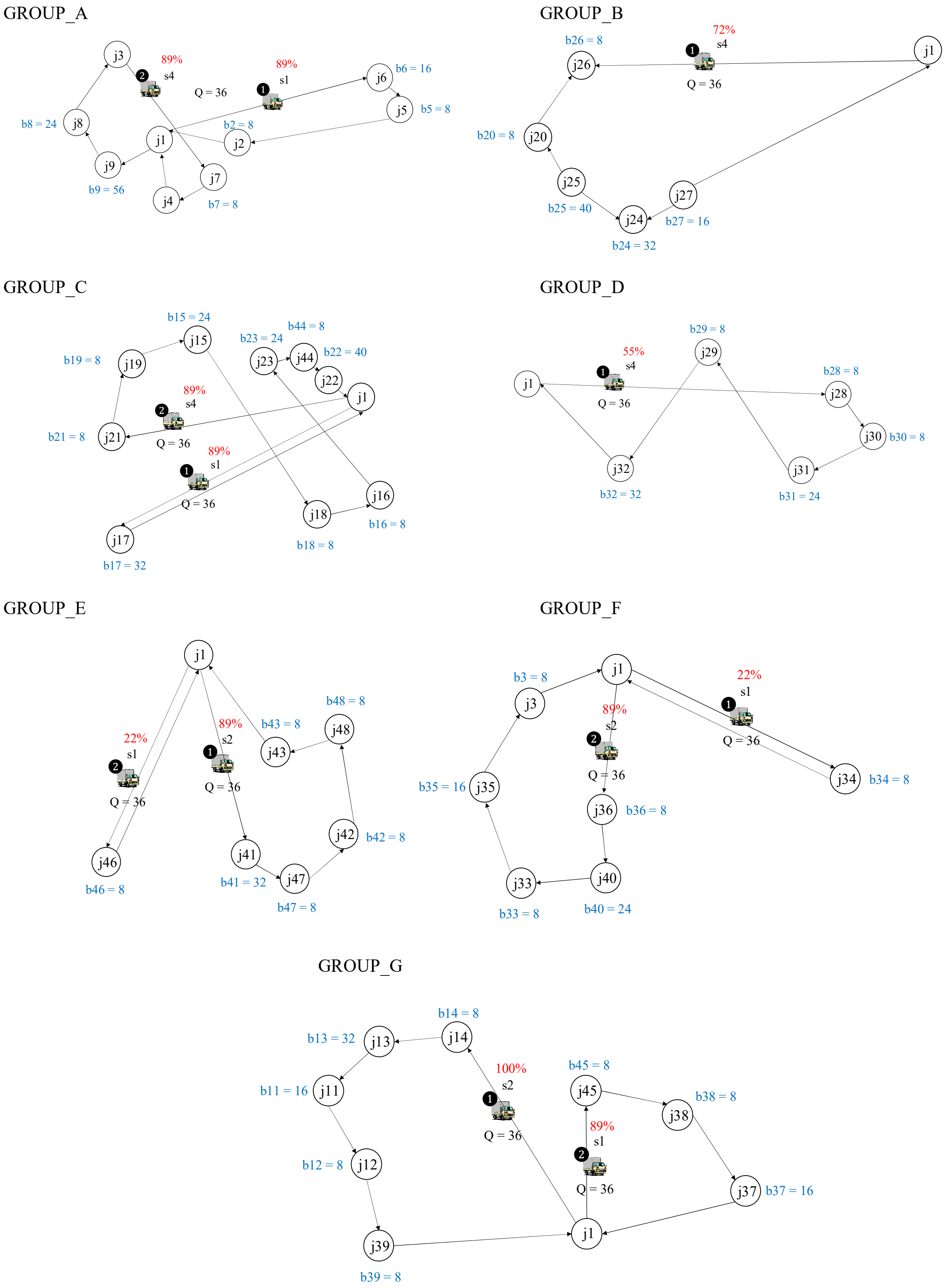
Source: elaborated by the author.

According to the table, the quantity of suppliers by group can vary from 5 (GROUP_B and GROUP_D) to 9 (GROUP_C). For each group, the number of nodes considered in the model is the total quantity of suppliers plus j_1 and j_{n+1}, where n represents the number of suppliers in the group. The nodes j_1 and j_{n+1} correspond to the manufacturer's plant. The application of the model was done for each group, and the detailed data and results are presented in APPENDIX A.

The model was applied to each group of suppliers, considering the same strategy used for the manufacturer to calculate route's planning. The results are shown in Figure 31 and Table 27. The figure demonstrates, for each group of suppliers, the number of routes, the pick-up frequencies and the sequence of each route, which created the demand for each supplier. The vehicle's efficiency is also demonstrated.

As explained before, each group of suppliers demonstrated in Figure 31 is separated considering the road location network. Complementary to the figure, Table 27 summarizes the performance indicators' information from each supplier's group. We will use some data from the table to compare the performance indicators with the real problem routing planning results.

Figure 31 - The result of real problem.



Source: elaborated by the author.

Table 27 - The result of real problem.

