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SOROCABA CAMPUS
CENTER OF SCIENCE ON MANAGEMENT AND TECHNOLOGY
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ESSAY ON GAMES, EVOLUTION, AND INCOME DISTRIBUTION

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ESSAY ON GAMES, EVOLUTION, AND INCOME DISTRIBUTION

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ABSTRACT

The growing economic inequality in the last decades drew attention from society and the academy. The efforts to understand the issue created an ample body of knowledge. Here, we aimed to expand on the vast literature on inequality by combining tools from distinct disciplines in a novel way. We propose an evolutionary approach using an agent-based model that combines tools from Game Theory and Complex Systems. This method explores how individual characteristics affect the interactions between agents, how such interactions result in behavioral patterns, and how these patterns influence the population. Referring to the payoffs of each game as proxies for the individual's income, the model generates information about the process of income distribution considering individual characteristics, control of assets, interactions among individuals, and environmental conditions. Thus, providing a picture of the underlying mechanism of the distribution of income. This mechanism goes through three stages, characterized by three factors: individual characteristics, assets, and networks. Initially, for a brief period, differences in individual characteristics result in an uneven distribution of income. With that uneven distribution, some agents can save and amass assets. The control of assets magnifies the initial inequality, splitting the agents into networks separated by wealth. Finally, these networks regulate the agents' possibilities of employing their abilities and assets in obtaining income. Savings and inheritance create a feedback mechanism that amplifies and maintains income inequality.

Key-words: Complex Systems, Income Distribution, Income Inequality, Game Theory, Evolutionarily Stable Strategies, Population Games, Self-reinforcement

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SUMMARY

1. Introduction

From award-winning movies to daily news, income inequality has been in the spotlight for quite some time now. Economists, philosophers, politicians, and lawmakers give their time to the theme. Society at large takes considers inequality its representatives -it is a recurring topic in political debates. Academia scrutinizes the problem. Most of the analysis focuses on historical trends and cross effects between economic growth and income inequality. Decades of hard work provided us with rich data and powerful insights. Our goal is to build upon these tools, to combine them in the hope of broadening our perspective.

The object of this paper is the income distribution. This investigation aims at analyzing the dynamics underlying the distribution of income amongst individuals of a group and throughout different groups within a population. The intention is to identify the main factors influencing income distribution. Is it possible to trace back a given level of income distribution to a handful of variables and processes? If so, can these factors be altered to produce a different outcome, a more balanced distribution? We begin with this question: *Is income inequality evolutionarily stable?* That is: *From an established level of income inequality, can an impoverished group reduce the income gap by its efforts?*

To this endeavor, we propose an agent-based model built with the tools of complex systems and game theory. The former present the means to analyze the intricate relationships behind income distribution; the latter establishes the patterns of interactions amongst individuals. This method allows the study of heterogeneous agents. It rules out the need for assumptions of complete information, perfect rationality, and the necessity of an equilibrium. Since there is no guaranty of equilibrium, the system's evolution becomes the focus of the analysis - we follow the different stages of the income distribution.

We use the theoretical body developed by sociologists, physicists, biologists, mathematicians, historians, and economists developed throughout the last decades as the blueprint for this project. This work is an attempt to expand on the vast literature on inequality by combining tools from distinct bodies of knowledge in a novel way. A pervasive issue, such as inequality, demands an ample and diverse approach. If we are to tackle this problem, we must understand it as well as we can. Our ultimate goal is to develop a method capable of identifying how the factors and circumstances that comprise an economy interact to bring about inequality. Drawing insight from distinct fields can help set up a model able to emulate the dynamics behind income distribution.

2. Literature Review

2.1 Inequality

In Politics, Aristotle voiced concerns about the social effects of economic inequality: "[…] For when some posses too much, and others nothing at all, the government must either be in the hands of the meanest rabble or else a pure oligarchy". (Aristotle, 384-322 BCE). Uneven distributions of income already existed way before this ancient writing. Evidence showed that some burials were much more lavish than others dating back to the last Ice Age (Scheidel, 2018). Economic inequality has been around for a long time. It has been influencing various aspects of life over vastly different cultures.

Inequality appears in endless social constructs - from political structures to burial rituals. To understand its roots and dynamics, we need to transcend signs that may be particular to some cultures. We need means of comparing income distributions throughout time and across societies. The World Inequality Database (WID.world) is an effort in providing such means. It is a collaborative database on the historical dynamics of the income distribution. Over a hundred researchers, from all continents, contribute to its existence - an extensive worldwide effort showcasing the issue's importance.

Alvaredo et al. (2018), using the WID.world, showed that income inequality increased over the world. In Europe, the top decile received 33% of the national income in 1980, which increased to 37% in 2016. In the US-Canada, the values are respectively, 34%, and 47%; 28% and 41% for China; 21% and 46% in Russia; and 32% to 55% in India (Alvaredo et al., 2018). The size and speed of variation differ. Their direction is the same.

The United States of America displays a similar pattern in the pre-World War II period. Piketty (2014) pointed out that in 1900 the top decile held 40% of the U.S.A.'s income, reaching 45% on the eve of World War II. The shock on capital caused by the war ensued a period of less inequality. The top decile's income share remained below 37% until the divergence process resumed in 1970 - 1980 (Piketty, 2014). Not every country experienced this redistribution. Brazil, the Middle East, and sub-Saharan Africa never experienced the post-war egalitarian regime. There, income inequality remained stable at extreme levels. They set the world inequality frontier where the top decile received, respectively 55%, 61%, and 54% of these regions' income in 2016 (Alvaredo et al., 2018).

The data showed a long-term trend broken by a shock only to regain momentum later on. Clashes in the forms of mass warfare, revolutions, state failure, and lethal pandemics have been the most potent income distribution levelers throughout history (Scheidel, 2018). Whether by relocating means of production, changing propriety rights, destroying capital, or whipping out part of the population, these kinds of shocks have a profound impact on the use and control of resources.

Resources are a central theme in the probe of inequality. To better grasp inequality, the WID.world includes data on wealth. Considering China, the U.S.A., and Europe, wealth is even more concentrated than income. The top 10% owns as much 70% of the wealth; in 2017 the top 1% held 33% of the total wealth, while the bottom 75% owned 11% of it; in 1980 these figures were respectively 28% and 9% (Alvaredo et al., 2018). The data on wealth is not as abundant as the data on income. However, we can see that wealth, like income, highly concentrated with a tendency towards even higher levels of consolidation.

The aggregation of assets in the form of wealth was not always the norm. In the early stages of human social organization, hunting and gathering were the main economic activities. Income stemmed from individuals' abilities to procure food and shelter. The growth of hunter-gatherers populations in a circumscript environment creates an imbalance between individuals and resources, which leads to changes in behavior, technological innovation, reorganization of labor, and the rise of new hierarchies (Price and Brown, 1985). Reductions in resources per capita pushed those communities to increase productivity. Agriculture was one of the novel technologies used in this venture.

Scheidel (2018) argues that "the domestication of plants and animals made it possible to accumulate and preserve productive resources." Such accumulation created wealth at unprecedented levels and brought about social norms regarding the rights over those resources. The control of resources generated surpluses beyond those achieved by hunting and gathering. Behavior and institutions evolved to govern access to such excesses. Thus, a sedentary agricultural lifestyle paved the way for the concentration of wealth.

Agriculture marks a tipping point in the relationship between individuals and wealth. If income was once the product of what the environment had to offer and individuals' abilities, it now must weigh the control of productive resources. Piketty (2014) proposed a fundamental relationship between income from productive resources and economic inequality. "If, moreover, the rate of return on capital remains above the growth rate for an extended period of time (which is more likely when the growth rate is low, though not automatic), the risk of divergence in the distribution of wealth is very high." (Piketty, 2014).

The return on capital is part of the growth rate. Economic growth stems from increments in production factors and productivity gains (Dornbusch et al., 2011). Marginal productivity encompasses both aspects, representing how much an extra unit of a resource increases production. For Piketty (2014), how assets are used (technology), and how much of it is available (abundance), impacts its rate of return and its marginal productivity. The asset must be employed as a production factor to generate profit. New technologies expand the possible employment of capital. Furthermore, the current stock of assets affects the impact of the next unit of a resource incorporated into a production process.

What sets capital apart from other economic growth factors is how fast it can compound. When the return on equity exceeds the economic growth, saving a small part of the income from capital is enough to make assets grow faster than the economy (Piketty, 2014). Since the savings turn into additional assets that also generate income, over time, the income from capital might exceed that of all other economic factors.

The possibility of savings implies the existence of surpluses. Only those whose income exceeds their consumption needs can save. Hence, the concentration of savings may follow the aggregation of income. The cumulative effect of inequality in savings is a concentration of an increasing proportion of income-yielding assets in the hands of groups with higher incomes (Kuznets, 1955). Market imperfections can intensify such consolidation of assets. In an imperfect credit market, there is an uneven distribution of investment opportunities (Galor and Zeira, 1993).

Some individuals have access to investments with higher returns, while others are left out of the capital market altogether.

There are opposing views on how income inequality affects the overall state of the economy. In the Classical approach, higher levels of inequality result in higher rates of economic growth (Galor, 2009; and Kaldor, 1957). The argument is that, since the propensity to save increases with higher incomes, inequality raises the overall level of savings, which expands capital accumulation. More capital boosts productivity. Therefore the economy grows.

In contrast, the modern perspective underscores the adverse consequences of income inequality for the economic development process (Galor, 2009). Economic growth suffers from sterile investment, social instability, and suboptimal investment in human capital (Aghion et al., 1999; and Todaro and Smith, 2011). Between these divergent viewpoints, we can find representative-agent models that rule out inequality from economic analysis. The contrast in these perspectives may be the product of different stages in economic development.

In the early stages, the accumulation of physical capital is the driving force of economic growth, as such cumulation progresses, human capital gains prominence (Galor and Zeira, 1993, and Galor and Moav, 2004). This proposition assumes that there are physiological constraints on human capital, which "subjects its accumulation at the individual level to diminishing returns" (Galor and Moav, 2004). Furthermore, market imperfections prevent extensive investment in human capital. There is an asymmetry in the accumulation of physical and human capital that bridges the gap between these opposing views.

The effect of inequality on growth changes as the primary source of development moves from physical to human capital. However, there is still no such unifying proposition when it comes to the effect of economic growth on income inequality. This relationship remains ambiguous. Kuznets (1955) emphasizes the importance of economic growth in countervailing inequality. He argues that in a dynamic, growing economy, new industries emerge. The rapid growth of new industries would rise de income of new entrepreneurs and new forms of capital. Thus, narrowing the wealth gap between the participants of old and new industries.

Adelman (1975) stated that high rates of economic growth are a necessary but not a sufficient condition for improvements in the relative income of the poor. Without strategies for a redistribution of income, there is no guarantee that the additional product will benefit the least favored. Piketty's hypothesis also reflects this nonlinear relationship. Inequality mighty rise, even in the presence of economic growth as long as the return on capital grows faster than the whole economy.

Inequality arises from the interplay of individual and environmental factors — some which generate feedback loops. Historical accidents seem to play significant roles. To understand this phenomenon is not enough to analyze one element in a single point of time. We need to look into how the components interact throughout time. We need to understand how a system evolves to produce economic inequality.

2.2 Complexity, Games, and Evolution

The interactions of agents and assets create the economic environment. Since individual actions can affect the constraints, beliefs, and preferences of others, aggregate results cannot be considered the sum of particular behaviors (Bowles, 2004). The relationships between economic growth and income inequality are examples of such nonlinearity among economic variables. Understanding the economic environment entails more than understanding its parts.

The income distribution results from the interplay of a series of economic factors. Like propensity to save and economic growth. Bar-Yam (1997, p. 1) defines a complex system as a system whose functioning is explained: "not only by the behavior of the parts but by how they act together to form the behavior of the whole." In a complex system, there are dependencies among its elements. Such dependencies are fundamental features of the system. The impact of removing one of these crucial elements goes beyond what is embodied by the particular deleted item. (Miller and Page, 2009). Thus, the connections among elements are critical characteristics of the system's behavior.

The connectedness refers to the social system's structure and the interdependence in the behavior of its elements (Easley and Kleinberg, 2010). The structure deals with the association among components: what connects to what. The shape of the network might account for market imperfections. Moreover, social networks are essential information channels in the labor market (Arrow and Borzekowski, 2001; Calvo-Armengol, 2004; and Topa, 2001). Social contacts can be sources of information for job opportunities. These opportunities can drive personal investments in human capital. The same dynamics can operate in markets other than labor. Driving different kinds of investment decisions

Networks bear information beyond economic opportunities. In situations involving uncertainty, close communities are essential structures for reducing risks (Coleman, 1990). A community conveys information about its members. It implies links that go beyond a single interaction. Thus, controlling risks. Homophily is one of the basic principles governing social networks' structures, working through selection and social influence (Easley and Kleinberg, 2010). Homophily is the tendency to develop associations based on similarities in characteristics and behavior.