Group name	Suppliers	Routes	Suppliers by route	# of suppliers	Route Efficiency	Average efficiency	Total Cost	Earliness Cost	Tardiness Cost	Distance	Time	Process Time (min)
GROUP_A	8	2	j1-j7-j6-j2-j1 j1-j10-j4-j8-j5-j9-j1	3 5	89% 89%	89%	39620	12480	0	732	640	7,3
GROUP_B	5	1	j1-j26-j20-j25-j24-j27-j1	5	72%	72%	23440	2400	0	552	516	0,0
GROUP_C	9	2	j1-j17-j1 j1-j21-j19-j15-j18-j16-j23-j44-j22-j1	1 8	89% 89%	89%	33120	7200	0	671	593	74,9
GROUP_D	5	1	j1-j28-j30-j31-j29-j32-j1	5	55%	55%	49280	2400	0	1844	1588	0,0
GROUP_E	6	2	j1-j41-j47-j42-j48-j43-j1 j1-j46-j1	5 1	89% 22%	67%	32180	10560	0	706	611	0,2
GROUP_F	6	2	j1-j34-j1 j1-j36-j40-j33-j35-j3-j1	1 5	22%	67%	34100	10560	0	1342	1166	0,2
GROUP_G	8	2	j1-j14-j13-j11-j12-j39-j1 j1-j45-j38-j37-j1	5 3	100% 89%	96%	36460	17280	0	584	508	9,2
Total	47	12		47		76%	$248.200,00	$62.880,00	$ -	6431	5622	91,8

Source: elaborated by the author.

According to the table, the 47 suppliers attended by a total of 12 routes. The number of suppliers assigned on each route varies from 1 to 8. The maximum number of routes opened for each group was 2. The route's efficiency varies from 22% to 100%. The average efficiency, considering each group of suppliers, varies from 55% to 96%, where the best route efficiency was allocated to suppliers' GROUP_G. The total average efficiency is 76%.

The total transportation cost result is $248,200.00, where $62,880.00, or 25%, is related to earliness penalty costs, or inventory level at the manufacturer plant, and there is no cost related to tardiness. The total distance executed was 6,431 kilometers and the total time spent was 5,622 minutes, or 93.7 hours. The total processing time for all groups is 91.8 minutes, or 1.32 hours.

Compared with the real problem, results demonstrate that the MRPPL-AB is suitable for the manufacturer's operation. Analyzing the number of trips, the model proposed a total of 27 trips, considering 12 routes, and the manufacturer planned 15 routes and 35 trips. Also, the result will impact vehicles' efficiency. The average vehicle's efficiency for the MRP-LL is 27.85 pallets/trip or 76% while the manufacturer planned 21.48 pallets/trip, representing 59% efficacy.

An important point to be observed is that the MRPPL-AB model considers an inventory cost of $62,880.00, which represents 25% of the total logistics costs. Despite the correct value of this cost being difficult to calculate, since it represents the necessary area inside the manufacturer's plant to keep the stock, it is an important factor to be considered while analyzing the trade-off between VRP and lean logistics goals.

Another important factor to be analyzed is the resolution timing. Since the timing spent from the manufacturer to calculate the route planning can vary from 3-5 days according to the complexity of monthly schedule changing, the proposed model can achieve the optimum result in a computational timing of 91.8 minutes, or 1.32 hours.

As a result, we can conclude that the MRPPL-AB can be fitted to the real problem and generate a greater and faster result for the routing planning problem. Despite the inventory level increasing compared with the real problem, the total logistics cost has decreased when utilizing the MRPPL-AB.

6. CONCLUSION

We addressed the application of the Vehicle Routing Problem (VRP) to a milk-run pick-up system under the lean manufacturing approach. The aim was to evaluate the trade-offs of lean performance criteria applied to an automotive manufacturer, specifically analyzing the relationship between the milk-run routes pick-up frequency and the inventory levels at the manufacturer plant.

As a first step, we presented a literature review exposing the lean logistics overall concepts and its applications to the milk-run. The state of the art of the VRP application to lean logistics was also presented. After all, a Systematic Literature Review (SLR) was conducted aiming to identify what is being studied when combining the VRP to the lean logistics.

As a result of the SLR, we identified that the mathematical model proposed by Mao et al. (2020) was the one that most fit the real problem presented in the case study. After adapting this model to properly represent the real problem, it was noted that the number of indices and binary variables negatively affected the computational performance of the solver when using this model.

Based on this observation, we proposed a model based on different types of variables and constraints. The results of computational experiments considering a sample of fictitious data demonstrated that the MRPPL-AB led to proven optimal solutions in 8% of the time required when using the MRPPL-RB.

The MRPPL-AB was then used for a better understanding of the relationship between the model parameters variation and the variable's result under a lean approach. As a conclusion, described parameters demonstrate the trade-off between the number of routes and vehicles used, the number of trips, related to the route pick-up frequency definition and the inventory levels at the manufacturer plant.

After applying MRPPL-AB model to the real problem, we could verify that the MRPPL-AB model leads to proven optimal solutions in a shorter computation time than the method used by the manufacturer, despite the inventory level having some increase. The result demonstrated that the parameter of earliness arrival θ, instead of being difficult to calculate, has an important role for lean logistics performance criteria judgment. In this sense, we can also conclude that the P-LANE has an important role in the lean logistics approach while acting as a buffer between external and internal logistics.

Future perspectives instigate us to include environmental aspects to the proposed model to analyze the impacts of, for example, carbon emissions, on the routing planning sequence and, consequently, the lean logistics goals impacts. Another important point to explore is, despite the problem, the solution was found for groups of suppliers considering the same road network, how should the model's performance and the result be considered the total problem resolution.

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APPENDIX A – MRPPL-AB APPLICATION TO THE REAL PROBLEM

In this appendix, we presented the detailed data and the main results for the resolution of the real problem, divided into seven supplier groups, considering the same configuration of the manufacturer for routing planning.

GROUP_A

The first group of suppliers is composed of 8 suppliers and 10 nodes. The P-LANE is composed of 8 lanes. The distance d_{ij} in kilometers and the travel time σ_{ij} in minutes are presented in Table A1 and Table A2, respectively.

Table A1 – Distance in kilometers between the suppliers of GROUP_A.

	j1	j2	j4	j5	j6	j7	j8	j9	j10	jn+1
j1	0	95	26	37	87	81	53	5	8	0
j2	96	0	76	91	95	90	96	94	103	96
j4	21	74	0	27	65	60	32	19	28	21
j5	34	87	28	0	60	55	13	30	41	34
j6	87	90	67	60	0	10	49	86	95	87
j7	84	86	64	57	5	0	45	82	91	84
j8	56	94	35	11	51	45	0	54	43	56
j9	4	90	21	32	82	76	48	0	9	4
j10	8	99	30	41	91	85	57	9	0	8
jn+1	0	95	26	37	87	81	53	5	8	0

Source: elaborated by the author.

Table A2 – Travel time in minutes between the suppliers of GROUP_A.

	j1	j2	j4	j5	j6	j7	j8	j9	j10	jn+1
j1	0	82	23	32	75	70	46	5	7	0
j2	83	0	66	78	82	78	83	81	89	83
j4	18	64	0	24	56	52	28	17	24	18
j5	30	75	24	0	52	48	12	26	36	30
j6	75	78	58	52	0	9	42	74	82	75
j7	72	74	55	49	5	0	39	71	78	72
j8	48	81	30	10	44	39	0	47	37	48
j9	4	78	18	28	71	66	42	0	8	4
j10	7	85	26	36	78	73	49	8	0	7
jn+1	0	82	23	32	75	70	46	5	7	0

Source: elaborated by the author.

The variable transportation cost per kilometer VC was considered \$20.00 monetary units. The fixed transportation cost per each travel executed by one route FC was considered as \$2,500.00 monetary units. The penalty costs by earliness or tardiness of arrival at the P-LANE were considered, respectively, as ear \$1.00 and tar \$100.00 monetary units per minute. The penalty for the delayed arrival is higher than anticipation because it can impact the parts fulfillment to the line side and consequently line stop due to parts shortage. On the other hand, the anticipation of the arrival will impact the component's stock volume at the manufacturer plant.

The unloading time per volume of parts collected from the supplier l_j and is fixed and equal to 15 minutes per pallet. It was considered a homogeneous fleet with vehicles' capacity Q of 36 cubic meters (one cubic meter can be considered to be the equivalent volume of one pallet). The above-mentioned parameters are considered the same for all groups of suppliers. The demand in pallets for each supplier and lane is presented in Table A3.

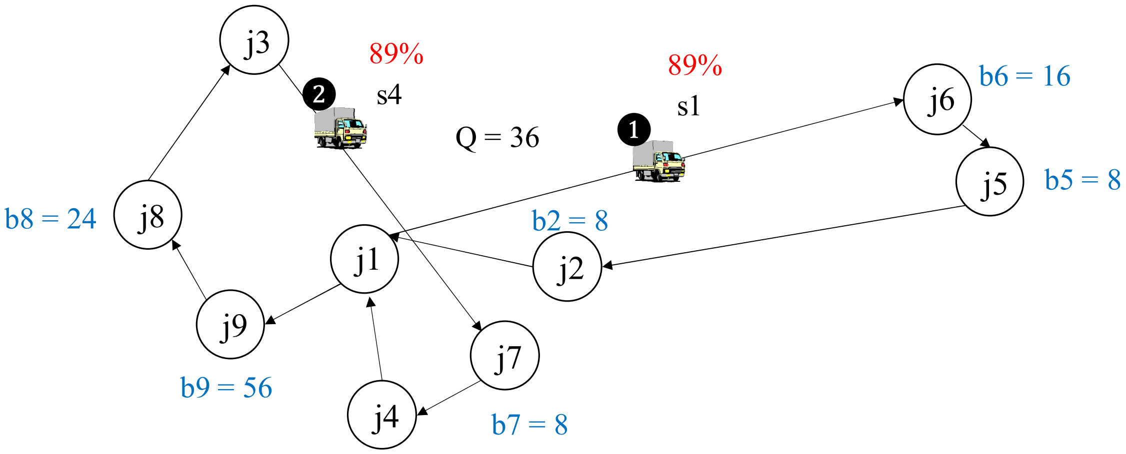
Table A3 – Demand in pallets per lane of the suppliers of GROUP_A.

	p1	p2	p3	p4	p5	p6	p7	p8
j1	0	0	0	0	0	0	0	0
j2	1	1	1	1	1	1	1	1
j4	1	1	1	1	1	1	1	1
j5	1	1	1	1	1	1	1	1
j6	2	2	2	2	2	2	2	2
j7	1	1	1	1	1	1	1	1
j8	3	3	3	3	3	3	3	3
j9	7	7	7	7	7	7	7	7
j10	4	4	4	4	4	4	4	4
jn+1	0	0	0	0	0	0	0	0

Source: elaborated by the author.

The optimal solution for GROUP_A was found after 436.52 seconds. The result is presented in Figure A1.

Figure A1 – Routing result for GROUP_A.



Source: elaborated by the author.

As shown in the figure, there were two routes created to attend the total of 8 suppliers. The first route has 3 suppliers allocated to it, a pickup frequency of 1 trip, with an efficiency of 89%. The second route has 5 suppliers allocated, a pickup frequency of 4 trips and an efficiency of 89%. The average vehicle's efficiency to attend GROUP_A is 89%.

There were a total of 5 trips, corresponding to a total of 732 kilometers and 640 minutes. Regarding the costs, the total cost for this group is \$39,620.00 monetary units, where \$12,480.00 corresponds to the earliness arrival at P-LANE (and represents the inventory cost at the manufacturer's plant). There was no tardiness penalty cost.