The selection mechanism acts upon immutable characteristics (Easley and Kleinberg, 2010). Similar individuals tend to interact more. Bowles and Gintis (2002) highlighted race, geographic location, height, physical appearance, health status, personality, and cognitive skills as individual traits that affect income. Some of these traits can be bases for selecting the members of a network. Thus, immutable characteristics might have direct and indirect impacts on one's income.

The behavioral level of connectedness regards the consequences of individual actions to the whole system. That is the level at which the social influence mechanism operates. Social influence affects mutable characteristics, shaping the behavior of the network's elements (Easley and Kleinberg, 2010). When the outcome of an action depends on someone else's decision, it can be useful to spread behavioral patterns. Social influence plays a role in spreading these patterns, intertwining the behaviors of individuals and the system.

In a network, one element's behavior may have direct and indirect impacts on the functioning of others. Friedman (1977) termed this situation as behavioral interdependence. In the presence of behavioral dependence, decisions must consider the possible actions of others. Hence, behavior gains a strategic character in social networks.

Similarities drive the building of a network. Its links influence behavior, connecting the actions of its members. "Games are a way of modeling strategic interactions, that is, situations in which the consequences of individuals' actions depend on the actions taken by others, and this mutual interdependence is recognized by those involved" (Bowles, 2004, p. 31). A social network places its elements in a game setting.

Social interactions arise from the fact that individuals have interests unfulfilled by the resources they currently control (Coleman, 1990). To fulfill these interests, agents engage in the game embedded in social networks. It is not necessary to assume that individuals are entirely rational or have complete information about the game (Axelrod, 2009; Bowles, 2004; and Sandholm, 2010). The social network can dictate strategies through social influence.

It is not necessary to assume that each individual solves complex maximization problems before every interaction. The actions taken are not necessarily conscious or rational. The players need not be trying to maximize their outcomes. They can be only imitating, following habits, rules of thumb, instincts, or reflecting standard procedures (Axelrod, 2009). Since the actions do not necessarily follow a maximizing direction, the resulting outcomes are not necessarily optimal. In the absence of perfect rationality and maximization behavior, the resulting payoffs are not necessarily optimal. Different habits might lead to variations in the individual's payoffs.

In a bounded rationality context, as described by Axelrod (2009), the repetition of interactions becomes essential. Once the utility-maximization principle does not guide the decisions, an alternative mechanism such as imitation and learn-by-doing come into play, helping individuals to design their strategies. Bowles (2004, p. 10) termed these alternative mechanisms behavioral responses: "Individuals intentionally pursue their objectives, but they do this more often by drawing on a limited repertoire of behavioral responses acquired by past experience than by engaging in the cognitively demanding forward-looking optimizing processes […]".

Throughout time, the players will gradually adjust their behavior in response to the system. This evolution brings about a spontaneous order in which the player's "behavior will be as though they had consulted the game theory book" (Binmore, 1991, p. 397). The system attains equilibrium once there are no intrinsic incentives for change. That is a state in which most players adopt a strategy, and an alternative cannot replace it. The adaptation process establishes an evolutionarily stable strategy (E.S.S.). Bowles (2004, p.12) highlighted the role of positive feedbacks in determining an E.S.S.: "the payoff to taking action is increasing in the number of people taking the same action." Thus, the fact that one specific strategy is the most popular is enough to increase the number of its user to a point where no other action is viable.

There are numerous ways of reaching a steady-state. Even erratic behaviors are possible: in addition to a great many possible equilibrating processes, there is no guaranty that an equilibrium exists (Binmore, 1991). Small variations in the environment or the equilibrating processes might lead the same game to converge to different equilibria. Modest changes in the economic context, social structure, or social behavior, can result in much different income distributions. Positive feedbacks create economic environments in which these small chance events have durable consequences over very long-time frames. Furthermore, initial conditions may have persistent so-called lock-in effects. The poverty traps faced by peoples and nations, as well as the virtuous circles of affluence enjoyed by others, exhibit the effects of these influences (Bowles, 2004).

Positive feedbacks magnify small variations, altering the dynamics of a system. Systems with self-reinforcement mechanisms display: (i) multiple equilibria; (ii) possible inefficiency; (iii) lock-in; and (iv) path dependence. (Arthur 1988)

Path dependence suggests that historical accidents might have lasting impacts. The early history of the system, small events, and chance circumstances can determine the outcome. Shocks, like wars, can determine the income distribution long after they happened. Which leads to the possibility of multiple equilibria, "[…] quite different outcomes are possible for two populations with identical preferences, technologies, and resources but with different histories" (Bowles, 2004, p. 42). Once the system reaches one of the possible equilibria, the coordination effects may lock the population into it. There is no guaranty that it will be the most efficient. Since "[…] in evolutionary models, none of the actors has preferences defined over aggregate outcomes" (Bowles, 2004, p. 58) That is, there is no active mechanism working towards a global social optimum.

In the presence of positive feedbacks, an active, directed measure is necessary to change a current equilibrium. Arthur (1988, p. 13) finds a parallel with physical systems: "[w]hen a nonlinear physical system finds itself occupying a local minimum of a potential function, 'exit' to a neighboring minimum requires sufficient influx of energy to overcome the 'potential barrier' that separates the minima." If a disadvantaged group hopes to reduce the income gap, a 'sufficient influx of energy' is necessary to overcome the barrier imposed by the economic advantages that prompted the evolutionarily stable strategy. The source and direction of this 'energy' need further testing.

We can summarise the main characteristics of an evolutionary system as:

Adaptative agents: individuals react strategically to the environment;

Connectedness: the structure and behavior of the system affects how individuals interact, prompting adaptation;

Replication: strategies are transmitted or learned to spread practices through the population;

Out-of-equilibrium dynamics: time is an explicit part of the model, accounting for the populational movements that lead to equilibrium. Based on individual traits and behavior, it is possible to track social interactions over time – "who meets whom, to do what, with what payoffs, with what information, and the like." (Bowles, 2004, p. 48);

Chance: historical accidents may have lasting impacts.

Establishing a game setting representing a social system allows the observation of how behaviors and payoffs evolve. The equilibrating process can reflect economic, social, and biological characteristics (Binmore, 1991). Environmental conditions, individual traits, and social interactions can be computed and measured to identify how each contributes to the resulting state. Following the evolution of the system provides insights into the origins of income gaps and their propagation factors.

With the payoffs representing income, its values show how a player fares - given her characteristics and the environmental conditions. The payoffs' evolution pattern generates information about the behaviors linked to different levels of the income distribution. The income distribution reflects how the characteristics of groups affect the interactions among individuals. Finally, the divisions among groups show the income concentration of the social system.

An evolutionary approach provides an analysis of income distribution's mechanics starting at the individual level, going through the group level, and, finally, showing the social system's patterns. Which can be a way to overcome one of the current challenges faced by Economics, as stated by Easley and Kleinberg:

> Economics has developed rich theories for the strategic interactions among small numbers of parties, as well as for the cumulative behavior of large, homogeneous populations. The challenge it faces is that much of economic life takes place in the complex spectrum between these extremes, with macroscopic effects that arise from an intricate pattern of localized interactions (EASLEY; KLEINBERG, 2010, p. 6)

The focus on interactions encompasses the roles of individual traits and macroeconomic conditions while makes room for the consideration of feedback mechanisms and the dynamic aspect of the income distribution. This perspective broadens the take on the issue, providing a more comprehensive range of factors influencing income inequality, increasing the understanding of income inequality. Therefore, enhancing the chance of overcoming it.

3. The Basic Model

3.1 Games

If we are to understand if a disadvantaged group can reduce the income gap by its efforts, it is necessary to analyze how the differences emerge in the first place. How the system's structure and behavior affect the arrangement of different income groups? Understanding the economy as a complex system makes it possible to consider the effects of individual traits, environmental conditions, and the individuals' interactions on the evolution of income groups. Moreover, approaching these interactions as a game allows an analysis of behavior without the assumptions of perfect rationality and utility-maximizing decisions.

The basic representation of a game setting is the *normal-form game*. In the normal-form, it is possible to identify how individual decisions affect one another's payoffs. "In the normal-form representation of a game, each player simultaneously chooses a strategy, and the combination of strategies chosen by the players determines a payoff for each player" (Gibbons, 1992, p. 2). A bi-matrix represents the problem with the payoffs of each combination of strategies. Given the *strategy profiles* player 1 = (cooperate - C, defect - D) and player 2 = (cooperate - C, defect - D), the bi-matrix is:

Table 1 - The Normal-Form Game

	Player 2			
	Cooperate		Defect	
Player 1	R	R	S	T
	S	T	P	P

Table 2 - Payoffs

R	Mutual Cooperation Reward
P	Punishment for Mutual Defection
S	Sucker Payoff - the payoff for cooperating while the other player defects
T	Temptation Payoff - the payoff for defecting while the other player cooperates

In this game, each player has two possible strategies: to cooperate or not to cooperate (defect). We call each of these actions a *Pure Strategy* respectively denoted by s_1 and s_2, part of the set S_i. It is also possible to associate a probability distribution over pure strategies. In this case, the player is working with *Mixed Strategies*, as defined by Narahari (2014, p. 93) "given a player *i* with $\mathrm{S_i}$ as the set of pure strategies, a mixed strategy (also called a randomized strategy) σ_i of a player, i is a probability distribution over $\mathrm{S_i}$", such that:

$$\sum_{s_i \in S_i} \sigma_i(s_i) = 1 \qquad (1)$$

These strategies result in the payoffs π_1 and π_2. The first column (Table 1) represents Player 1's payoff, and the second column (Table 1) shows Player 2's payoffs. Where:

$$T > R > P > S$$

The players have incentives to pursue T and avoid S, which makes the Mutual Defection the only possible equilibrium. Therefore, in this situation, cooperation is not possible. What makes collaboration feasible is the fact that players might meet again. The future can affect current strategic positions. (Axelrod, 2009)

Nonetheless, the future is less important than the present for two reasons: (1) players tend to attribute higher value for current payoffs; and (2) there is always the possibility that the players do not meet again. To account for the "shadow that the future cast upon the present" (Axelrod, 2009), it is necessary to consider the impact of both factors on the importance of the next move. The payoff of the next step is *weighted* by its relative importance to the current payoff using the *discount factor* u. Let π_{i1} denote the payoff of the player i in the current period, the subsequent periods' payoffs are $\pi_{i2} = u\pi_{i1}, u_{i3} = u^2\pi_{i1}$.

This connection with the future is a two-way street, the future can affect current strategic decisions, and the present can transmit information to the future. The *replicator* is the mechanism responsible for such a transmission:

> The central actor in an evolutionary system is the replicator—an entity having some means of making approximately accurate copies of itself. The replicator can be a gene, an organism (defining "accurate copy" appropriately in the case of sexual reproduction), a strategy in a game, a belief, a technique, a convention, or a more general institutional or cultural form. A replicator system is a set of replicators in a particular environmental setting with a structured pattern of interaction among agents. (GINTIS, 2000, p. 271)

3.2 The Board

This game takes place in a complex system. Bar-Yam (1997) lists elements, diversity, variability, formation, interactions, activities, and environment as the central properties of complex systems. These properties are useful guidelines for describing the model.

The elements of the model are a finite number (N) of heterogeneous economic agents (i). These agents control resources (w_i^t) and possess innate abilities (h_i^t). The variable (w_i^t) represents any assets that may provide revenue directly or help obtain earnings, such as tools, buildings, machinery, and education. The variable (h_i^t) stands for all abilities that might be useful in producing income; it is a general indicator of *economic prowess*. To grant equal probabilities for every level of capacity, we randomly assign skills to each individual from a discrete uniform distribution. The randomness allows for *mutations* within individuals. The agents receive their resources from the previous generation except for the first generation. We deliberately attribute assets to each individual in the first generation, which allows testing the impact of initial conditions in the system's evolution. The rate (μ) accounts for the degree to which physical capital wears or technology becomes obsolete - it is the depreciation rate.

These mechanics confer diversity to the system, with the agents varying in resources and innate abilities. There is a stochastic source of variability governing the inherent talents. Furthermore, there is an endogenous variation coming from the capital accumulation process. The *propensity to save* (r_i^t) works as a *replicator*. Since the present analysis focus on income distribution, the replicator reflects factors capable of affecting the income of the players' descendants: wealth, education, and contacts, as examples. At the end of each generation (t), new individuals replace their ancestors. Each individual transmits part of her income (y_i^t) to her descendants in the form of wealth (w_i^t). The size of the transmission depends on the propensity to save.

$$w_i^t = r_i^{t-1} y_i^{t-1} \mu \qquad (2)$$

Since the propensity to save increases as income grows, the replicator varies with income. Differential replicators give direction to evolutionary processes. (Bowles, 2004). Those who obtain higher earnings can increase the transmission of wealth to their descendants. In doing so, they increase their descendants' earnings capacity and adds momentum to the feedback circle.