GROUP_B

GROUP_B is composed of 5 suppliers and 7 nodes. The distance d_{ij} in kilometers and the travel time σ_{ij} in minutes are presented in Table A4 and Table A5, respectively.

Table A428 – Distance in kilometers between the suppliers of GROUP_B.

	j1	j20	j24	j25	j26	j27	jn+1
j1	0	53	52	53	52	52	0
j20	52	0	5	5	5	5	52
j24	51	10	0	10	10	10	51
j25	52	10	10	0	10	10	52
j26	51	10	10	10	0	10	51
j27	51	10	10	10	10	0	51
jn+1	0	53	52	53	52	52	0

Source: elaborated by the author.

Table A5 – Travel time in minutes between the suppliers of GROUP_B.

	j1	j20	j24	j25	j26	j27	jn+1
j1	0	46	45	46	45	45	0
j20	45	0	10	10	10	10	45
j24	44	10	0	10	10	10	44
j25	45	10	10	0	10	10	45
j26	44	10	10	10	0	10	44
j27	44	10	10	10	10	0	44
jn+1	0	46	45	46	45	45	0

Source: elaborated by the author.

The demand in pallets for each supplier and lane is presented in Table A6.

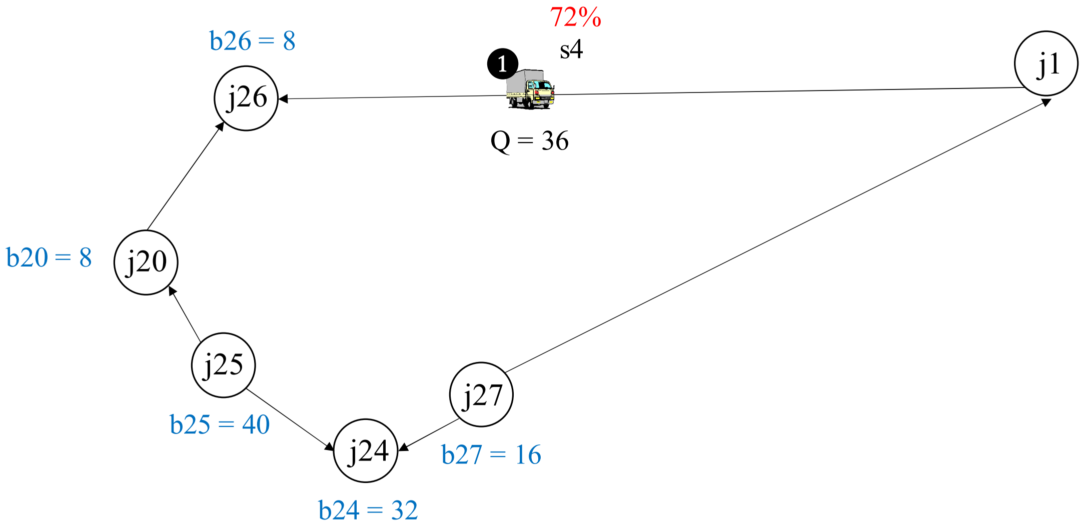
Table A6 – Demand in pallets per lane of the suppliers of GROUP_B.

	p1	p2	p3	p4	p5	p6	p7	p8
j1	0	0	0	0	0	0	0	0
j20	1	1	1	1	1	1	1	1
j24	4	4	4	4	4	4	4	4
j25	5	5	5	5	5	5	5	5
j26	1	1	1	1	1	1	1	1
j27	2	2	2	2	2	2	2	2
jn+1	0	0	0	0	0	0	0	0

Source: elaborated by the author.

The optimal solution for GROUP_B was found after 1.23 seconds. The result is presented in Figure A2.

Figure A2 – Routing result for GROUP_B.



Source: elaborated by the author.

As shown in the figures, one route was created to attend a total of 5 suppliers. The route has a pickup frequency of 4 trips, with an efficiency of 72%. There were a total of 4 trips, which corresponded to 552 kilometers and 516 minutes. Regarding the costs, the total cost for this group is $23,440.00 monetary units, where $2,400.00 corresponds to the earliness arrival at P-LANE (and represents the inventory cost at the manufacturer's plant). There was no tardiness penalty cost.

GROUP_C

GROUP_C is composed of 9 suppliers and 11 nodes. The distance d_{ij} in kilometers and the travel time σ_{ij} in minutes are presented in Table A7 and Table A8, respectively.

Table A7 – Distance in kilometers between the suppliers of GROUP_C.

	j1	j15	j16	j17	j18	j19	j21	j22	j23	j44	jn+1
j1	0	51	50	35	47	46	21	31	33	33	0
j15	51	0	5	50	2	4	36	59	25	25	51
j16	50	7	0	48	5	8	34	58	23	23	50
j17	36	50	49	0	46	45	16	44	30	30	36
j18	50	3	5	48	0	5	35	58	24	24	50
j19	51	1	5	50	4	0	36	59	25	25	51
j21	22	36	35	14	32	32	0	31	16	16	22
j22	34	63	62	47	59	58	33	0	33	33	34
j23	32	25	30	28	26	25	14	27	0	5	32
j44	32	25	30	28	26	25	14	27	5	0	32
jn+1	0	51	50	35	47	46	21	31	33	33	0

Source: elaborated by the author.

Table A8 – Travel time in minutes between the suppliers of GROUP_C.

	j1	j15	j16	j17	j18	j19	j21	j22	j23	j44	jn+1
j1	0	44	43	30	41	40	18	27	29	29	0
j15	44	0	5	43	2	4	31	51	22	22	44
j16	43	6	0	42	5	7	30	50	20	20	43
j17	31	43	42	0	40	39	14	38	26	26	31
j18	43	3	5	42	0	5	30	50	21	21	43
j19	44	1	5	43	4	0	31	51	22	22	44
j21	19	31	30	12	28	28	0	27	14	14	19
j22	30	54	54	41	51	50	29	0	29	29	30
j23	28	22	26	24	23	22	12	24	0	5	28
j44	28	22	26	24	23	22	12	24	5	0	28
jn+1	0	44	43	30	41	40	18	27	29	29	0

Source: elaborated by the author.

The demand in pallets for each supplier and lane is presented in Table A9.

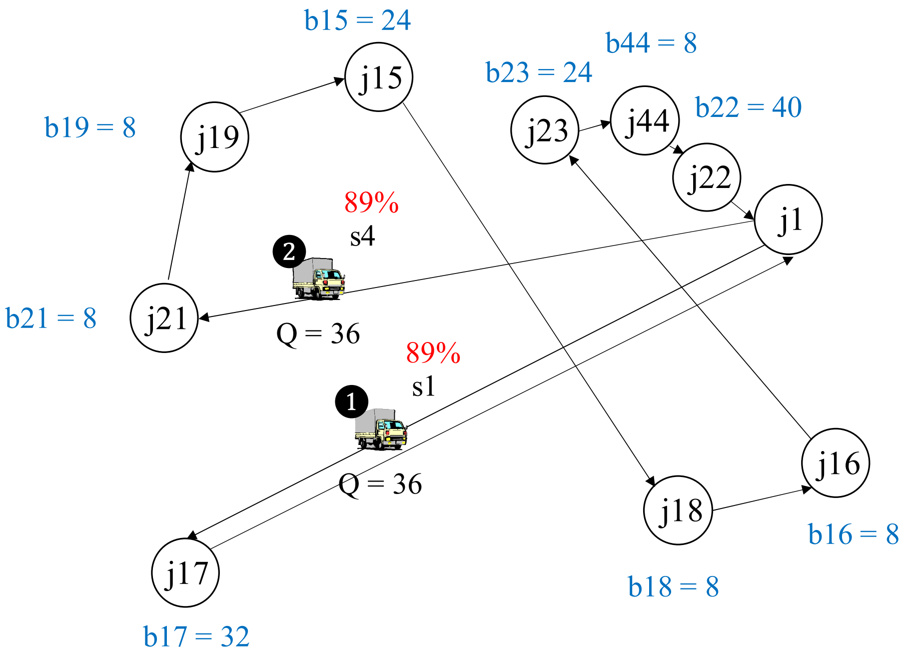
Table A9 – Demand in pallets per lane of the suppliers of GROUP_C.

	p1	p2	p3	p4	p5	p6	p7	p8
j1	0	0	0	0	0	0	0	0
j15	3	3	3	3	3	3	3	3
j16	1	1	1	1	1	1	1	1
j17	4	4	4	4	4	4	4	4
j18	1	1	1	1	1	1	1	1
j19	1	1	1	1	1	1	1	1
j21	1	1	1	1	1	1	1	1
j22	5	5	5	5	5	5	5	5
j23	3	3	3	3	3	3	3	3
j44	1	1	1	1	1	1	1	1
jn+1	0	0	0	0	0	0	0	0

Source: elaborated by the author.

The optimal solution for the GROUP_C was found after 4,491.41 seconds. The result is presented in Figure A3.

Figure A3 – Routing result for GROUP_C.



Source: elaborated by the author.

As shown in the figure, there were two routes created to attend the total of 9 suppliers. The first route has only one supplier allocated to it, a pickup frequency of 1 trip, with an efficiency of 89%. The second route has 8 suppliers allocated, a pickup frequency of 4 trips and an efficiency of 89%. The average vehicle's efficiency to attend the GROUP_C is 89%.

There were a total of 5 trips, corresponding to a total of 671 kilometers and 593 minutes. Regarding the costs, the total cost for this group is $33,120.00 monetary units, where $7,200.00 corresponds to the earliness arrival at P-LANE (and represents the inventory cost at the manufacturer's plant). There was no tardiness penalty cost.

GROUP_D

GROUP_D is composed of 5 suppliers and 7 nodes. The distance d_{ij} in kilometers and the travel time σ_{ij} in minutes are presented in Table A10 and Table A11, respectively.

Table A10 – Distance in kilometers between the suppliers of GROUP_D.

	j1	j28	j29	j30	j31	j32	jn+1
j1	0	212	212	238	239	164	0
j28	212	0	5	15	16	50	212
j29	212	5	0	16	16	50	212
j30	241	19	19	0	2	69	241
j31	240	18	18	2	0	68	240
j32	164	53	53	65	65	0	164
jn+1	0	212	212	238	239	164	0

Source: elaborated by the author.

Table A11 – Travel time in minutes between the suppliers of GROUP_D.

	j1	j28	j29	j30	j31	j32	jn+1
j1	0	182	182	204	205	141	0
j28	182	0	5	13	14	43	182
j29	182	5	0	14	14	43	182
j30	207	17	17	0	2	60	207
j31	206	16	16	2	0	59	206
j32	141	46	46	56	56	0	141
jn+1	0	182	182	204	205	141	0

Source: elaborated by the author.

The demand in pallets for each supplier and lane is presented in Table A12.