The wealth also plays a role in the formation of the system. Each agent i interacts with agents j from the same population N. That is, i stands for any given agent under analysis and j represents her counterpart in one specific interaction. Homophily drives these interactions: similar levels of wealth increase the probability of a connection between i and j. Wealth shapes the system's networks by increasing the likelihood of a link (${l_{ij}}^t$) between individuals who possess resembling resources. This probability is given by:

$$ {l_{ij}}^t = \frac{\sqrt{\left(w_i^t - w_j^t\right)^2}}{w_i^t + w_j^t} \qquad (3) $$

The more similarities in resources, the higher the possibility of interactions. At the beginning of each generation, we build a network based on these probabilities. l defines a binary selection: connection or no connection — j is or is nor a neighbor of i. The resources in common determine the agent's neighbors (G_i^t):

$$ \mathbb{1}_{G_i^t}(j) = \begin{cases} 1, & with\, probability\; {l_{ij}}^t \\ 0, & with\, probability\; (1 - {l_{ij}}^t) \end{cases} \qquad (4) $$

Once we have the neighbors for all agents in N, the intersections among neighbors form a network. Then we can calculate the *embeddedness* (σ_{ij}^t) of the links. The embeddedness of a relationship is the number of shared neighbors shared by two linked individuals. An embedded edge connecting two individuals makes it easier for them to trust one another (Easley and Kleinberg, 2010). Which affects the confidence in transactions within the network.

Since the possibility of future interactions impacts present strategies (Axelrod, 1981), we want to assess how likely are future connections between agents. For that, we use the strength of the current link as a proxy. As ties strengthen, the number of neighbors shared by the tied individuals also increases (Easley and Kleinberg, 2010). The *neighborhood overlap* (σ_{ij}^t) is the ratio:

$$ \sigma_{ij}^t = \frac{\text{neighbors of both } i^t \; and \; \text{j}^t}{\text{neighbors of } i^t \; or \; \text{j}^t} = \frac{|G_i^t \cap G_j^t|}{|G_i^t \cup G_j^t|} = \sigma_{ji}^t \qquad (5) $$

The higher the value of σ_{ij}^{t}, the higher the probability of a long term connection between i^{t} and j^{t}. Which increases the possibility of cooperation between them. The players will cooperate with a chance σ_{ij}^{t}. Thus, the strategies (s_{ij}^{t}) for i^{t} when interacting with j^{t} are:

$$s_{ij}^{t} = \begin{cases} C, with\ probability\ \sigma_{ij}^{t} \\ D, with\ probability\ (1 - \sigma_{ij}^{t}) \end{cases} \qquad (6)$$

The shape of the network influences not only if interactions occur, but how they transpire. The environment affects decisions. In the game amongst individuals, σ_{ij}^{t} stands for the possibility of cooperation in the set of mixed strategies. The focus here is not on any particular kind of activity. The players could be applying for a job, trading securities, buying commodities, acquiring capital. It could be any economic activity able to generate income.

The spotlight is on the balance of outcomes in each interaction. This abstraction increases the model's generality. Allowing for adaptations and broadening the possibilities of activities. We do not restrict individuals to a single market or economic activity. Instead, the agents' places in the networks bound their options. Every interaction is a possible source of income. The players receive payoffs (π_{ij}^{t}) dependent on their strategies. The payoffs are percentages, and their relative values follow the structure of the Prisoner's Dilemma:

$$\pi_{ij}^{t} = \begin{cases} T, & if\ s_{ij}^{t} = D \wedge s_{ij}^{t} = C \\ R, & if\ s_{ij}^{t} = C \wedge s_{ij}^{t} = C \\ P, & if\ s_{ij}^{t} = D \wedge s_{ij}^{t} = D \\ S, & if\ s_{ij}^{t} = C \wedge s_{ij}^{t} = D \end{cases} \qquad (7)$$

The income from each interaction (y_{ij}^{t}) is:

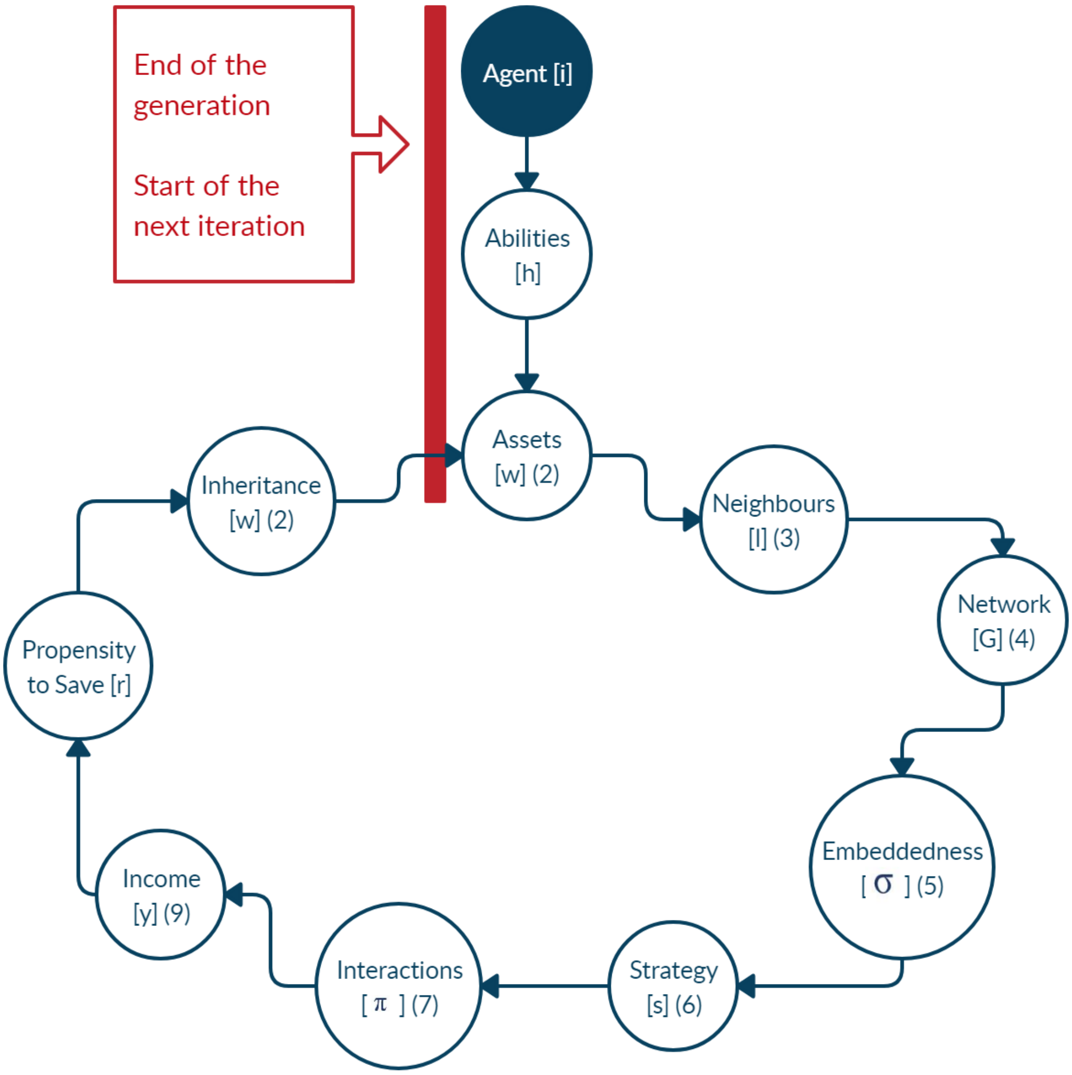
$$y_{ij}^{t} = (w_{i}^{t} + h_{i}^{t})\pi_{ij}^{t} \qquad (8)$$

The total income of an agent (y_{i}^{t}) is the sum of the earnings obtained from all interactions. We attribute the individuals' propensity to save (r_{i}^{t}) based on this total income. The higher the percentile, the higher the savings. The wealth amassed, after the effect od depreciation, constitutes the resources for the descendants ($w_{i}^{t+1}\mu$).

$$y_i^t = \sum_{j = inf\ G_i^t}^{G_i^t} (w_i^t + h_i^t)\pi_{ij}^t \qquad (9)$$

The schematic (Figure 1) shows a representation of the process behind the computation of the income of one agent (i) for one generation (t). We repeat such process k times for all agents in N.

Figure 1: Diagram of One Iteration



3.2 The Plays

We propose a model built upon individuals. In an agent-based approach, the system's behavior emerges from the direct interaction of entities forming its basis. (Miller and Page, 2007). It is a bottom-up construction. We place abstractions over individuals, assuming that:

1. the agents engage in a game seeking unfulfilled interests;
2. resources are the drivers of homophily;
3. the network is the main determinant of trust and cooperation; and
4. there are physiological limitations to intrinsic abilities.

As opposed to creating general rules for the system to which agents must abide. That is, there is no imposed equilibrium or global optima. These states are possible, but they are not mandatory.

The goal is not to find a single, definitive solution. Instead, we aim to understand how the many components affect one another — testing the cumulative effects of small changes. Computational models embrace systems characterized by dynamics, heterogeneity, and interacting parts. (Miller and Page, 2009). With this kind of tool, we can track changes in each parameter and check its long-term effects on the system's state.

Repeating the simulations, we can identify universalities behind the distribution of income. Running the model with different parameters and varying the starting points can produce insights into the dynamics behind income distribution. Ideas that can be useful in designing mechanisms for countervailing income inequality.

4. Results

The model we propose allows for a vast number of combinations of parameters. It would be impractical, and out of the scope of this work, to exhaust the analysis of all meaningful combinations of settings. In this section, we present a few illustrative examples, using parameters and metrics that aim to reflect an economic environment as described by the theory. The goal is to test the model's general workings and how much of the theory this system embodies.

Table 3 displays the parameters of each scenario. Version A is our reference to which we compare the overall behavior of the other runs of the model. At this point, we do not aim to reflect specific economic conditions; we want to understand how the relationships among the parameters affect the system.

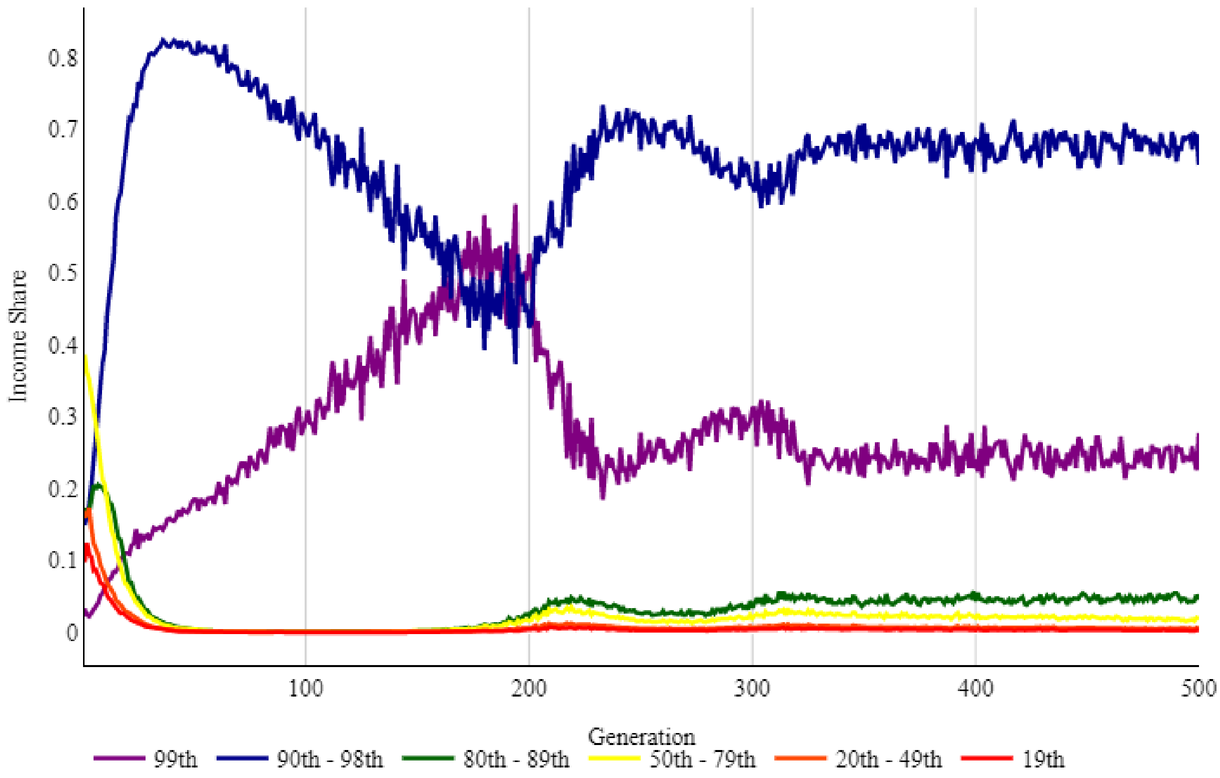
Table 3 – Parameters

Scenarios	A	B	C	D	E	F
Population (N)	400	**100**	400	400	400	400
Intrinsic Abilities (h_i^t)	1 - 100	1 - 100	1 - 100	**1 – 1000**	1 - 100	1 - 100
Depreciation Rate (μ)	10%	10%	10%	10%	10%	10%
Savings 99th (r_i^t)	80%	80%	80%	80%	**50%**	80%
Savings 90th-98th (r_i^t)	60%	60%	60%	60%	**30%**	60%
Savings 80th-90th (r_i^t)	40%	40%	40%	40%	**20%**	40%
Savings 80th (r_i^t)	0	0	0	0	0	0
T (π_{ij}^t)	2.1%	2.1%	**21%**	2.1%	2.1%	**1.1%**
R (π_{ij}^t)	1.1%	1.1%	**11%**	1.1%	1.1%	**0.6%**
P (π_{ij}^t)	0.1%	0.1%	**1%**	0.1%	0.1%	0.1%
S (π_{ij}^t)	0	0	0	0	0	0

Scenario A uses a gap of 2% between the temptation payoff and the non-cooperation payoff - the temptation payoff generates a 2,1% return on the individual's assets. In contrast, non-cooperation payoff yields 0,1%. The return on cooperation is the average between these payoffs. In this simulation, the population consists of four hundred individuals. All individuals start with the same amount of resources - 1. The initial level of assets changes every generation under the effects of savings and depreciation. The resources become obsolete or wear out at a rate of 10% per generation. There is a concentration of savings in the upper-income groups. Those in the 80th percentile do not earn enough to save, while the 99th percentile saves 80% of its income. The top 10% and 20% save respectively 60% and 40% of their proceeds. Moreover, each of the four hundred agents receives randomly a value between 0 - 100, from a discrete uniform distribution, representing their economic prowess.

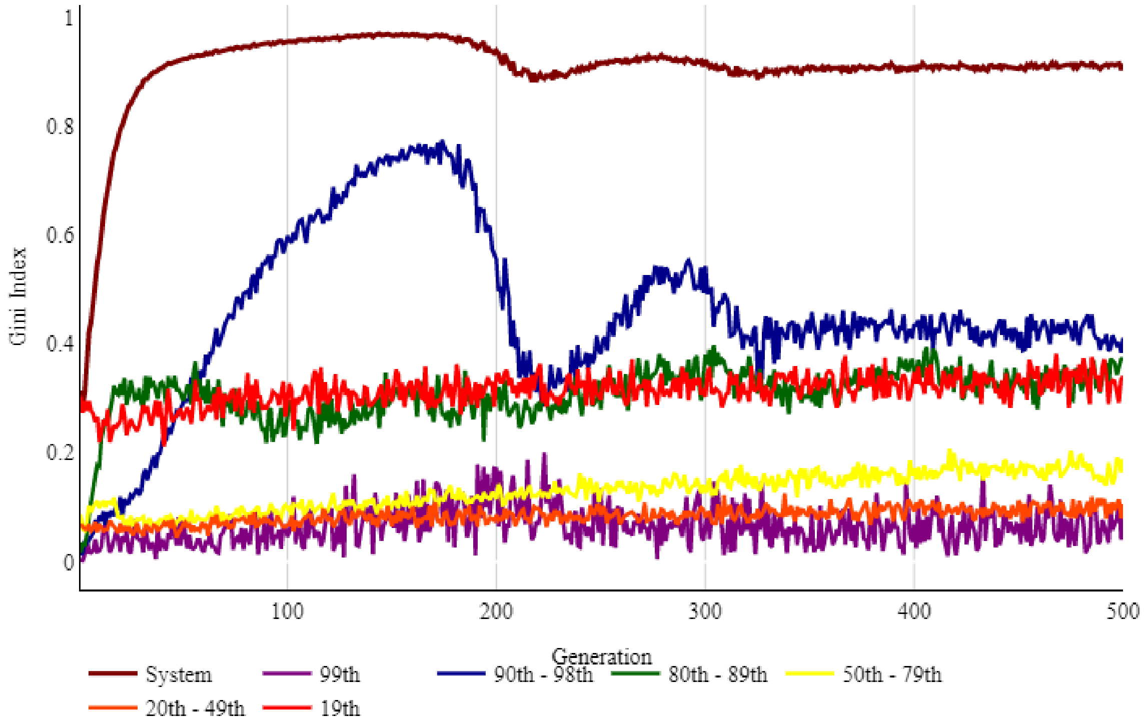
These parameters are the backdrop for the income share per bracket that figure 2 expresses. In the first generation, the top bracket (99th percentile) captures 3% of the total income; the group between the 90th and 98th percentiles receives 15%; and 17% between 80th and 89th percentiles. The richer classes earn 35% of the income, comprising 20% of the population. The bracket between 50th and 79th percentiles receive 39% of the income. The agents within the 20th and 49th percentiles share 16% of the proceeds, leaving 10% of the earnings for the most disadvantaged group (19th percentile). By the tenth iteration, the 90th - 98th bracket received 39% of the income, and the 99th percentile receives as much as the 19th percentile - 6%. In the twentieth generation, the top 10% earners captured 80% of the total income. The tendency of concentration continues. The 90th - 98th group riches its highest share in the thirty-sixth generation, receiving 82% of the income. From this point, the concentration of income in the top 1% gains momentum. In generation 194, the 99th percentile amassed 60% of the total income. Which preceded a tendency reversal, leading to a slight reduction in the concentration on earns in the upper brackets. In the last iteration, the top 1% received 28% of the total income; the next 9%

Figure 2: Share of Income per Bracket (Scenario A)



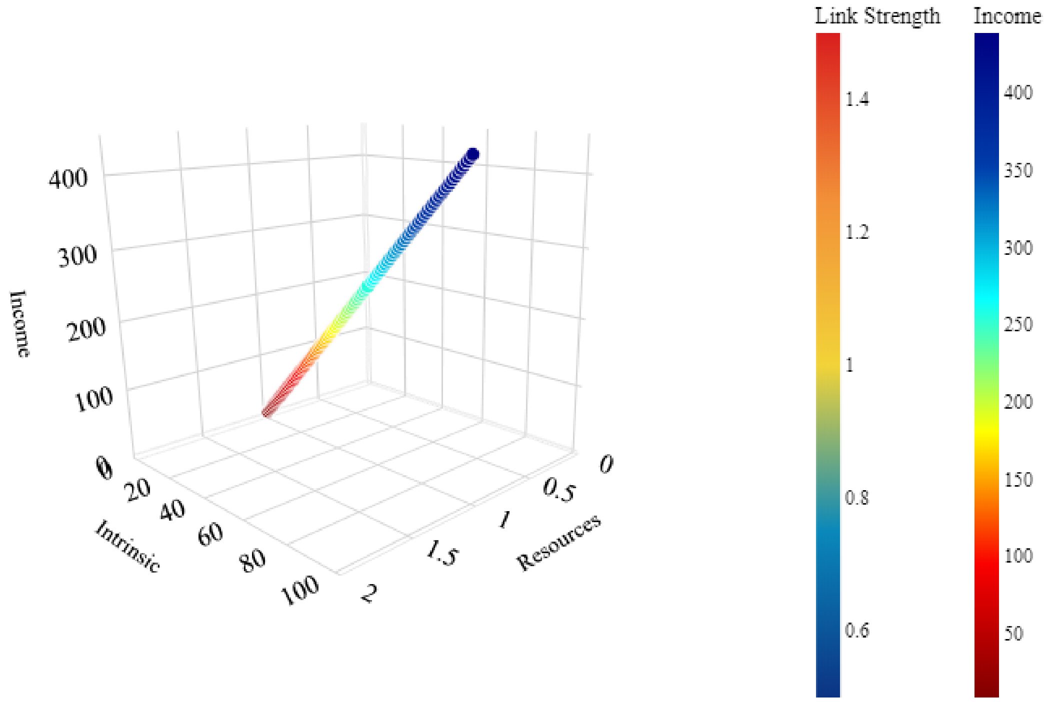
got 65% of it; and the 80th - 89th bracket received 5% of the proceeds. The remaining 80% of the population shares 2% of the total income.

Figure 3: Gini Index (Scenario A)



There is inequality even within brackets. Figure 3 shows the Gini Index for the whole system and each group. For all agents, we have a Gini index of 0.32 in the first generation; in the tenth iteration, its value reaches 0.57; and cross the threshold of 0.9 by the thirty-fifth generation. After the fast initial concentration of income, the model displays a slower pace of growth in inequality, reaching its peak in the iteration 182 with a Gini of 0.97. Following the peak, the index oscillates until it reaches some stability around 0.9 by the end of the simulations. Understanding such oscillation can clarify the functioning of the model, and lead to insights on the mechanics behind income distribution. Breaking down the scenario into generations can help in this process.

Figure 4: 1st Generation Network (Scenario A)



In the first generation (figure 4), every individual has the same amount of resources. They are all connected. Therefore every interaction is cooperative. Disparities in income emerge from differences in intrinsic abilities. The top decile captures 18.3% of the total income, while the bottom 50% received 25.7%. An individual in the top 10% earns 3.5 times as much as someone in the bottom 50%. A result of intrinsic abilities 3.6 times as high in the top decile when compared with the bottom 50%. By the fourth generation (figure 5), the top 10% and the bottom 50% received 23% and 25% of the total income. The former bracket earns, on average, 4.6 times as much as the later. With the ratio of 1.9 between average intrinsic abilities. The contrast regarding resources becomes significant. The upper bracket possesses 14.8 times more resources than the lower group. The number and nature of interactions counteract, to some extent, this discrepancy in sources of income. A member of the top decile has, on average, 107 connections, of which 30% are cooperative. For the bottom 50%, the comparative values are 209 and 88%.

In the eleventh generation (figure 6), the top 10% amasses 51% of the income, while the bottom 50% receives a 12% ratio of 21.16. On average, the upper group has 1.77 times more intrinsic abilities and 109.18 times as many resources. An individual in the top decile averages 61 interactions, 27% of those are cooperations. Someone in the bottom 50% averages 168 connections, with 85% of collaboration. At this point, a more extensive and more cooperative network is not enough to countervail the concentration of resources.

Figure 5: 4th Generation Network (Scenario A)