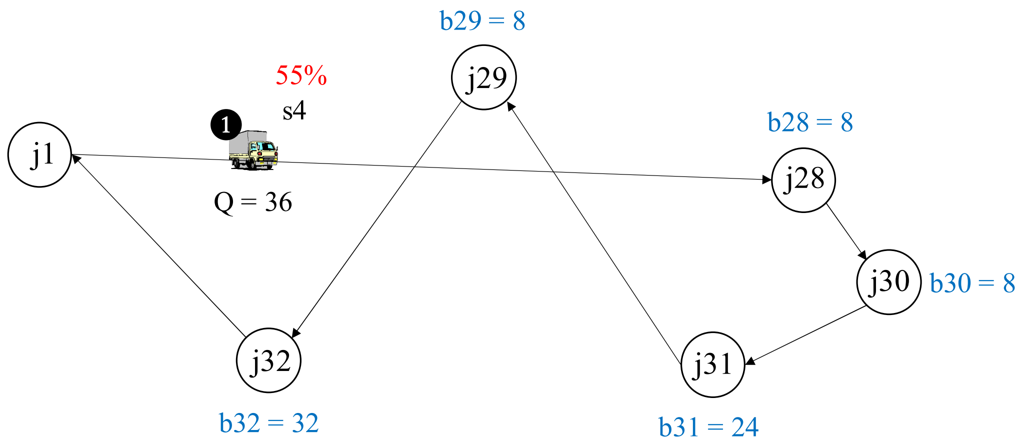
Table A12 – Demand in pallets per lane of the suppliers of GROUP_D.

	p1	p2	p3	p4	p5	p6	p7	p8
j1	0	0	0	0	0	0	0	0
j28	1	1	1	1	1	1	1	1
j29	1	1	1	1	1	1	1	1
j30	1	1	1	1	1	1	1	1
j31	3	3	3	3	3	3	3	3
j32	4	4	4	4	4	4	4	4
jn+1	0	0	0	0	0	0	0	0

Source: elaborated by the author.

The optimal solution for the GROUP_D was found after 1.44 seconds. The result is presented in Figure A4.

Figure A4 – Routing result for GROUP_D.



Source: elaborated by the author.

As shown in the figure, one route was created to attend the total of 5 suppliers. The route has a pickup frequency of 4 trips, with an efficiency of 55%. There were a total of 4 trips, corresponding to a total of 1,844 kilometers and 1,588 minutes. Regarding the costs, the total cost for this group is $49,280.00 monetary units, where $2,400.00 corresponds to the earliness arrival at P-LANE (and represents the inventory cost at the manufacturer's plant). There was no tardiness penalty cost.

GROUP_E

GROUP_E is composed of 6 suppliers and 8 nodes. The distance d_{ij} in kilometers and the travel time σ_{ij} in minutes are presented in Table A13 and Table A14, respectively.

Table A13 – Distance in kilometers between the suppliers of GROUP_E.

	j1	j41	j42	j43	j46	j47	j48	jn+1
j1	0	113	134	119	73	133	131	0
j41	116	0	30	22	50	29	27	116
j42	132	31	0	17	66	1	4	132
j43	117	20	16	0	58	17	10	117
j46	83	40	60	54	0	60	58	83
j47	131	30	1	19	65	0	7	131
j48	130	28	4	11	63	5	0	130
jn+1	0	113	134	119	73	133	131	0

Source: elaborated by the author.

Table A14 – Travel time in minutes between the suppliers of GROUP_E.

	j1	j41	j42	j43	j46	j47	j48	jn+1
j1	0	97	115	102	63	114	113	0
j41	100	0	26	19	43	25	24	100
j42	114	27	0	15	57	1	4	114
j43	101	18	14	0	50	15	9	101
j46	72	35	52	47	0	52	50	72
j47	113	26	1	17	56	0	6	113
j48	112	24	4	10	54	5	0	112
jn+1	0	97	115	102	63	114	113	0

Source: elaborated by the author.

The demand in pallets for each supplier and lane is presented in Table A15.

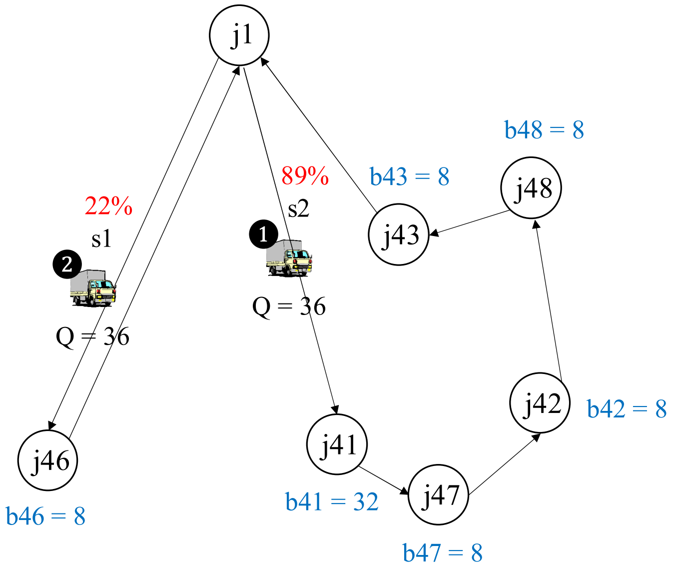
Table A15 – Demand in pallets per lane of the suppliers of GROUP_E.

	p1	p2	p3	p4	p5	p6	p7	p8
j1	0	0	0	0	0	0	0	0
j41	4	4	4	4	4	4	4	4
j42	1	1	1	1	1	1	1	1
j43	1	1	1	1	1	1	1	1
j46	1	1	1	1	1	1	1	1
j47	1	1	1	1	1	1	1	1
j48	1	1	1	1	1	1	1	1
jn+1	0	0	0	0	0	0	0	0

Source: elaborated by the author.