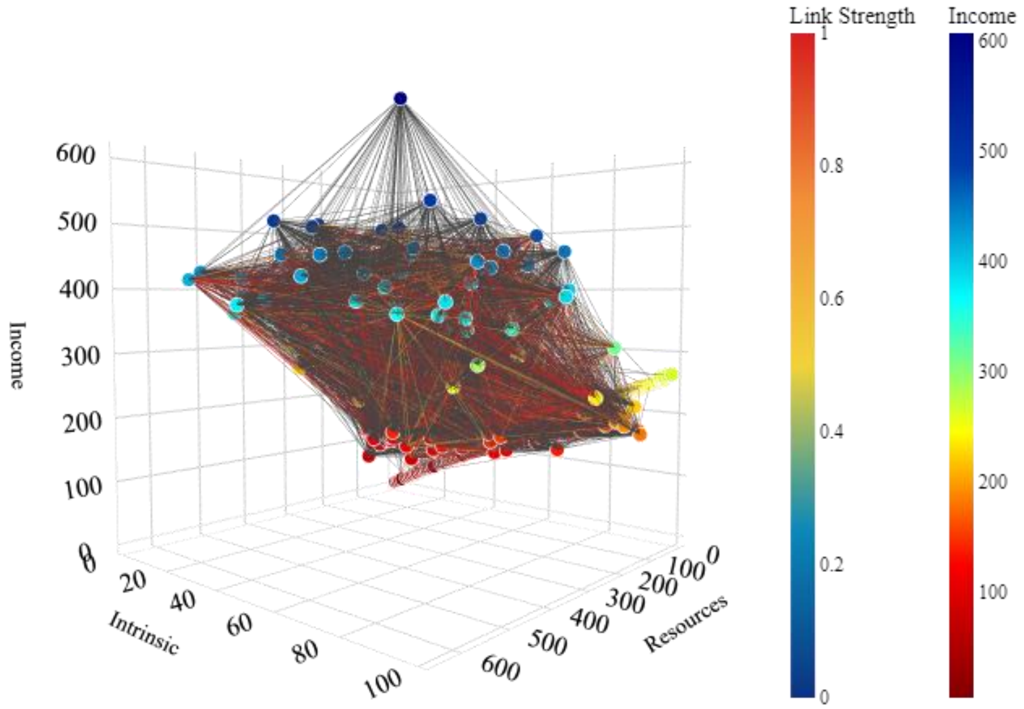
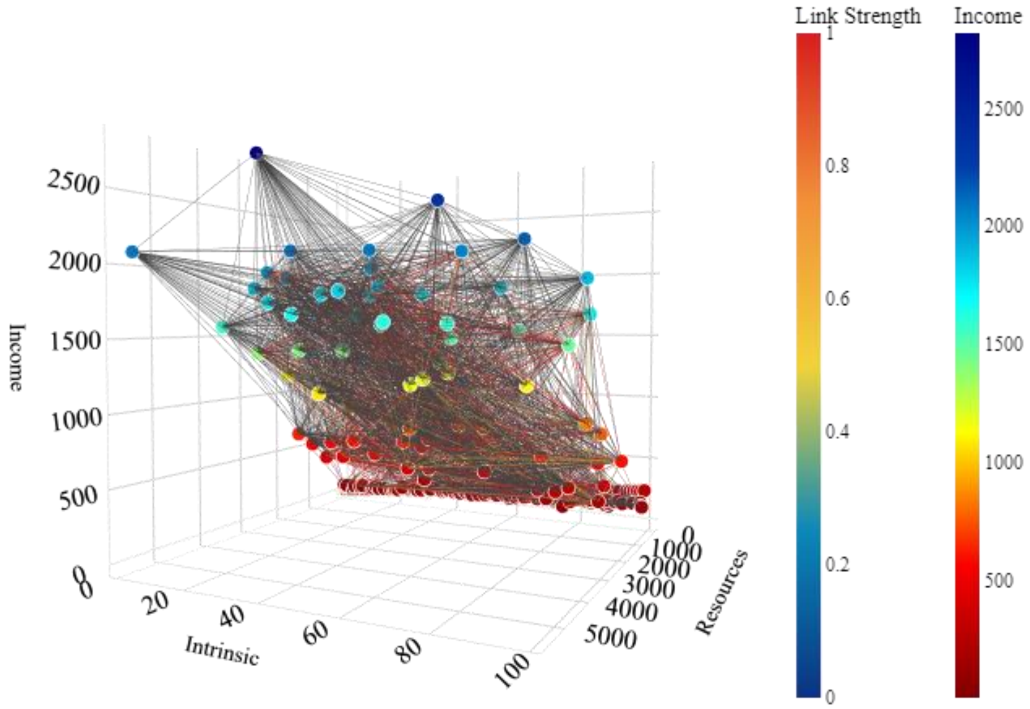
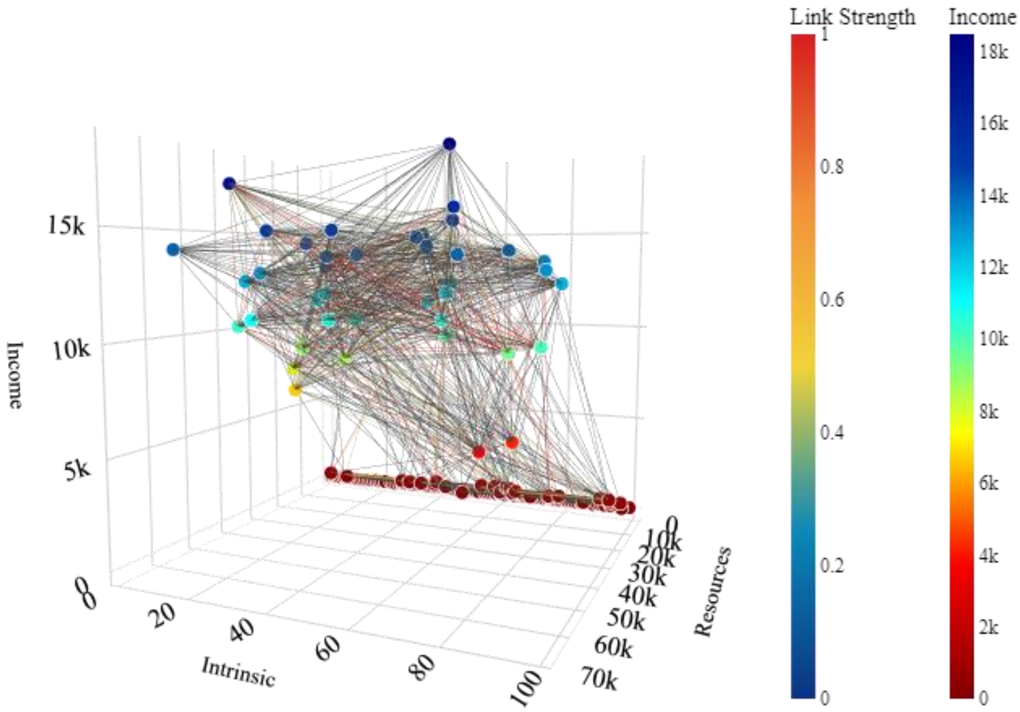


Figure 6:11th Generation Network (Scenario A)



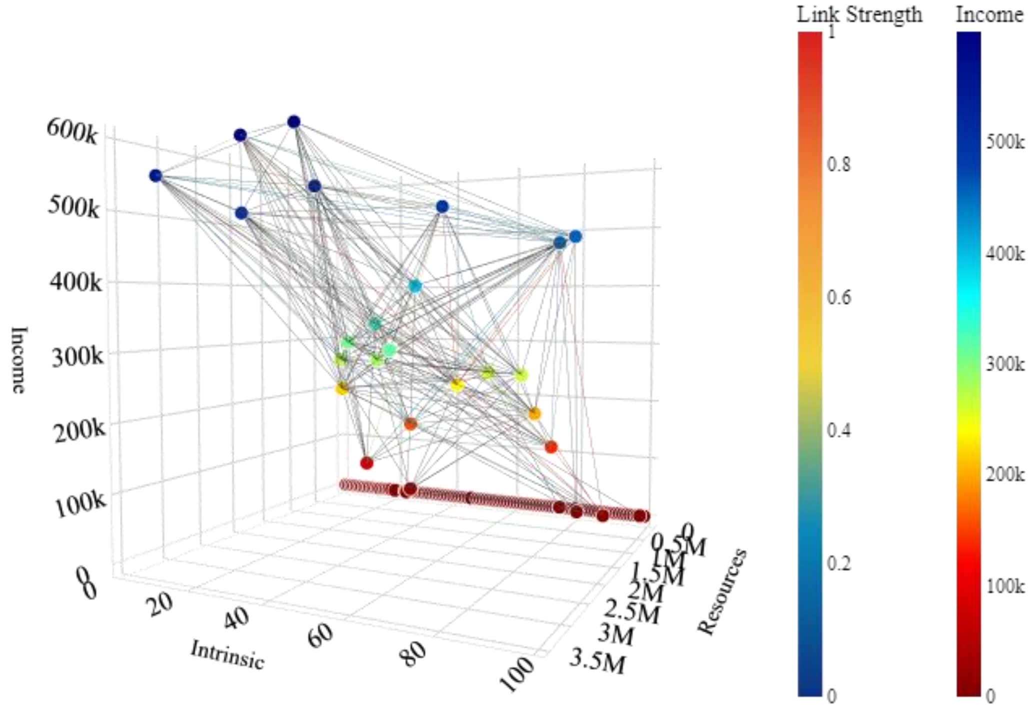
The trend of accumulation continues. When the system reaches the thirtieth generation (figure 7), 93% of the proceeds are in the hands of the top 10%, while the bottom 50% holds 1.77%. The top decile receives, on average, 263.08 times more income, averaging 1.52 times more intrinsic abilities and 1,488.98 times more resources. The network at the top becomes smaller and more secluded, with its members averaging 37 connections, of which 36.34% are cooperative. In the bottom, the average number of interactions and the percentage of cooperation decreased to 120 and 48%. At the peak of concentration (figure 8) - in generation 108, the upper decile receives 99.87% of the income - the top is densely connected and distant from the rest of the network.

Figure 7: 30th Generation Network (Scenario A)



This distance reflects the growing inequality within the upper brackets (figure 3). There is a steep increase in inequality within the 90% - 98% bracket, reaching a Gini Index of 0.7753. In the first generations, differences in income derive from differences in intrinsic abilities. Higher inherent skills result in higher revenues, enabling the accumulation of resources. After a few generations, resources become the prominent driver of earnings. With a high rate of savings (80% - 60%), assets become the primary source of income for the upper brackets. Since assets are the basis for homophily that divides the population into groups, the distance among the bracket grows as the accumulation of resources hasten. The increasing gap amongst groups, and individuals, change the amount and nature of the interactions.

Figure 8:108th Generation Network (Scenario A)



The Gini index of the 80th - 89th percentiles increases fast through the first seventeen generations and then oscillates around 0.3. In the early iterations, income stems from the cooperative use of intrinsic abilities. Therefore, the Gini index is a product of the distribution of inherent capacities, as savings turn income into capital, the dynamics behind income changes. Figures 9 and 10 show the contribution of each factor to the total earnings per bracket. By the third generation, 76% of the 80th - 89th percentile's income comes from resources - 35% from temptation payoffs and 41% from cooperation payoffs (figure 10). Intrinsic abilities promptly lost room in this bracket. The concentration of saving creates distance amongst individuals, a process in which temptation payoffs are both cause and consequence. Temptation payoffs generate higher yields for only one of the players; this higher income increases the savings. That is, temptation payoffs contribute to the concentration of resources. Moreover, the clustering of resources decreases the connections amongst individuals, which increases the possibilities of defections, including one-sided defections - temptation payoffs. In the fifteenth generation, 59% of the 80th - 89th bracket stems from temptation payoffs over resources.

Figure 9: Sources of Income - Upper Brackets (Scenario A)

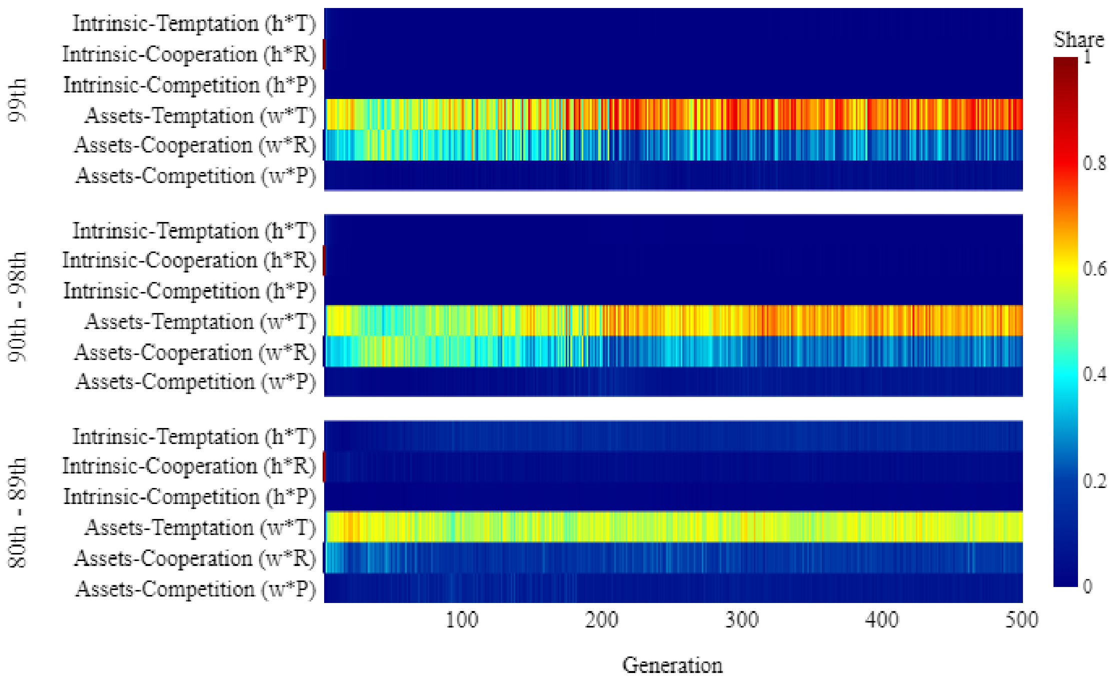
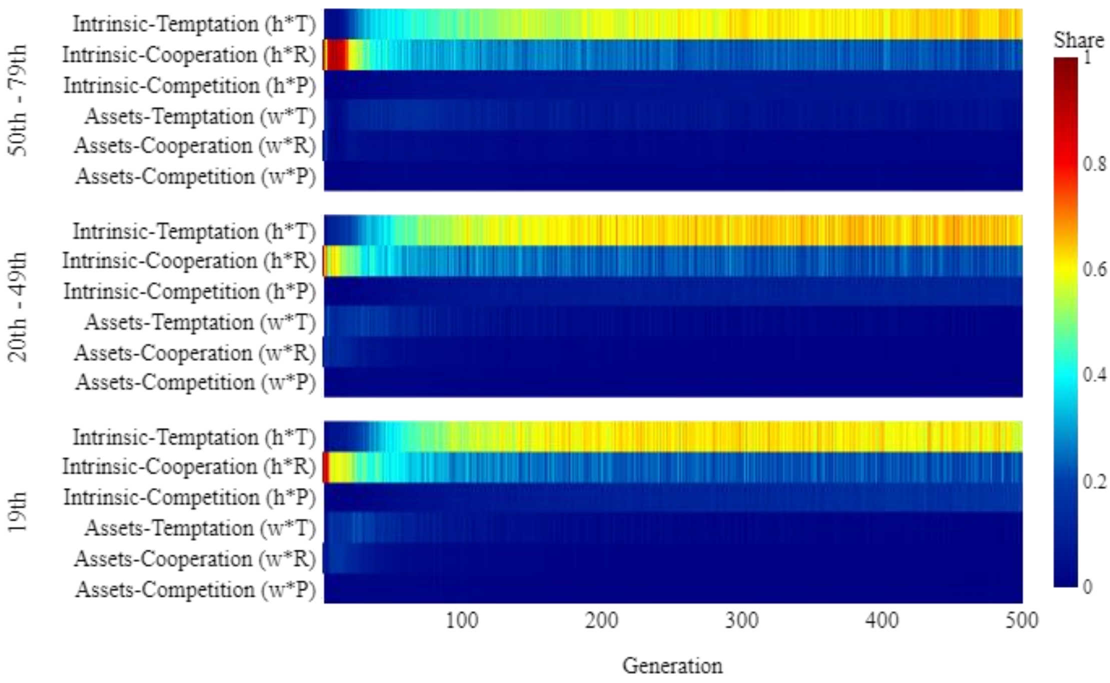


Figure 10: Sources of Income - Lower Brackets (Scenario A)



One-sided defections are not the full explanation for the rapid growth of inequality. The mobility amongst groups also plays a role—figures 11 and 12 display the origins of each bracket's members. Until the fifteenth generation, the 80th - 89th percentile receives members from upper and lower brackets, then, for the next twentytwo generations, gets only bottom brackets new-members. The individuals from top brackets come holding resources, and those from lower groups derive their incomes from their intrinsic abilities and the extent of their networks (figure 13). The former maintain, or increase, the group's average level of resources, with the same effect in income; the latter has much fewer resources than the bracket's average. Their savings from one period in this bracket diminish their network. The additional resources distance these individuals from impoverished members and are not enough to connect with the richer. Besides, the added resources do not generate enough income to keep them within the group. Thus, there is a constant turnover in the lower limit of the bracket that prevents its income from growing. At the same time, the wealthier members of the group keep amassing resources and increasing their income. These dynamics result in the initial fast growth of the Gini index.

Figure 11: Mobility - Upper Brackets (Scenario A)

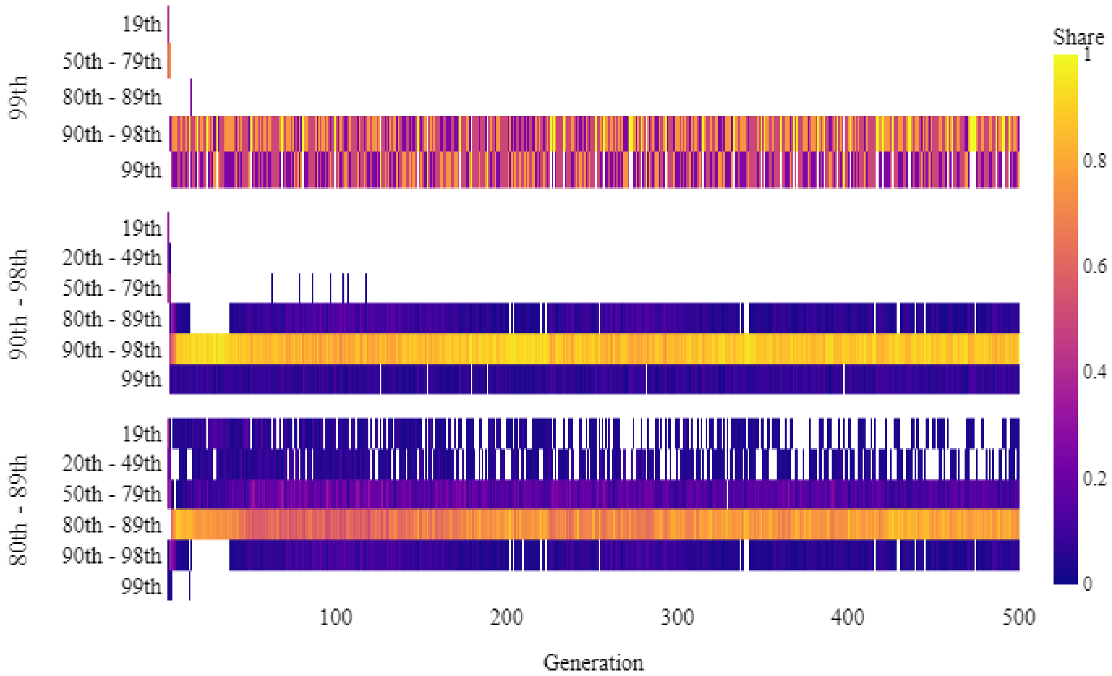
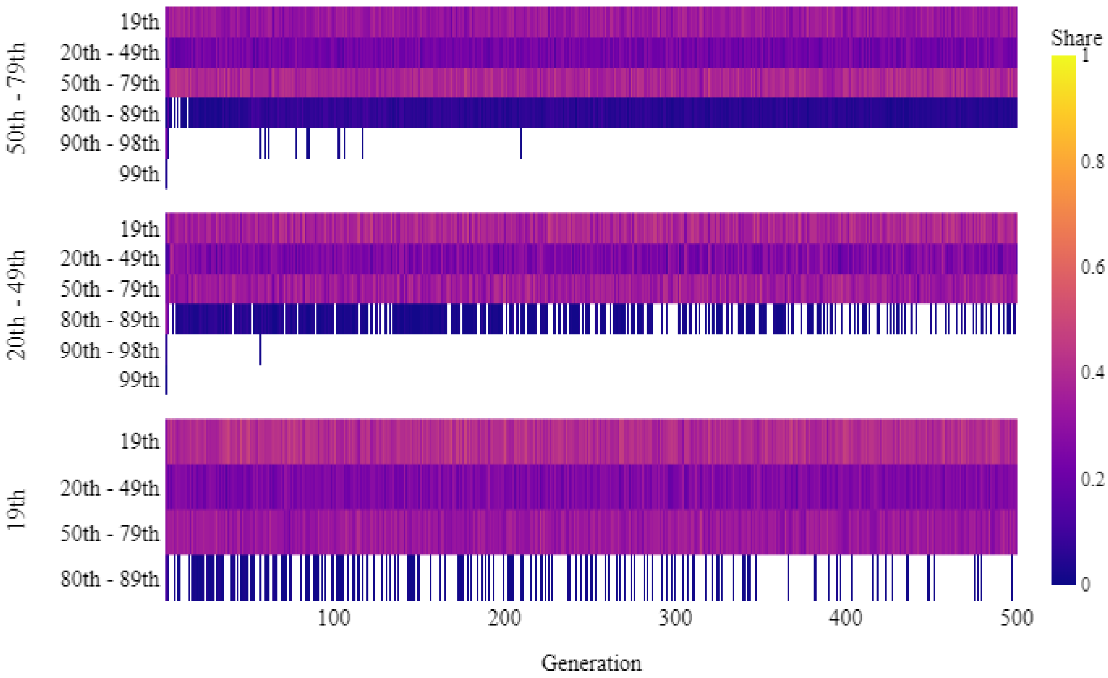


Figure 12: Mobility - Lower Brackets (Scenario A)



Even within brackets, the income variation is unequal due to differences in individuals' endowments, networks, and strategies. The savings pile up these differences in the form of assets. A concentration of income follows a concentration of resources, which, in turn, increases the aggregation of resources. Savings create a feedback mechanism in income distribution mechanics. With resources as the driver of homophily, the networks get restricted to those who possess similar amounts of resources, reducing the overall number of interactions an individual might have. Fewer interactions mean fewer opportunities for employing resources and intrinsic abilities into the production of income. In figures 14 and 15, we see the share of possible connections that come to fruition amongst brackets. For the 80th - 89th percentiles, the Gini index's rapid growth reflects the number of links within the group: by the sixteenth generation, no more than 60% of the possible connections happen. There is also a decline in the number of relationships with upper brackets from the seventeenth generation, which explains the halt in the Gini index increase. The affluent members of one group interact with individuals of top groups. Their networks migrate from connections with less prosperous members of their group to relationships with impoverished individuals of top brackets. When this movement is no possible - because they do not have enough resources - their network shrinks, leading to a decrease in income.

Figure 14: Sources of Income of New Members - Upper Brackets (Scenario A)

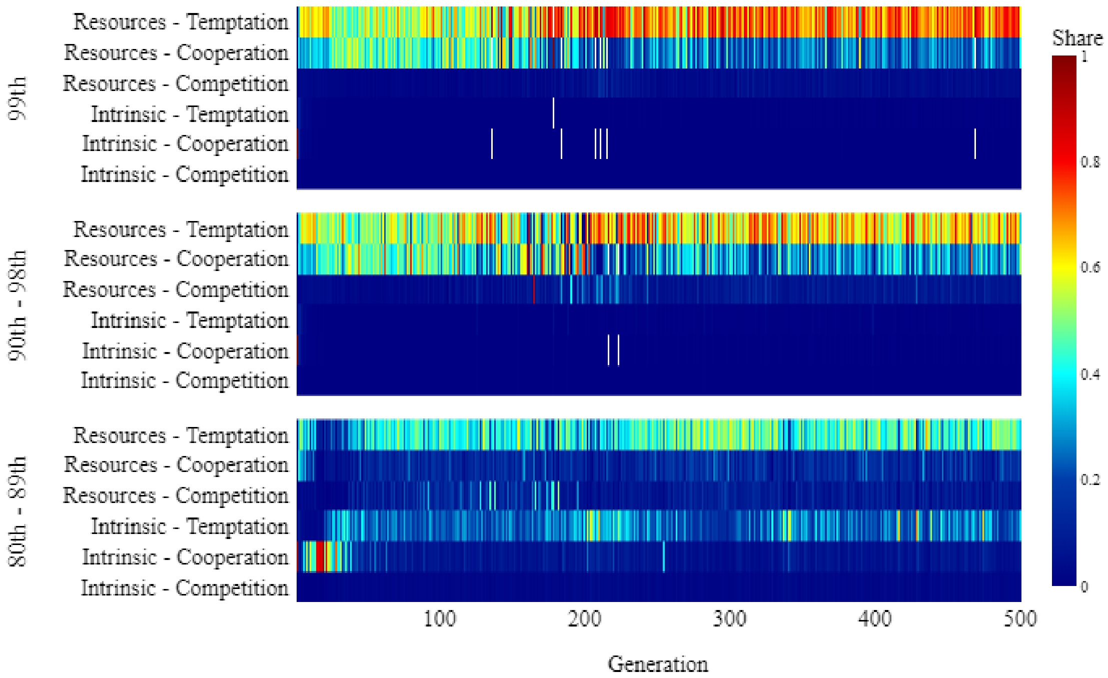
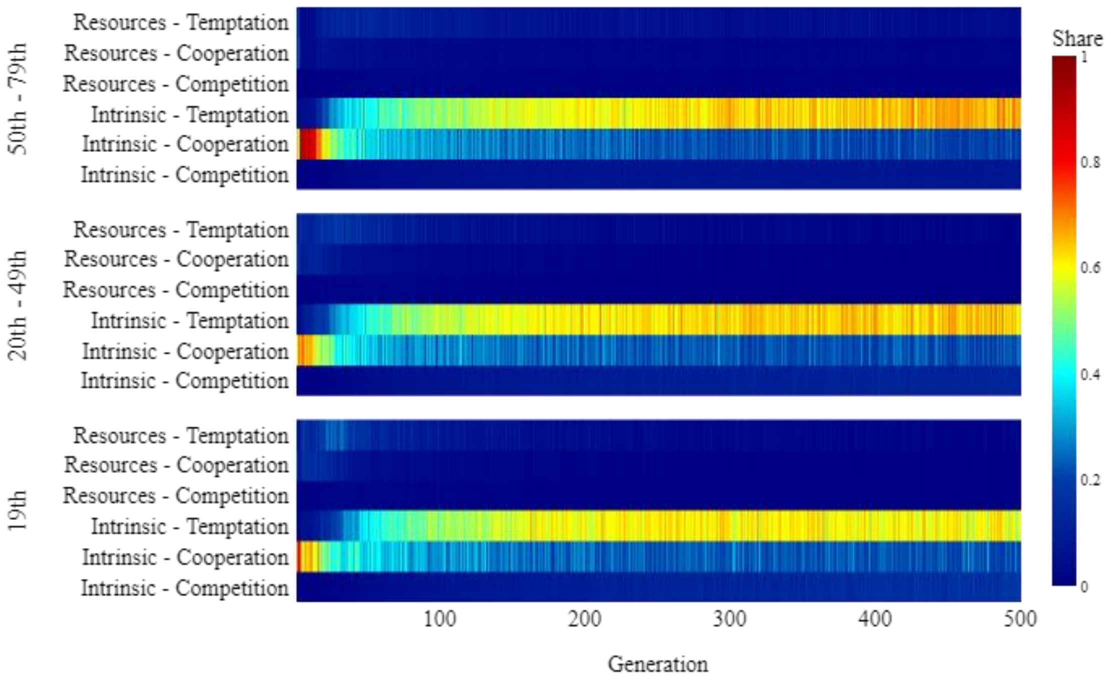


Figure 13: Sources of Income of New Members - Lower Brackets (Scenario A)



The concentration of savings creates more cloistered networks, to a point where the high amount of resources is insufficient to offset the low number of connections. At this stage, with diminishing income, the savings do not suffice to replenish the depreciation of capital. Hence, reverting the concentration process. The Gini index of the 90th 98th percentiles reflects these dynamics. In the first generations, the index grows slowly with high levels of cooperation - 63.9% of income comes from cooperation over resources in the forty-fourth generation. As the concentration of savings develops, the number of connections decreases, reaching 27% of possible links within the 114th generation's bracket. As the Gini index approaches its peak (0.7753), the interactions with the upper bracket also diminish. Thus, reducing the income of the most affluent individuals, which begets the withering of the Gini index. As the connections grow back, the process of concentration resumes, the Gini index reaches a new high of 0.5534 before oscillates around 0.4. The process is faster the second time. Since it starts from a higher level of inequality, the networks are already smaller. Therefore, there are fewer interactions available, which reduces the attainable levels of income and savings.