The optimal solution for GROUP_E was found after 12.55 seconds. The result is presented in Figure A5.

Figure A5 – Routing result for GROUP_E.



Source: elaborated by the author.

As shown in the figure, there were two routes created to attend the total of 6 suppliers. The first route has 5 suppliers allocated to it, a pickup frequency of 2 trips, with an efficiency of 89%. The second route has 1 supplier allocated, a pickup frequency of 1 trip and an efficiency of 22%. The average vehicle's efficiency to attend GROUP_E is 67%.

There were a total of 3 trips, corresponding to a total of 706 kilometers and 611 minutes. Regarding the costs, the total cost for this group is $32,180.00 monetary units, where $10,560.00 corresponds to the earliness arrival at P-LANE (and represents the inventory cost at the manufacturer's plant). There was no tardiness penalty cost.

GROUP_F

GROUP_F is composed of 6 suppliers and 8 nodes. The distance d_{ij} in kilometers and the travel time σ_{ij} in minutes are presented in Table A16 and Table A17, respectively.

Table A16 – Distance in kilometers between the suppliers of GROUP_F.

	j1	j3	j33	j34	j35	j36	j40	jn+1
j1	0	112	127	132	124	128	131	0
j3	111	0	17	22	13	18	23	111
j33	122	13	0	30	1	10	13	122
j34	130	22	34	0	30	37	42	130
j35	122	12	4	30	0	11	14	122
j36	126	21	10	39	11	0	5	126
j40	129	24	13	42	14	10	0	129
jn+1	0	112	127	132	124	128	131	0

Source: elaborated by the author.

Table A17 – Travel time in minutes between the suppliers of GROUP_F.

	j1	j3	j33	j34	j35	j36	j40	jn+1
j1	0	96	109	114	107	110	113	0
j3	96	0	15	19	12	16	20	96
j33	105	12	0	26	1	9	12	105
j34	112	19	30	0	26	32	36	112
j35	105	11	4	26	0	10	12	105
j36	108	18	9	34	10	0	5	108
j40	111	21	12	36	12	9	0	111
jn+1	0	96	109	114	107	110	113	0

Source: elaborated by the author.

The demand in pallets for each supplier and lane is presented in Table A18.

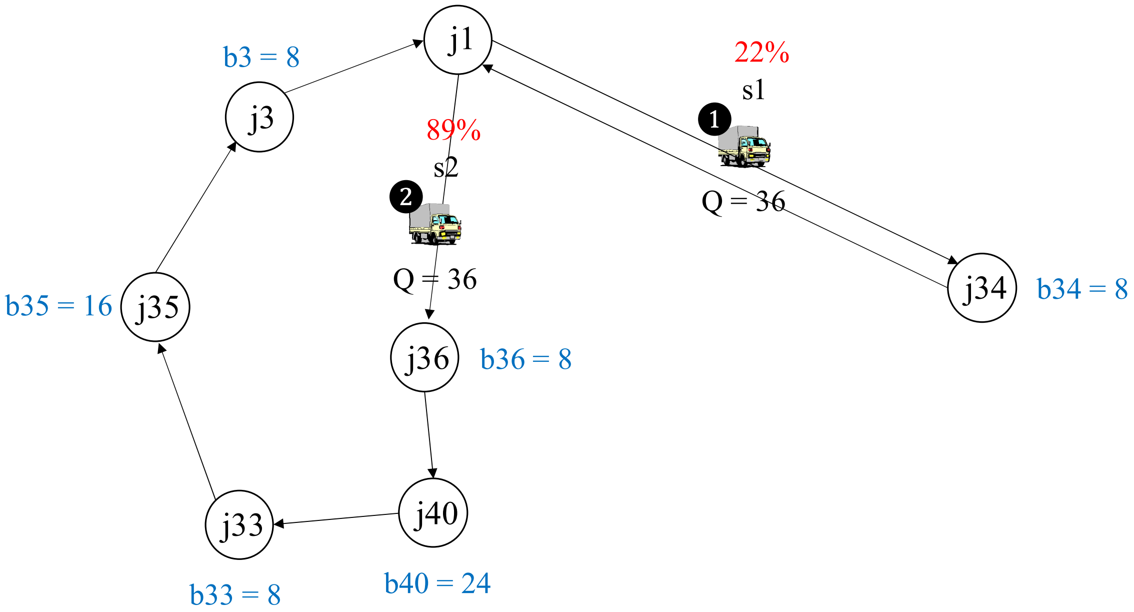
Table A18 – Demand in pallets per lane of the suppliers of GROUP_F.

	p1	p2	p3	p4	p5	p6	p7	p8
j1	0	0	0	0	0	0	0	0
j3	1	1	1	1	1	1	1	1
j33	1	1	1	1	1	1	1	1
j34	1	1	1	1	1	1	1	1
j35	2	2	2	2	2	2	2	2
j36	1	1	1	1	1	1	1	1
j40	3	3	3	3	3	3	3	3
jn+1	0	0	0	0	0	0	0	0

Source: elaborated by the author.

The optimal solution for the GROUP_F was found after 12.92 seconds. The result is presented in Figure A6.

Figure A6 – Routing result for GROUP_F.



Source: elaborated by the author.

As shown in the figure, there were two routes created to attend the total of 6 suppliers. The first route has 1 supplier allocated to it, a pickup frequency of 1 trip, with an efficiency of 22%. The second route has 5 suppliers allocated, a pickup frequency of 2 trips and an efficiency of 89%. The average vehicle's efficiency to attend the GROUP_F is 67%.