Figure 15: Possible vs Actual Connections - Upper Brackets (Scenario A)

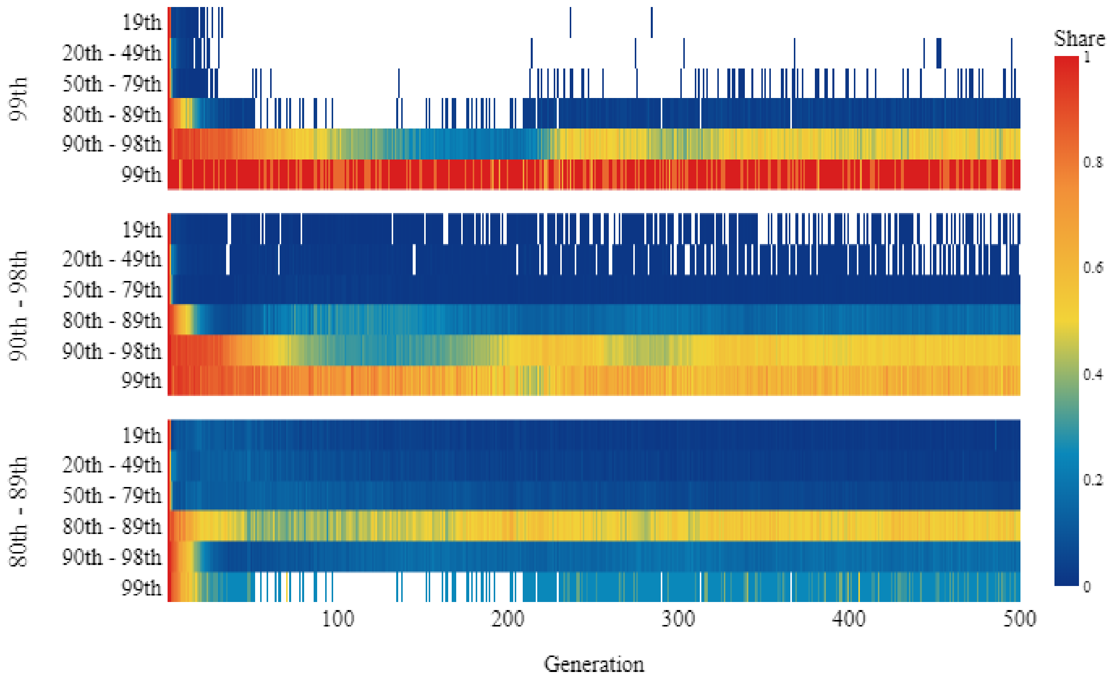


Figure 16: Possible vs Actual Connections - Lower Brackets (Scenario A)

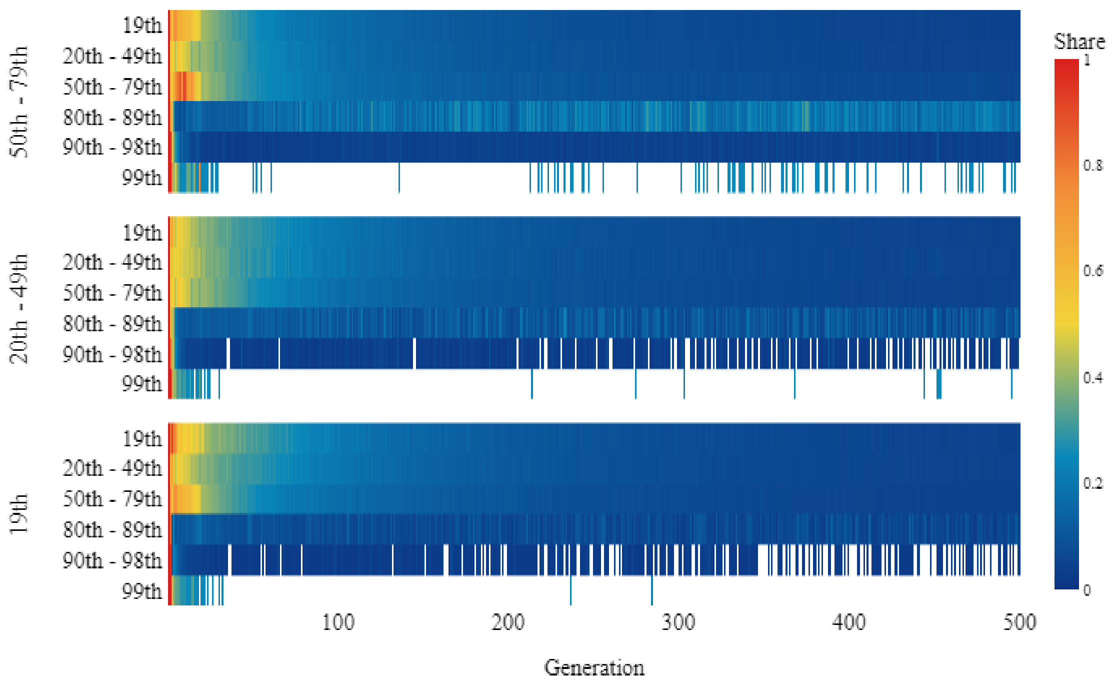


Figure 17: Average Income (Scenario A)

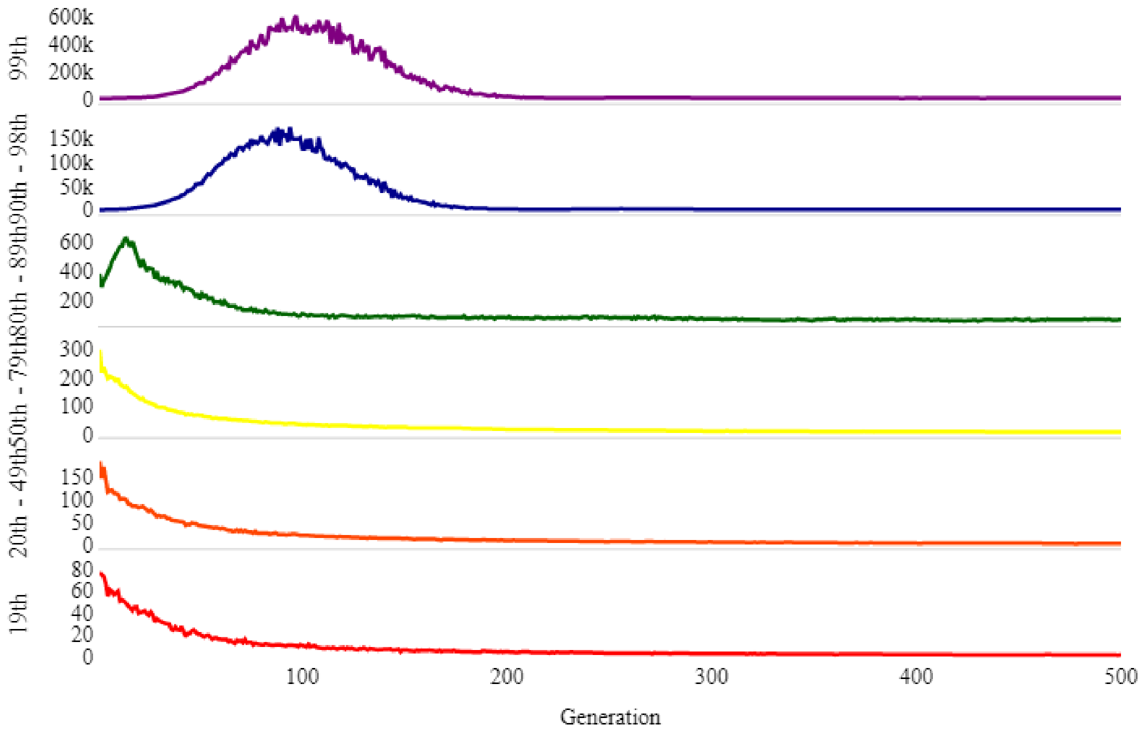
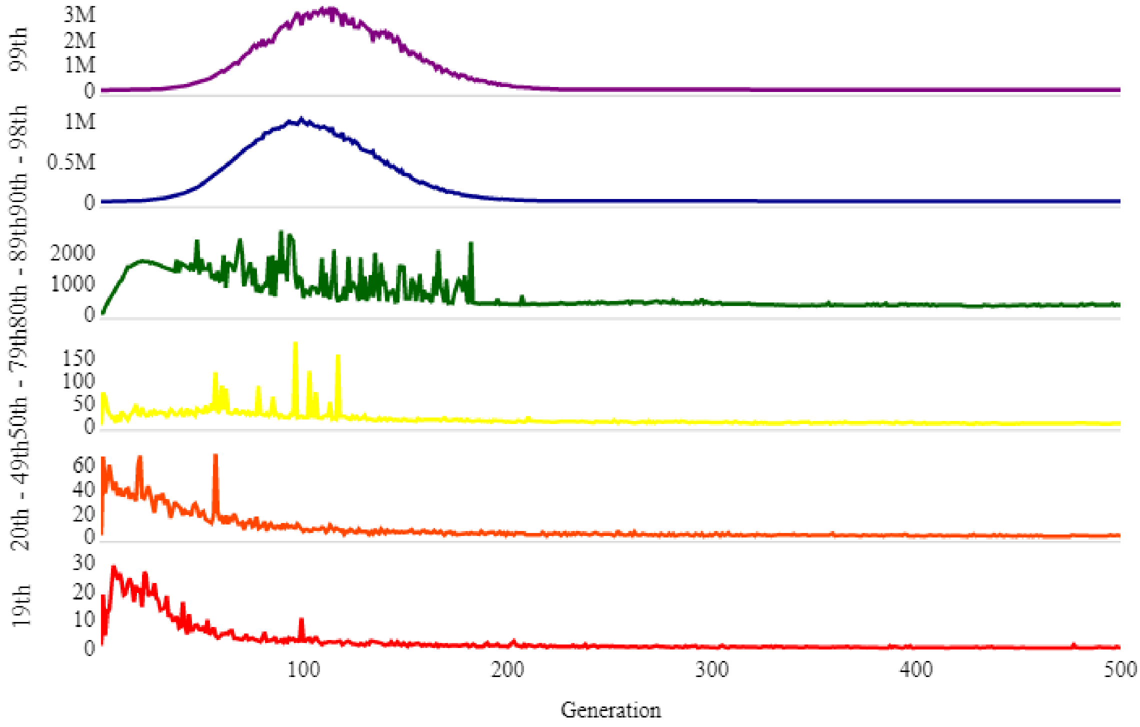


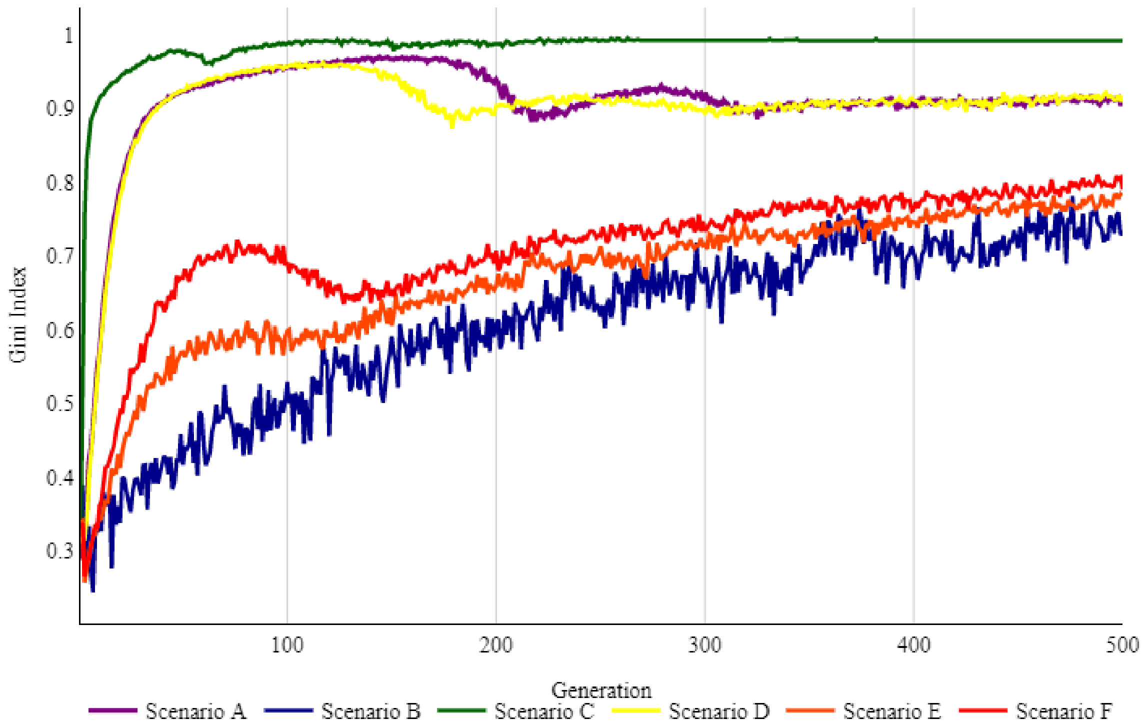
Figure 18: Average Wealth (Scenario A)



A sparse network results in lower average income for every bracket (figure 17). Starting from a dense network allows for the initial income growth, with a similar effect on the resources (figure 18). Differences in endowments lead to uneven income growth, and the concentration of saving builds up these differences. The combination of endowments and concentration of savings creates a feedback mechanism that, at first, speeds up the production of income, and, ultimately, reshapes the network in a way that restricts the level of income.

Figure 19 compares the Gini index of scenario A with the remaining scenarios. Such a comparison is a way of accessing how the parameters affect the overall state of the system. There is a tendency towards growing inequality in all versions of the model. However, the speed of concentration of income varies. Scenario B displays the slowest pace of growth in inequality. This account has one-quarter of the original population, which increases the impacts of chance events - as we can see in the frequency and amplitude of oscillations between iterations. The smaller network limits the attainable income, which reduces the accumulation of and transmission of assets. With fewer assets concentrated, the random intrinsic abilities play a bigger role in the production of earnings. A small network keeps the concentration of resources in check, leaving more room to chance and individuality. Still, the tendency to concentrate on assets and income remains. On the opposite side of the spectrum, scenario C - with payoffs tenfold over other versions - reaches a Gini index of 0.9 by the eighth generation and finds stability around 0.99.

Figure 19: Gini Index



Increasing the range of intrinsic abilities (scenario D) leads to a pattern inequality very similar to scenario A, except that the reduction in the index happens earlier. The reduction in the propensity to save (scenario D) increases the time it takes to amass enough resources to render intrinsic abilities irrelevant to attain higher levels of income. Thus, slowing the pace of the concentration of income. The smaller payoffs in scenario F cause a similar effect. In both cases, there is an initial reduction in the index, followed by rapid growth. This fast growth - as we have seen in scenario A - contracts the network. Wich diminishes the income at the top, reducing inequality. However, unlike scenario A, scenarios E and F do no reach stability. Instead, they undergo a slower and steady growth in the Gini index.

Figure 21: Averge Income

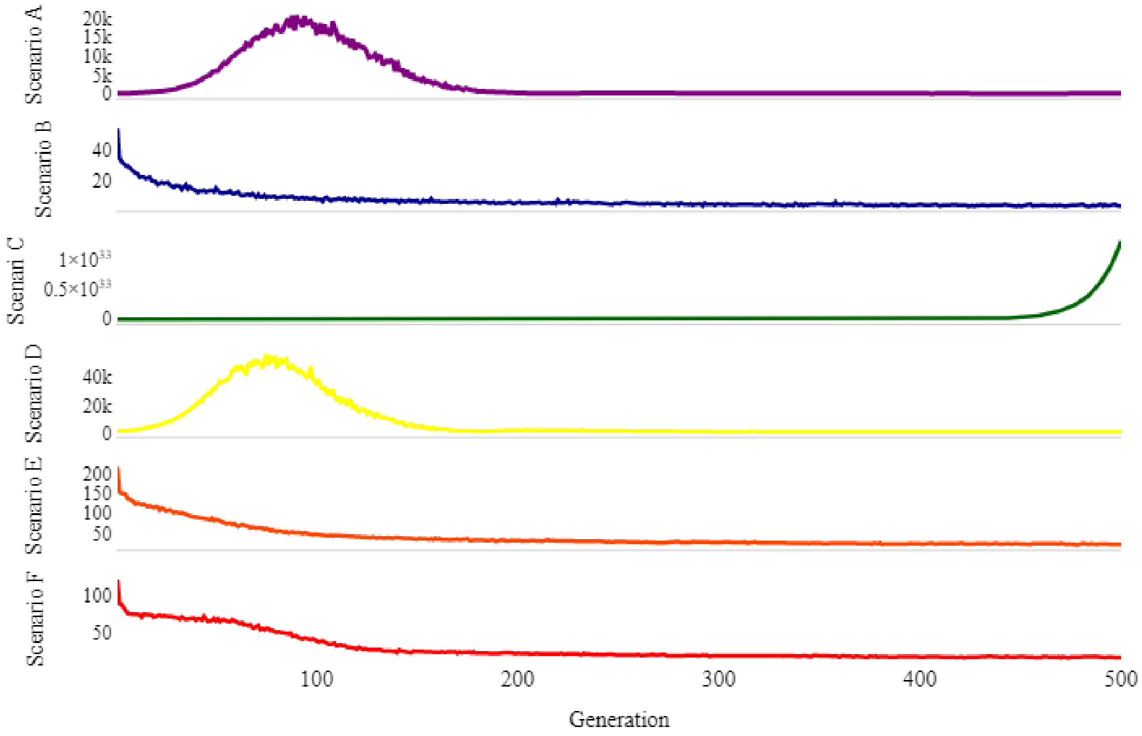
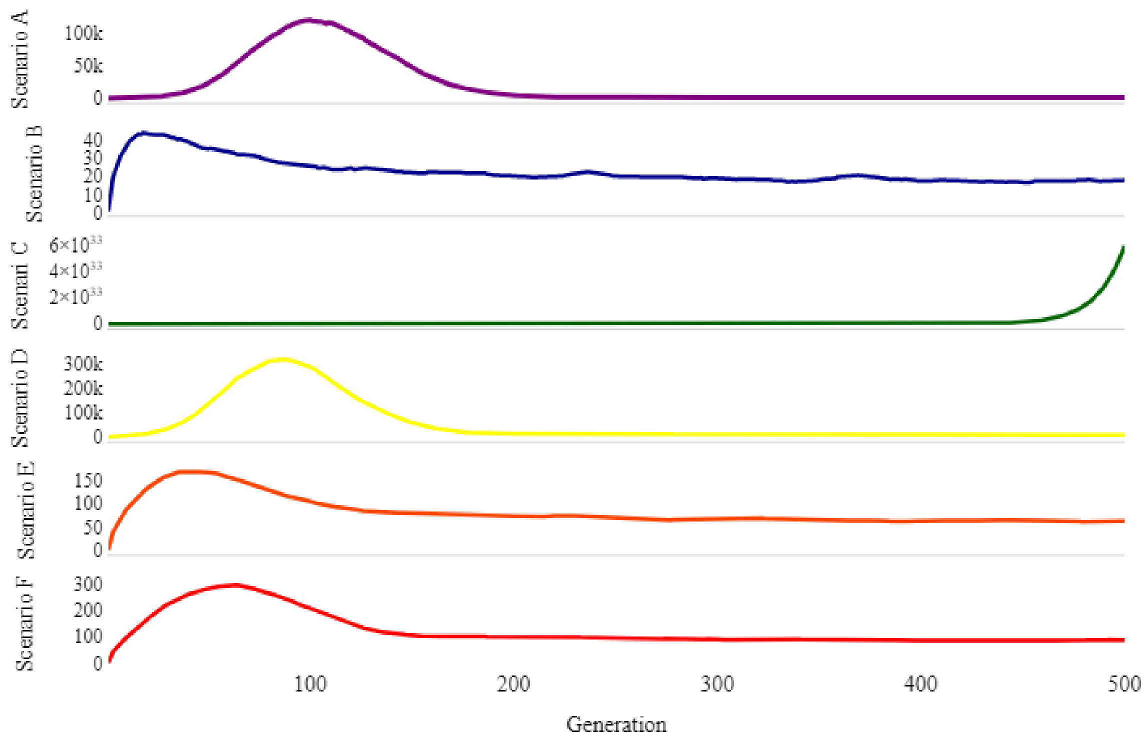


Figure 20: Average Wealth



In the present model, the values of income and wealth convey no meaning per se. They are useful only as a means of comparison. Figure 20 compares the average income across scenarios. Scenarios A and D follow similar patterns, but, having a larger range of abilities, scenario D attains twice as much average income. The size of the network constrains the income in scenario B. Figure 21 shows that there is not a sustained accumulation of assets capable of increasing income. Nevertheless, the aggregation of resources is enough to diminish the network. As a result, income reduces over time. Scenarios E and F went through similar processes driven by different factors. In scenario C, the magnitude of payoffs causes such rapid growth of income and assets, that the network's contraction does not affect the aggregation of resources.

5. Concluding Remarks

In an evolving system, the properties of the whole emerge from the characteristics of heterogeneous individuals. It rules out the need for homogeneity, complete information, perfect rationality, and other assumptions that might force the arrangement into equilibrium. Without assuming the existence of a single optimal stability point, the analysis gains scope. We can follow the setting through its different stages. That allows the consideration of the mechanics underlying the results, the paths that lead the system to a particular state.

As the proposed system develops, it shows that the opposing theories regarding the effect of inequality on growth reflect different evolution stages. At first, the average income grows with the concentration of resources - a product of an uneven initial distribution of income magnified by the concentration of savings. Those who hold resources capitalize on them, starting a feedback mechanism of creation and concentration through savings, income, and wealth. The feedback speeds the process creating many isolated groups within society, separated by wealth.

The isolation advances along with the aggregation of wealth. For a while, the accumulation of resources compensates for the smaller network - the contracted economic activity caused by isolation. In the absence of a designed mechanism to keep the concentration in check, one emerges. The separation reaches a point where the high amount of resources is underused due to the low number of interactions. With fewer interactions, the capital does not produce as much income as it could. Less income leads to fewer savings. The savings become insufficient to replenish the depreciation of resources or upgrade the technology — a shock on capital that also reshapes the network.

After the shock, the process restarts happening faster each time. Since the destruction of capital is not abrupt nor complete, the network does not build back its original form - with all possible connections. With fewer connections, neither income nor wealth restores its previously high levels. There are continuous decreases in both through the cycles. That is, in later stages, high levels of inequality bring about lower levels of income.

The aftereffect of high inequality affects those who possess wealth. It is a reduction in inequality stemmed from reductions in the upper-brackets' income. It does not entail increments in impoverished-groups' income. Those in the bottom derive their incomes from their intrinsic abilities and the extent of their networks. Without saving, they are unable to engage in the feedback mechanism responsible for income growth over time. Lower bracket experience decreases in income as the network becomes more sparse, with no underlying structure in the system available to revert the situation.

Early, the savings set the separation between upper e lower brackets. Those who have high intrinsic abilities at the start of the setting make use of the dense network to obtain high earnings. Their descendants receive part os these earnings in the form of wealth, which also provides income. Therefore, the primary advantages compound over time. Within a few generations, the effect of inheritance surpasses intrinsic abilities to the production of income.

Later, the savings drive the networks' shapes, which operate as barriers for mobility amongst brackets. Those who have high intrinsic abilities, but are part of a scattered network, are unable to employ the full potential of their capabilities. Thus, obtaining low incomes when compared to those positioned in denser networks. That is, inherent characteristics are determining to the extent that they occur in the initial stages of the system in individuals with favorable positions within the network. In advanced phases of evolution, inherited resources are decisive factors, not only because they can be transmitted but also because they compound. Therefore, a disadvantaged group cannot reduce the income gap by their efforts since they do not have access to the central piece of the dynamics that brought the gap about: savings.

This work is an attempt to combine tools from different theories to understand a pervasive issue better. Although it fulfills its role as an initial venture in this particular approach, it falls short in essential aspects. Limited by computational capabilities, we worked with small numbers of agents, which may exacerbate the impact of chance events. Chance events are an explicit part of the model. Nonetheless, it necessary to test its behavior on a larger scale, which dilutes chance events. On the same account, improved computational capabilities would allow a more significant number of simulations, increasing the confidence in the results.

It is also possible to amplify the model's scope, including devices such as taxation, government, and education systems. We can test the effects of direct shocks on capital, sudden changes in the networks, increments in human and social capital, amongst other relevant economic factors. The goal is to create an ample and flexible tool to improve our grasp on a fundamental problem. There is still a long way to go.

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