There were a total of 3 trips, corresponding to a total of 1,342 kilometers and 1,166 minutes. Regarding the costs, the total cost for this group is $34,100.00 monetary units, where $10,560.00 corresponds to the earliness arrival at P-LANE (and represents the inventory cost at manufacturer's plant). There was no tardiness penalty cost.

GROUP_G

GROUP_G is composed of 8 suppliers and 10 nodes. The distance d_{ij} in kilometers and the travel time σ_{ij} in minutes are presented in Table A19 and Table A20, respectively.

Table A19 – Distance in kilometers between the suppliers of GROUP_G.

	j1	j11	j12	j13	j14	j37	j38	j39	j45	jn+1
j1	0	98	73	74	74	81	75	8	37	0
j11	97	0	20	31	31	65	73	89	97	97
j12	84	23	0	18	18	92	86	68	85	84
j13	72	28	19	0	5	76	70	66	72	72
j14	72	28	19	5	0	76	70	66	72	72
j37	80	80	94	78	79	0	15	88	61	80
j38	73	68	87	71	71	10	0	80	54	73
j39	8	91	67	63	63	86	79	0	41	8
j45	33	94	87	70	70	57	51	40	0	33
jn+1	0	98	73	74	74	81	75	8	37	0

Source: elaborated by the author.

Table A20 – Travel time in minutes between the suppliers of GROUP_G.

	j1	j11	j12	j13	j14	j37	j38	j39	j45	jn+1
j1	0	84	63	64	64	70	65	7	32	0
j11	84	0	18	27	27	56	63	77	84	84
j12	72	20	0	16	16	79	74	59	73	72
j13	62	24	17	0	5	66	60	57	62	62
j14	62	24	17	5	0	66	60	57	62	62
j37	69	69	81	67	68	0	13	76	53	69
j38	63	59	75	61	61	9	0	69	47	63
j39	7	78	58	54	54	74	68	0	36	7
j45	29	81	75	60	60	49	44	35	0	29
jn+1	0	84	63	64	64	70	65	7	32	0

Source: elaborated by the author.

The demand in pallets for each supplier and lane is presented in Table A21.

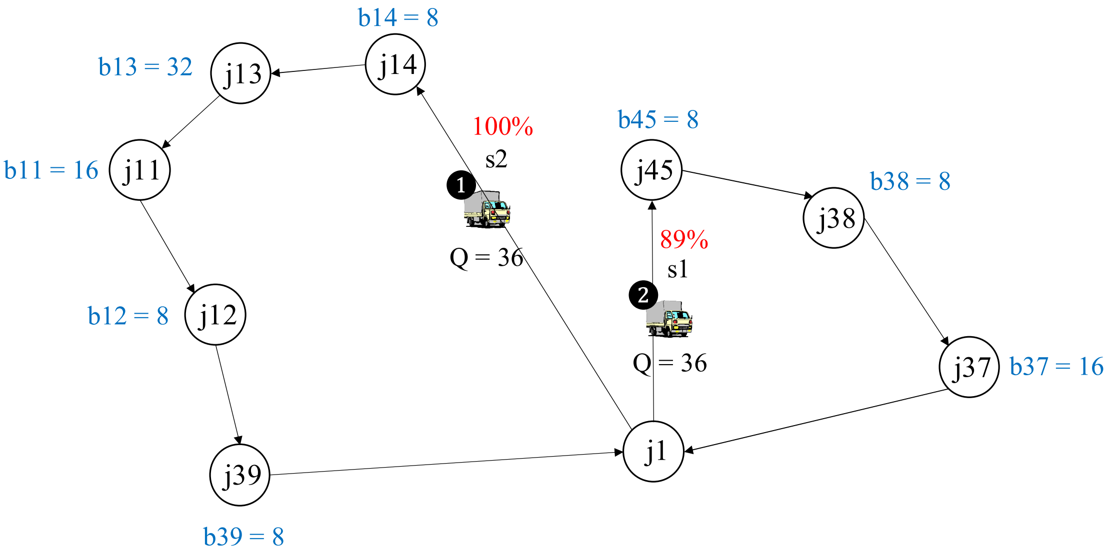
Table A21 – Demand in pallets per lane of the suppliers of GROUP_G.

	p1	p2	p3	p4	p5	p6	p7	p8
j1	0	0	0	0	0	0	0	0
j11	2	2	2	2	2	2	2	2
j12	1	1	1	1	1	1	1	1
j13	4	4	4	4	4	4	4	4
j14	1	1	1	1	1	1	1	1
j37	2	2	2	2	2	2	2	2
j38	1	1	1	1	1	1	1	1
j39	1	1	1	1	1	1	1	1
j45	1	1	1	1	1	1	1	1
jn+1	0	0	0	0	0	0	0	0

Source: elaborated by the author.

The optimal solution for the GROUP_G was found after 552.28 seconds. The result is presented in Figure A7.

Figure A7 – Routing result for GROUP_G.



Source: elaborated by the author.

As shown in the figure, there were two routes created to attend the total of 8 suppliers. The first route has 3 suppliers allocated to it, a pickup frequency of 1 trip, with an efficiency of 89%. The second route has 5 suppliers allocated, a pickup frequency of 4 trips and an efficiency of 100%. The average vehicle's efficiency to attend the GROUP_G is 96%.

There were a total of 3 trips, corresponding to a total of 584 kilometers and 508 minutes. Regarding the costs, the total cost for this group is $36,460.00 monetary units, where $17,280.00 corresponds to the earliness arrival at P-LANE (and represents the inventory cost at the manufacturer's plant). There was no tardiness penalty cost.